Control of Bending Vibrations within Subdomains of Thin Plates by Piezoelectric Actuation

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DIRECT and INVERSE PROBLEMS in PIEZOELECTRICITY
Miniworkshop

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INTRODUCTION

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Krommer, M., Varadan, V.V.: **Control of Bending Vibrations within Sub – Domains of Thin plates; Part I: Theory and Exact Solution**. Journal of Applied Mechanics **72**(3), 2005, pp. 432-444.

PART I

Krommer, M.: **The Significance of Non-Local Constitutive Relations for Composite Thin Plates Including Piezoelastic Layers with Prescribed Electric Charge**. Smart Materials and Structures **12**, 2003, pp. 318-330.

PART II

Krommer, M., Varadan, V.V.: **Control of Bending Vibrations within Sub – Domains of Thin plates; Part II: Piezoelectric Actuation and Approximate Solution**. Scheduled for publication January 2006: Journal of Applied Mechanics.

PART III

PART I:
Bending Vibrations of Thin Plates

$$A: \quad div(div\mathbf{M}) + p_z = P\ddot{w}_0$$

$$C: \quad \left[div\mathbf{M}\cdot\mathbf{n} + \nabla(\mathbf{Mn}\cdot\mathbf{s})\cdot\mathbf{s} - \bar{q}\right]\delta w_0 = 0\,, \quad (\mathbf{Mn}\cdot\mathbf{n} - \bar{m})(\nabla\delta w_0\cdot\mathbf{n}) = 0$$

$$P: \quad (\mathbf{Mn}\cdot\mathbf{s})\delta w_0\Big]_{P^+}^{P^-} = 0$$

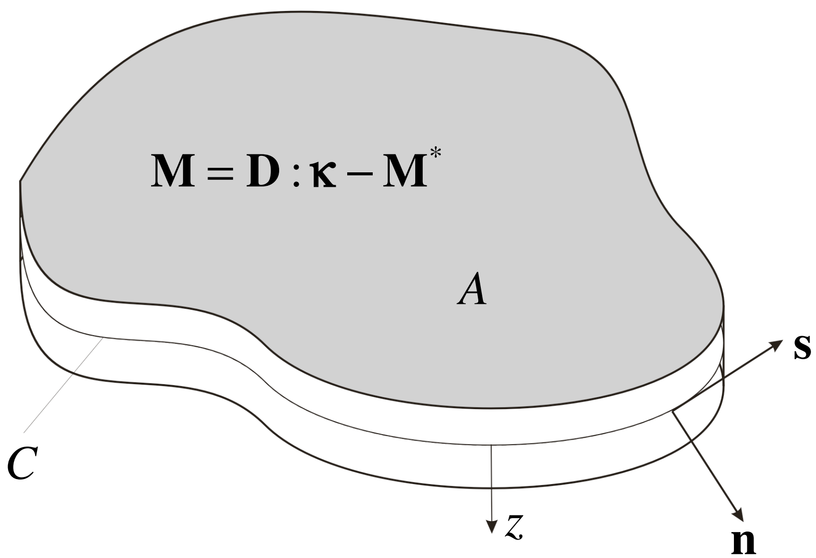
$\mathbf{M}$ ... bending moment tensor

$\mathbf{M}^*$ ... self moment tensor

$\boldsymbol{\kappa} - \frac{1}{2}\left[\nabla\nabla w_0 + (\nabla\nabla w_0)^T\right]$ ... curvature tensor

w_0 ... deflection

p_z ... transversee force loading



$$\int_A \hat{p}_z^d \hat{w}_0 dA - \int_{C_\psi} \left(\hat{\mathbf{M}}^d \mathbf{n} \cdot \mathbf{n}\right) \hat{\bar{\psi}} dC + \int_{C_w} \left[div \hat{\mathbf{M}}^d \cdot \mathbf{n} + \nabla \left(\hat{\mathbf{M}}^d \mathbf{n} \cdot \mathbf{s}\right) \cdot \mathbf{s} \right] \hat{\bar{w}}_0 dC =$$

$$= \int_A \hat{\bar{p}}_z \hat{w}_0^d dA + \int_A \hat{\mathbf{M}}^* : \hat{\boldsymbol{\kappa}}^d dA - \int_{C_m} \hat{\bar{m}} \left(\nabla \hat{w}_0^d \cdot \mathbf{n}\right) dC + \int_{C_q} \hat{\bar{q}} \hat{w}_0^d dC + \sum_i \left[\hat{\mathbf{M}}^d \mathbf{n} \cdot \mathbf{s} \hat{w}_0 \right]_{P_{wi}^+}^{P_{wi}^-}$$

- $\hat{\bar{p}}_z = \hat{p}_z + P\left(s w_0 (t=0) + \dot{w}_0 (t=0)\right)$ ... accounts for initial conditions

- "dummy" loading case: $\begin{cases} \text{homogenous initial conditions} \\ \text{homogenous kinematical and dynamical bc's} \\ \text{no self - moment applied: } \mathbf{M}^{*d} = \mathbf{0} \end{cases}$

$$\int_A \hat{p}_z^d \hat{w}_0 dA - \int_{C_\psi} \left(\hat{\mathbf{M}}^d \mathbf{n}\cdot\mathbf{n}\right)\hat{\bar{\psi}} dC + \int_{C_w}\left[div\hat{\mathbf{M}}^d\cdot\mathbf{n} + \nabla\left(\hat{\mathbf{M}}^d\mathbf{n}\cdot\mathbf{s}\right)\cdot\mathbf{s}\right]\hat{\bar{w}}_0 dC =$$

$$= \int_A \hat{\bar{p}}_z \hat{w}_0^d dA + \int_A \hat{\mathbf{M}}^* : \hat{\boldsymbol{\kappa}}^d dA - \int_{C_m} \hat{\bar{m}}\left(\nabla\hat{w}_0^d\cdot\mathbf{n}\right) dC + \int_{C_q}\hat{\bar{q}}\hat{w}_0^d dC + \sum_i \left[\hat{\mathbf{M}}^d\mathbf{n}\cdot\mathbf{s}\hat{w}_0\right]_{P_{wi}^+}^{P_{wi}^-}$$

- kinematical bc's ... $\begin{cases} C = C_w \cup C_q\,, \quad C = C_\psi \cup C_m \\ C_w : w_0 = \bar{w}_0\,, \quad C_\psi : \nabla w_0\cdot\mathbf{n} = \bar{\psi} \end{cases}$

- C_w and C_ψ such that, if $\bar{w}_0 = 0$ and $\bar{\psi} = 0$, then no rigid body motion can occur!

