AM-FEM: Goal-oriented combined model and discretization error estimates in nonlinear and linear structural mechanics

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Abstract

In our previous work[1,2], residual-based goal-oriented combined model and discretization error estimators were derived that are based on improved finite element solutions, using improved interface boundary tractions analyzed with Neumann problems on the element level. In particular, these estimators also controll the error obtained while expanding the model from 1D- to 2D- and from 2D- to 3D-elastic theories.

Different from local model adaptivity — e.g. by Oden et al., for analyzing smaller scales of RVE's with microheterogeneous materials, or by Larsson et al., for detecting evolving local plastic deformations within an elastic system — structural model adaptivity is characterized by the fact that the domains and the dimensions and thus the test and solution spaces of the models considered are changing during the deformation process and thus are different, e.g. in the case of 3D-boundary layers.

This has the important consequence for model error estimates that consistent prolongations of discrete kinematic and static nodal quantities of a simpler model m to those of a hierarchically refined model $m + 1$ are necessary. The approximate solution of model m – e.g. a 2D-shell theory – can thus be transferred into the solution space of model $m + 1$ – e.g. a 3D-continuum theory –, such that reasonable model error norms can be defined by differences of solutions in the same function space.

Furthermore, duality techniques are used for the goal-oriented discretization and model error estimates, using — system and load depending — sensitive quantities of interest. However, one has to take into account that the Ritz-Galerkin orthogonality does not hold for the model error. Due to the properties of the error estimate of the considered quantities of interest it is not necessary to prolongate the finite element solutions and their improvements into the function space of a refined model[3].

Special attention is paid to the prolongation operators in the case of model error estimates. An example demonstrates the expected effects, realizing verification and even validation in structural mechanics, i.e. approaching a new paradigm in Computational Mechanics[1,2,3].

References

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