- Decomposition of deflection, $w_0 = \breve{w}_0 + \tilde{w}_0$, with: $\begin{cases} C_w : \breve{w}_0 = \bar{w}_0 \\ C_\psi : \nabla\breve{w}_0\cdot\mathbf{n} = \bar{\psi} \end{cases}$

$$\int_A \hat{p}_z^d \hat{\tilde{w}}_0 dA = \int_A \hat{\tilde{p}}_z \hat{w}_0^d dA + \int_A \hat{\mathbf{M}}^* : \hat{\boldsymbol{\kappa}}^d dA - \int_{C_m} \hat{\tilde{m}} \left(\nabla \hat{w}_0^d \cdot \mathbf{n}\right) dC + \int_{C_q} \hat{\tilde{q}} \hat{w}_0^d dC$$

- $A: \quad \hat{\tilde{p}}_z = \hat{p}_z + P\left(s w_0(t=0) + \dot{w}_0(t=0)\right) - P s^2 \hat{\breve{w}}_0 + div(div\hat{\breve{\mathbf{M}}})$

- $\begin{cases} C_m: \quad \hat{\tilde{m}} = \hat{\bar{m}} - \hat{\breve{\mathbf{M}}}\mathbf{n}\cdot\mathbf{n} \\ C_q: \quad \hat{\tilde{q}} = \hat{\bar{q}} - div\hat{\breve{\mathbf{M}}}\cdot\mathbf{n} - \nabla\left(\hat{\breve{\mathbf{M}}}\mathbf{n}\cdot\mathbf{s}\right)\cdot\mathbf{s} \end{cases}$ with $\breve{\mathbf{M}} = \mathbf{D} : \breve{\boldsymbol{\kappa}}$, $\quad \breve{\boldsymbol{\kappa}} = -\frac{1}{2}\left[\nabla\nabla\breve{w}_0 + \left(\nabla\nabla\breve{w}_0\right)^T\right]$

- statically admissible moment tensor $\mathbf{M}^p$: $\begin{cases} A: \quad div(div\mathbf{M}^p) + \tilde{p}_z = 0 \\ C_q: \quad div\mathbf{M}^p\cdot\mathbf{n} + \nabla\left(\mathbf{M}^p\mathbf{n}\cdot\mathbf{s}\right)\cdot\mathbf{s} = \tilde{q} \\ C_m: \quad \mathbf{M}^p\mathbf{n}\cdot\mathbf{n} = \tilde{m},\ \bar{P}_i: \quad \left[\mathbf{M}^p\mathbf{n}\cdot\mathbf{s}\right]_{\bar{P}_i^+}^{\bar{P}_i^-} = 0 \end{cases}$

$$\int_0^t \int_A p_z^d(\mathbf{x},\tau)\tilde{w}_0(\mathbf{x},t-\tau)\,dA\,d\tau = \int_0^t \int_A \left[\mathbf{M}^p(\mathbf{x},\tau)+\mathbf{M}^*(\mathbf{x},t-\tau)\right]:\boldsymbol{\kappa}^d\,dA\,d\tau$$

Due to the arbitriness of p_z^d the deflection $\tilde{w}_0 = w_0 - \breve{w}_0$ vanishes identically, if $\mathbf{M}^p + \mathbf{M}^* = \mathbf{0}$. Therefore, applying control by means of a self moment can be used to elliminate the bending motion $\tilde{w}_0$. The motion $\breve{w}_0$, which we account to the non homogenous kinematical bc's is not elliminated.

Hence, $w_0 = \breve{w}_0$.

- statically admissible moment tensor $\mathbf{M}^p = m^p\mathbf{I}$: $\begin{cases} A: & \Delta m^p + \tilde{p}_z = 0 \\ C_q: & \nabla m^p \cdot \mathbf{n} = \tilde{q} \\ C_m: & m^p = \tilde{m} \end{cases}$

$\rightarrow$ solution for $\mathbf{M}^p$ exists, but may be non unique!

PART I: Subdomain control

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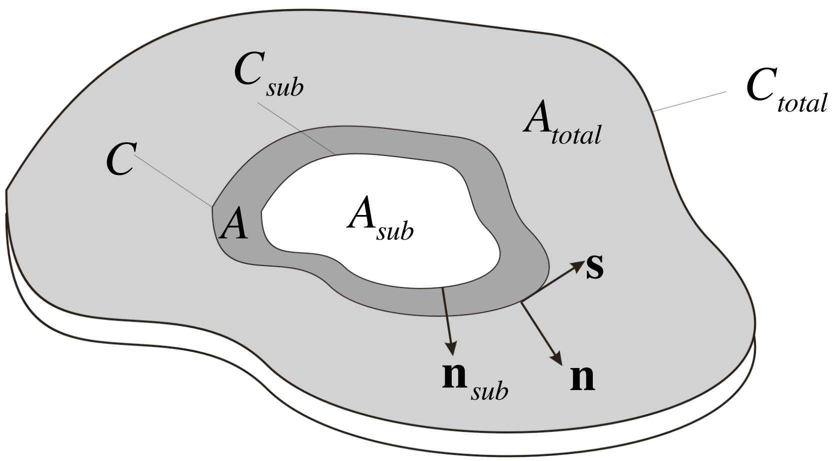


- Boundary conditions for free body A:

$$C: \begin{cases} w_0 = \bar{w}_0\,, \quad \mathbf{Mn}\cdot\mathbf{n} = \bar{m} \\ div\mathbf{M}\cdot\mathbf{n} + \nabla(\mathbf{Mn}\cdot\mathbf{s})\cdot\mathbf{s} = \bar{q}\,, \quad \nabla w_0\cdot\mathbf{n} = \bar{\psi} \\ w_0 = \bar{w}_0\,, \quad \nabla w_0\cdot\mathbf{n} = \bar{\psi} \end{cases}$$

- Desired deflection $\breve{w}_0$:

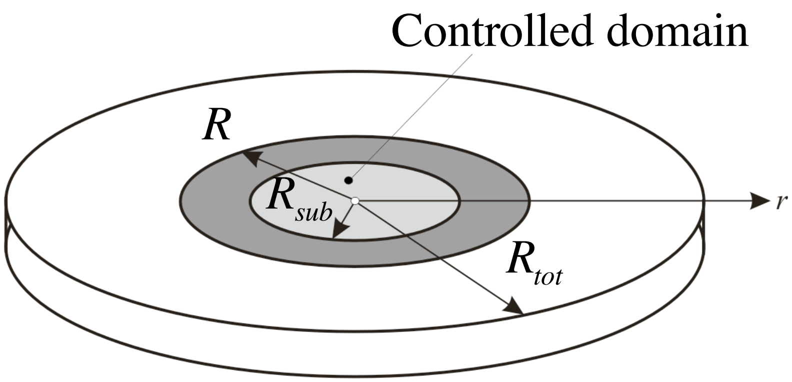
$$A_{sub}: \quad \breve{w}_0 = 0\,, \quad C_{sub}: \quad \breve{w}_0 = 0\,, \quad \nabla\breve{w}_0\cdot\mathbf{n}_{sub} = 0$$

$$C: \begin{cases} \breve{w}_0 = \bar{w}_0 \\ \nabla\breve{w}_0\cdot\mathbf{n} = \bar{\psi} \\ \breve{w}_0 = \bar{w}_0\,, \quad \nabla\breve{w}_0\cdot\mathbf{n} = \bar{\psi} \end{cases}$$

- Statically admissible moment tensor:

$$A: \quad \Delta m^p + \tilde{p}_z = 0$$

$$C: \begin{cases} m^p = \tilde{m} \\ \nabla m^p\cdot\mathbf{n} = \tilde{q} \\ \textit{no boundary conditions to be satisfied} \end{cases}$$



- Clamped circular plate:

$$p_z(r,\varphi,t) = p_0(t)$$

- Boundary conditions for free subdomain $r \leq R$:

$$w_0 = \bar{w}_0 \,, \quad \partial w_0 / \partial r = \bar{\psi}$$

- Desired deflection $\breve{w}_0$:

$$r \leq R_{sub} : \quad \breve{w}_0 = 0$$

$$R_{sub} \leq r \leq R : \quad \breve{w}_0 = C_1 + C_2 r^2 + C_3 r^4 + C_4 r^6 + C_5 r^8 + C_6 r^{10}$$

$$r = R_{sub} : \quad \breve{w}_0 = 0 \,, \quad \frac{\partial \breve{w}_0}{\partial r} = 0 \quad \text{and} \quad r = R : \quad \breve{w}_0 = \bar{w}_0 \,, \quad \frac{\partial \breve{w}_0}{\partial r} = \bar{\psi}$$

$\breve{w}_0$ is adjusted to the four conditions; hence four unknowns remain!

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- Statically admissible moment:

$$r \le R: \quad \frac{1}{r}\frac{\partial m^p}{\partial r} + \frac{\partial^2 m^p}{\partial r^2} + \tilde{p}_0 = 0$$

$$r \le R_{sub}: \quad \tilde{p}_0 = p_0 \,, \quad R_{sub} \le r \le R: \quad \tilde{p}_0 = p_0 - P\ddot{\breve{w}}_0 - D\Delta\Delta\breve{w}_0$$

No bc for m^p; hence, one more unknown!

If $\mathbf{M}^* = -\mathbf{M}^p = -m^p\mathbf{I}$ then the motion for $r \le R_{sub}$ vanishes;
for $R_{sub} \le r \le R$ the motion is $\breve{w}_0$, as it has been specified in the series.

Five unknowns present in the solution!

$\bar{w}_0, \bar{\psi}$, m^p is not unique , 2 constants from power series

- 2 from to be admissible
- 2 from post calculation: $\bar{w}_0, \bar{\psi}$ are deflection and slope of controlled plate at $r = R$
- 1 remains as additional degree of freedom

PART I:
Numerical results

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- Parameters:

$$D = 162.45\,Nm\,,\quad P = 8.1\,kgm^{-2}\,,\quad p_0(t) = P_0 e^{i\omega t}\,,\quad P_0 = 100\,Nm^{-2}$$

- Geometry: $R_{tot} = 0.2m\,,\quad R = 0.04m\,,\quad R_{sub} = 0.03m$

- Natural frequencies: $f_1 = 182.0\,Hz\,,\quad f_2 = 708.7\,Hz\,,\quad f_3 = 1588\,Hz$

$$w_0(r,t) = W(r)e^{i\omega t}\,,\quad \breve{w}_0(r,t) = \breve{W}_0(r)e^{i\omega t}$$

$$D\Delta\Delta W(r) - P\omega^2 W(r) = \begin{Bmatrix} 0 \le r \le R_{sub} : \quad = 0 \\ R_{sub} \le r \le R : \quad = -P\omega^2 \breve{W}_0 + D\Delta\Delta\breve{W}_0(r) \\ R \le r \le R_{tot} : \quad = P_0 \end{Bmatrix},\ r = R_{tot} : \quad W = 0\,,\ \frac{\partial W}{\partial r} = 0$$

$$W(r = R)e^{i\omega t} = \bar{w}_0 = \bar{W}_0 e^{i\omega t}\,,\quad \frac{\partial W}{\partial r}(r = R)e^{i\omega t} = \bar{\psi} = \bar{\Psi} e^{i\omega t}$$

PART I: Numerical results

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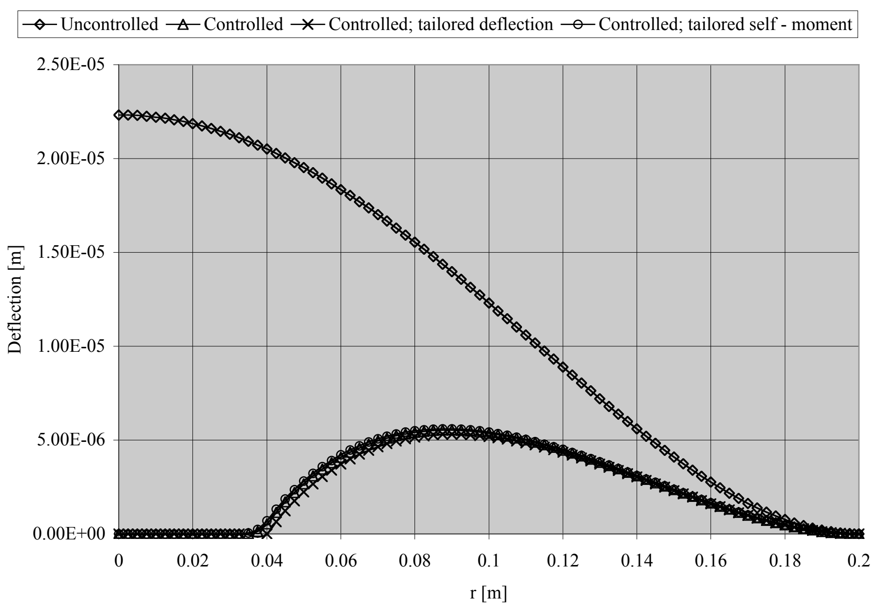

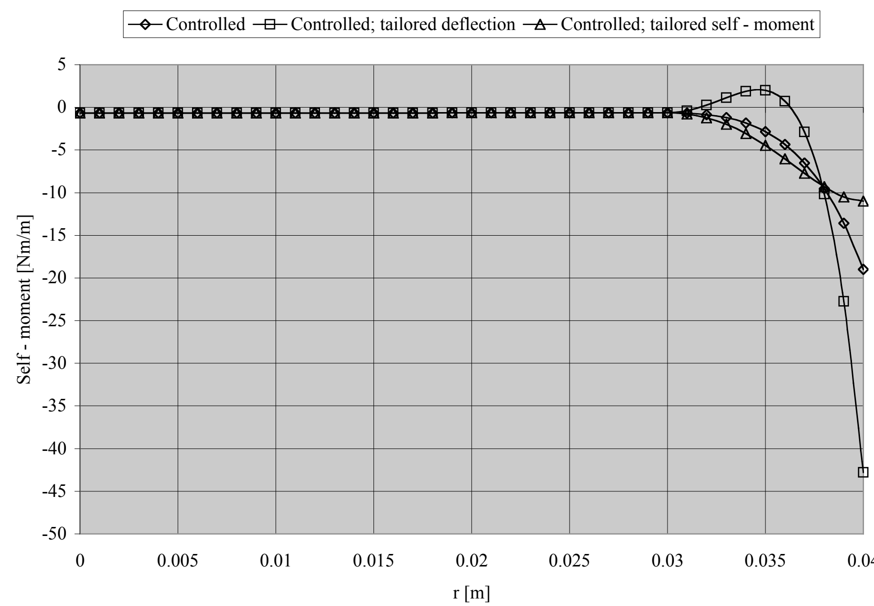

Deflection and self-moment for clamped plate: $\omega = 2\pi 100 s^{-1}$

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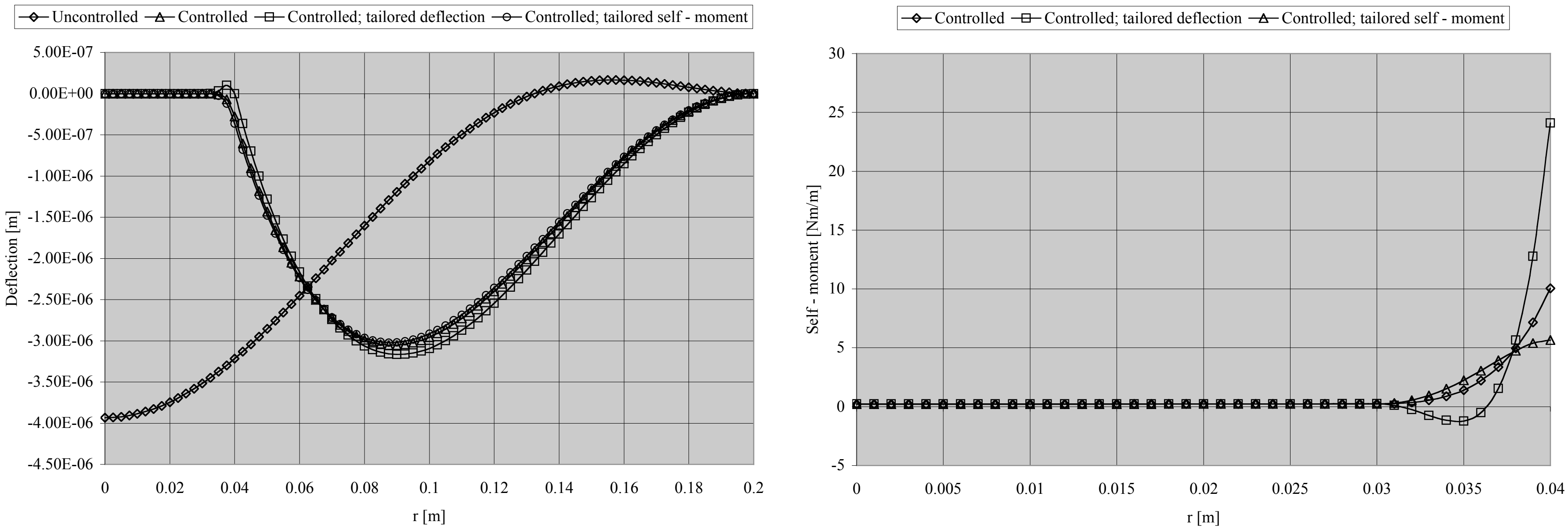

Deflection and self-moment for clamped plate: $\omega = 2\pi 500 s^{-1}$

PART I:
Numerical results

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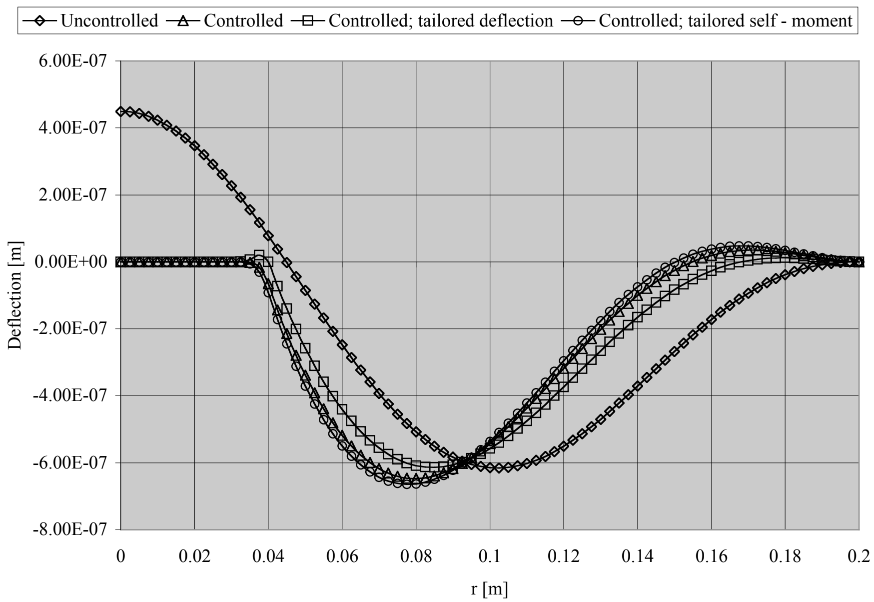

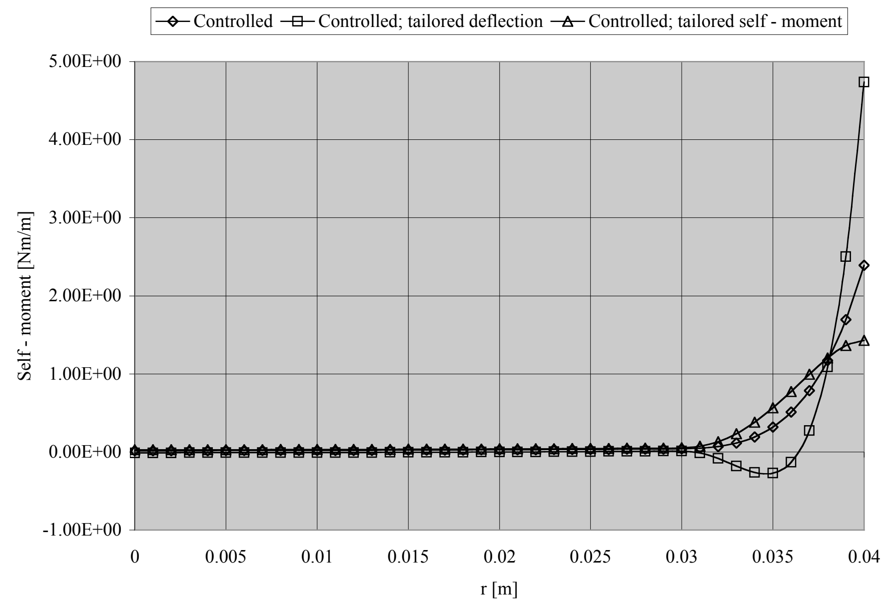

Deflection and self-moment for clamped plate: $\omega = 2\pi 1000 s^{-1}$

PART I:
Numerical results; control of alternative subdomains

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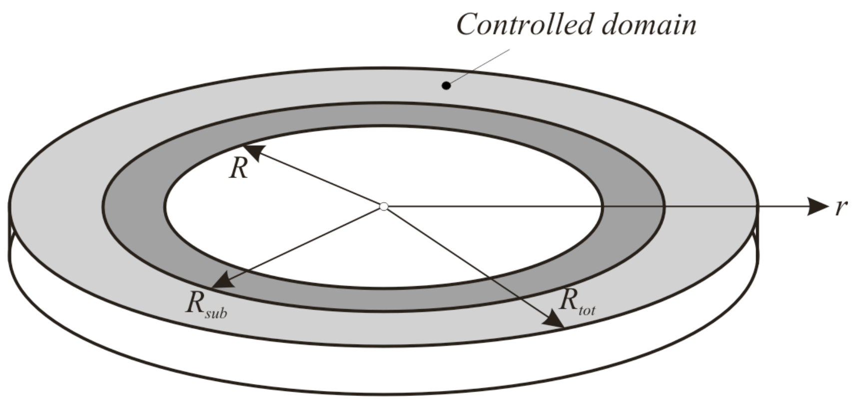

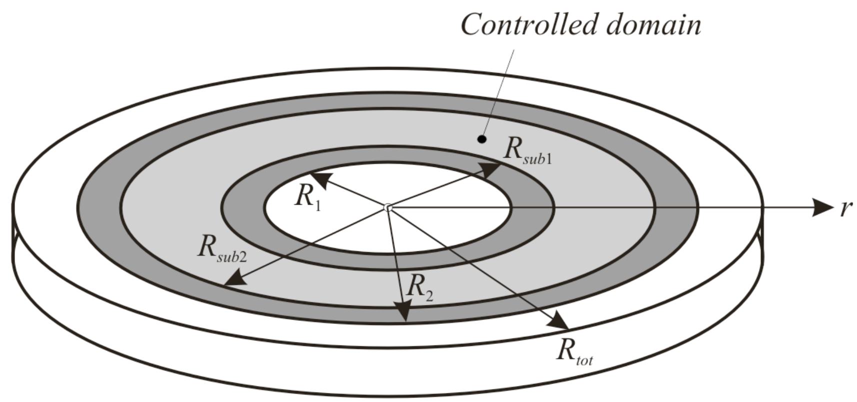

$$R_{tot} = 0.2m\,, \quad R = 0.16m\,, \quad R_{sub} = 0.17m$$

$$R_1 = 0.075m\,, \quad R_{sub1} = 0.085m$$
$$R_{sub2} = 0.115m\,, \quad R_2 = 0.125m$$

PART I:
Numerical results; control of alternative subdomains

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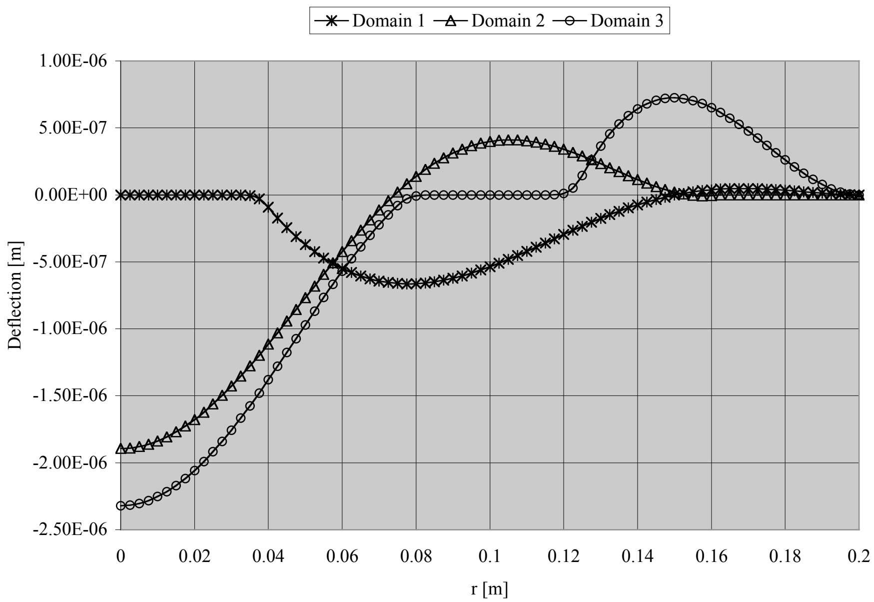

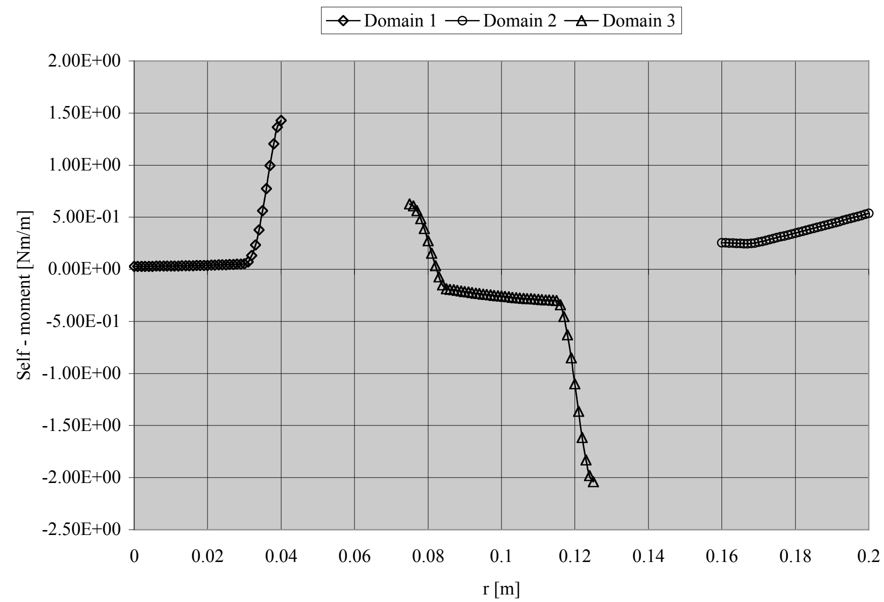

Deflection and self-moment for clamped plate: $\omega = 2\pi 1000 s^{-1}$

PART II:
Governing equations; mechanical

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$$A: \quad div(div\mathbf{M}) + p_z = P\ddot{w}_0$$

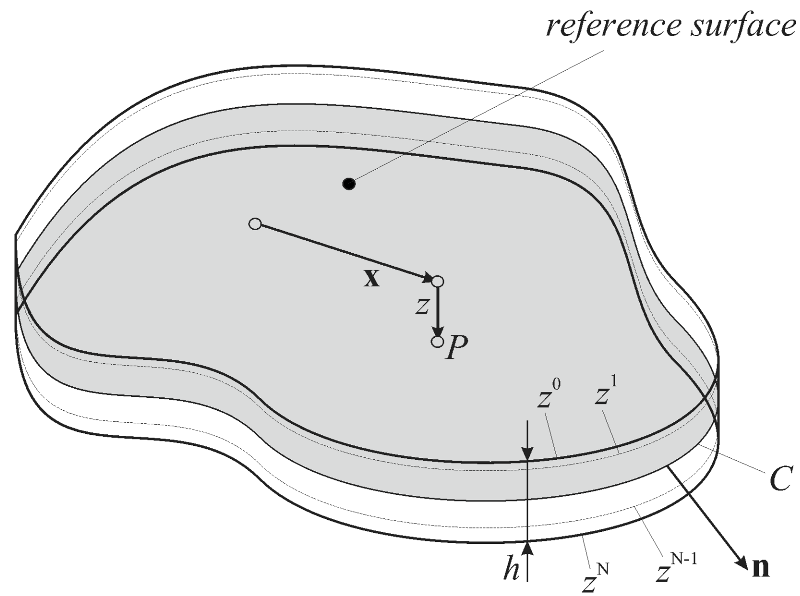
$$C: \quad \left[div\mathbf{M}\cdot\mathbf{n} + \nabla(\mathbf{Mn}\cdot\mathbf{s})\cdot\mathbf{s} - \bar{q}\right]\delta w_0 = 0\,, \quad (\mathbf{Mn}\cdot\mathbf{n} - \bar{m})(\nabla\delta w_0\cdot\mathbf{n}) = 0$$

$$P: \quad (\mathbf{Mn}\cdot\mathbf{s})\delta w_0\Big]_{P^+}^{P^-} = 0$$

$$\mathbf{M} = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\boldsymbol{\sigma}^k z\,dz\,, \quad P = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\rho^k dz$$

$\boldsymbol{\sigma}^k$ ... plane stress tensor

ρ^k ... density



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$$\boldsymbol{\sigma}^k = \mathbf{C}^k z : \boldsymbol{\kappa} - \mathbf{e}^k E_z^k = \mathbf{C}^k : \left(z\boldsymbol{\kappa} - \boldsymbol{\varepsilon}^{*k} \right), \quad \boldsymbol{\varepsilon}^{*k} \text{... eigenstrain}$$

$\mathbf{C}^k$ ... tensor of elastic moduli

$\mathbf{e}^k$ ... tensor of piezoelectric coefficients

$E_z^k = -\nabla\phi \cdot \mathbf{e}_z$ ... thickness component of electric field vector

$$\mathbf{M} = \mathbf{D} : \boldsymbol{\kappa} - \mathbf{M}^*$$

$$\mathbf{M} = \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \boldsymbol{\sigma}^k z dz = \underbrace{\left(\sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \mathbf{C}^k z^2 dz \right)}_{\mathbf{D}} : \boldsymbol{\kappa} - \underbrace{\sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \mathbf{e}^k E_z^k z dz}_{\mathbf{M}^*}$$

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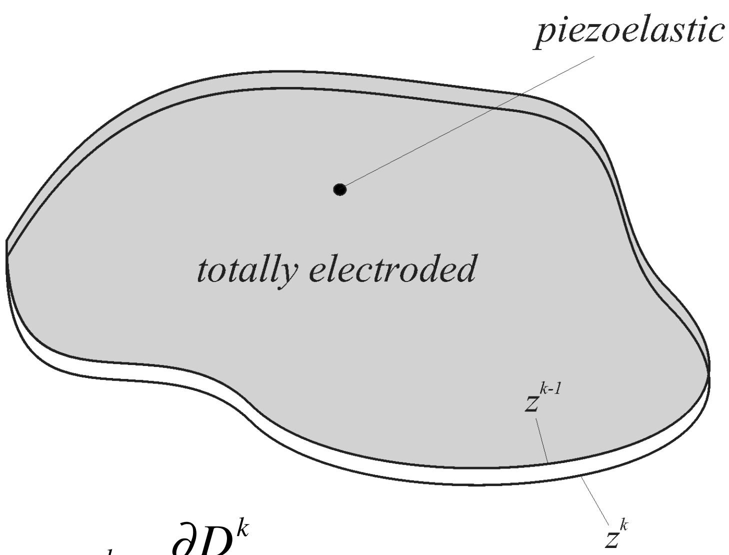

$$div\mathbf{D}^k + \frac{\partial D_z^k}{\partial z} = 0$$

$\mathbf{D}^k$ ... in - plane electric displacement vector

D_z^k ... thickness component of electric displacement vector

$$D_z^k = \mathbf{e}^k : z\boldsymbol{\kappa}^k + \eta^k E_z^k = \eta^k \left(E_z^k - E_z^{*k} \right)$$

η^k ... electric permittivity

$$E_z^{*k} = \frac{\mathbf{e}^k : z\boldsymbol{\kappa}^k}{\eta^k}$$ ... electric eigenfield

$\rightarrow$ due to the thinness of the layer we neglect $\mathbf{D}^k$

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$$\underbrace{\frac{\partial D_z^k}{\partial z}=0\,,\quad D_z^k=\eta^k\left(E_z^k-E_z^{*k}\right)}$$

$$D_z^k=const.=\frac{1}{z^k-z^{k-1}}\int_{z^{k-1}}^{z^k}D_z^k dz=\eta^k\frac{V^k}{z^k-z^{k-1}}-\frac{1}{z^k-z^{k-1}}\int_{z^{k-1}}^{z^k}\eta^k E_z^{*k}dz$$

- Relation between Q^k and V^k from Gauss law of electrostatics:

$$Q^k=\int_A D_z^k dA=A\eta^k\frac{V^k}{z^k-z^{k-1}}-\int_A\frac{1}{z^k-z^{k-1}}\int_{z^{k-1}}^{z^k}\eta^k E_z^{*k}dzdA$$

$$E_z^k=E_z^{k,c}+E_z^{*k}-\left[\frac{1}{z^k-z^{k-1}}\int_{z^{k-1}}^{z^k}E_z^{*k}dz-\kappa_Q^k\frac{1}{A}\int_A\frac{1}{z^k-z^{k-1}}\int_{z^{k-1}}^{z^k}E_z^{*k}dzdA\right]$$

$$E_z^{k,c}=\kappa_Q^k\frac{Q^k}{A\eta^k}+(1-\kappa_Q^k)\frac{V^k}{h^k}$$

$$\mathbf{M} = \mathbf{D} : \boldsymbol{\kappa} - \mathbf{M}^*$$

$$\mathbf{M}^* = \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \mathbf{e}^k E_z^k z dz , \quad E_z^k = E_z^{k,c} + E_z^{*k} - \left[\frac{1}{z^k - z^{k-1}} \int_{z^{k-1}}^{z^k} E_z^{*k} dz - \kappa_Q^k \frac{1}{A} \int_A \frac{1}{z^k - z^{k-1}} \int_{z^{k-1}}^{z^k} E_z^{*k} dz dA \right]$$

- Reformulation of constitutive relations: $\boxed{\mathbf{M} = \mathbf{D}^{eff} : \boldsymbol{\kappa} + \mathbf{d} : \bar{\boldsymbol{\kappa}} - \mathbf{M}^{*,eff}}$

$$\mathbf{D}^{eff} = \mathbf{D} + \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \frac{\mathbf{e}^k \otimes \mathbf{e}^k}{\eta^k} \left(z - \frac{1}{2}\left(z^k + z^{k-1}\right) \right) z dz , \quad \mathbf{d} = \sum_{k=1}^{N} \kappa_Q^k \int_{z^{k-1}}^{z^k} \frac{\mathbf{e}^k \otimes \mathbf{e}^k}{\eta^k} \frac{1}{2}\left(z^k + z^{k-1}\right) z dz$$

$$\bar{\boldsymbol{\kappa}} = \frac{1}{A} \int_A \boldsymbol{\kappa} dA$$ ... mean curvature tensor

$$\mathbf{M}^{*,eff} = \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \mathbf{e}^k E_z^{k,c} z dz$$ ... effective piezoelectric (self-) moment tensor

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- Transversally isotropic materials
- Isotropic plane parallel to reference surface

$$\mathbf{M} = D^{eff} \nu^{eff} \mathbf{I} tr\boldsymbol{\kappa} + D^{eff}\left(1-\nu^{eff}\right)\boldsymbol{\kappa} + d\mathbf{I} tr\bar{\boldsymbol{\kappa}} - m^{*,eff}\mathbf{I}$$

$$D^{eff} = D + D^{me}\,,\quad \nu^{eff} = \frac{D\nu + D^{me}}{D + D^{me}}\,,\quad d = \sum_{k=1}^{N} \kappa_Q^k \int_{z^{k-1}}^{z^k} \frac{e^k e^k}{\eta^k}\frac{1}{2}\left(z^k + z^{k-1}\right) z\,dz$$

$$D = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k} Y^k z^2 dz\,,\quad \nu = \frac{1}{D}\sum_{k=1}^{N}\int_{z^{k-1}}^{z^k} Y^k \nu^k z^2 dz\,,\quad D^{me} = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k} \frac{e^k e^k}{\eta^k}\left(z - \frac{1}{2}\left(z^k + z^{k-1}\right)\right) z\,dz$$

$$m^{*,eff} = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k} e^k\left(\kappa_Q^k \frac{Q^k}{A\eta^k} + (1-\kappa_Q^k)\frac{V^k}{h^k}\right) z\,dz$$

- sensor equation: $$y^k = Q^k - A\eta^k \frac{V^k}{h^k} = -\int_A e^k \Delta w_0 z_m^k dA$$

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- Simply supported edges
- Polygonal domain

$$\Delta\left(D^{eff}\Delta w_0\right)+P\ddot{w}_0=p_z-\Delta m^{*,eff}\ ,\quad at\ C:\quad w_0=0\,,\quad D^{eff}\Delta w_0+d\frac{1}{A}\int_A \Delta w_0 dA=-m^{*,eff}$$

- domain wise constant parameters

$$D^{eff}\Delta\Delta w_0+P\ddot{w}_0=p_z-D^{eff}\left(1-\nu^{eff}\right)\Delta\kappa^{eff}\ ,\quad at\ C:\quad w_0=0\,,\quad \Delta w_0=-\left(1-\nu^{eff}\right)\kappa^{eff}$$

$$\kappa^{eff}=\frac{m^{*,eff}+d\frac{1}{A}\int_A \Delta w_0 dA}{D^{eff}\left(1-\nu^{eff}\right)}=\frac{m^{*,eff}+m^{*,f}}{D^{eff}\left(1-\nu^{eff}\right)}$$

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- Frequency domain formulation: $p_z = \hat{p}e^{i\omega t}\,,\ m^{*,eff} = \hat{m}^{*,eff}e^{i\omega t}\,,\ w_0 = \hat{W}e^{i\omega t}$

$$D^{eff}\Delta\Delta\hat{W} - \mu\hat{W} = \hat{p} - D^{eff}\left(1-\nu^{eff}\right)\Delta\hat{\kappa}^{eff}\,,\quad at\ C:\quad \hat{W}=0\,,\quad \Delta\hat{W} = -\left(1-\nu^{eff}\right)\hat{\kappa}^{eff}$$

$$\mu = \omega^2 P\,,\quad \hat{\kappa}^{eff} = \frac{\hat{m}^{*,eff} + d\frac{1}{A}\int_A \Delta\hat{W}dA}{D^{eff}\left(1-\nu^{eff}\right)} = \frac{\hat{m}^{*,eff} + \hat{m}^{*,f}}{D^{eff}\left(1-\nu^{eff}\right)}$$

- Decomposition of fourth order problem into two second order problems of Helmholtz - Klein - Gordon type:

$$\hat{W} = \hat{W}_1 + \hat{W}_2\,,\quad \Delta\hat{W}_j - (-1)^j\alpha\hat{W}_j = (-1)^j\frac{\alpha}{2\mu}\hat{p} - \frac{\hat{m}^{*,eff} + \hat{m}^{*,f}}{2D^{eff}}\,,\quad at\ C_0:\quad \hat{W}_j = 0$$

$$\alpha = \sqrt{\frac{\mu}{D^{eff}}}\,,\quad \hat{m}^{*,f} = d\frac{1}{A}\int_A \Delta\hat{W}dA = d\frac{1}{A}\int_A \Delta\left(\hat{W}_1 + \hat{W}_2\right)dA$$

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- Green's function method for solution: $G_j^{\alpha}(\mathbf{x},\boldsymbol{\xi})$

$$\Delta\hat{W}_j-(-1)^j\alpha\hat{W}_j=0\,,\quad at\ C_0:\quad \hat{W}_j=0$$

- Solution is:

$$\hat{W}_j(\boldsymbol{\xi})=-\frac{\hat{m}^{*,f}}{2D^{eff}}\int_A G_j^{\alpha}(\mathbf{x},\boldsymbol{\xi})\,dA(\mathbf{x})+\int_A\left((-1)^j\frac{\alpha}{2\mu}\hat{p}(\mathbf{x})-\frac{\hat{m}^{*,eff}(\mathbf{x})}{2D^{eff}}\right)G_j^{\alpha}(\mathbf{x},\boldsymbol{\xi})\,dA(\mathbf{x})$$

- Integrate solution with respect to plate area A:

$$\hat{m}^{*,f}\left(1-\frac{\alpha}{2A}\frac{d}{D^{eff}+d}\int_A\left[\int_A\left(G_1^{\alpha}(\mathbf{x},\boldsymbol{\xi})-G_2^{\alpha}(\mathbf{x},\boldsymbol{\xi})\right)dA(\mathbf{x})\right]dA(\boldsymbol{\xi})\right)=$$

$$=\frac{1}{2}\frac{d}{D^{eff}+d}\frac{1}{A}\left(\int_A\left[\begin{array}{l}\displaystyle\int_A\hat{p}(\mathbf{x})\left(G_1^{\alpha}(\mathbf{x},\boldsymbol{\xi})+G_2^{\alpha}(\mathbf{x},\boldsymbol{\xi})\right)dA(\mathbf{x})+\\ \displaystyle+\int_A\alpha\hat{m}^{*,eff}(\mathbf{x})\left(G_1^{\alpha}(\mathbf{x},\boldsymbol{\xi})-G_2^{\alpha}(\mathbf{x},\boldsymbol{\xi})\right)dA(\mathbf{x})-2\hat{m}^{*,eff}(\boldsymbol{\xi})\end{array}\right]dA(\boldsymbol{\xi})\right)$$

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FREE VIBRATIONS (for $\hat{m}^{*,f}=0$ classical problem)

- characteristic equation for $\hat{m}^{*,f}\neq 0$:

$$2A=\alpha\frac{d}{D^{eff}+d}\int_A\left[\int_A\left(G_1^{\alpha}(\mathbf{x},\xi)-G_2^{\alpha}(\mathbf{x},\xi)\right)dA(\mathbf{x})\right]dA(\xi),\quad \omega_i=\alpha_i\sqrt{\frac{D^{eff}}{P}}$$

- Eigenfunctions:

$$\hat{W}^{\alpha_i}(\xi)=\int_A\left(G_1^{\alpha_i}(\mathbf{x},\xi)+G_2^{\alpha_i}(\mathbf{x},\xi)\right)dA(\mathbf{x})$$

FORCED VIBRATIONS

$$y^k=-e^k z_m^k\left\{\int_A\left[\alpha\left(\hat{W}_2(\xi)-\hat{W}_1(\xi)\right)-\frac{\hat{m}^{*,eff}(\xi)}{D^{eff}}\right]dA(\xi)-\frac{\hat{m}^{*,f}}{D^{eff}}A\right\}$$

PART II: Numerical example

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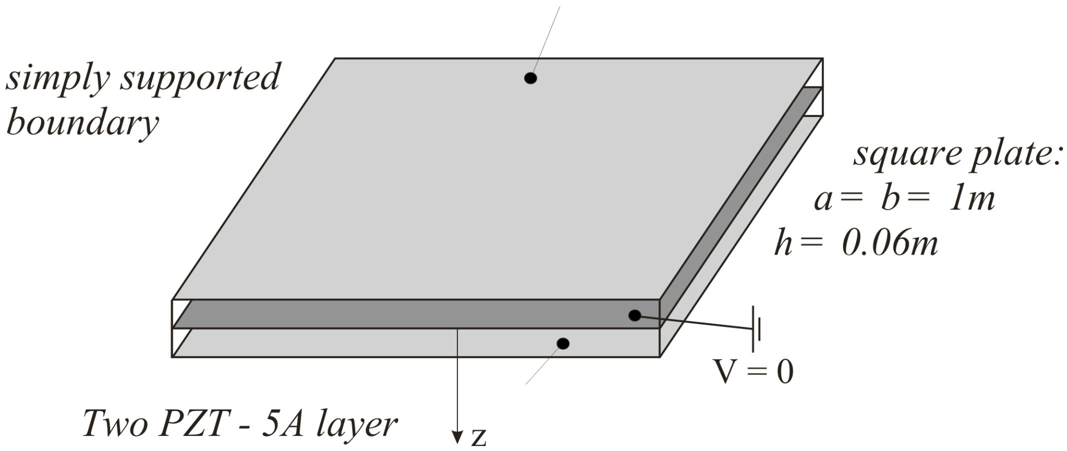

- SQUARE PLATE: a = b = 1m and h = 0.06m
- TWO identical PZT-5A layers: interface is grounded and outer sides are electroded
- Electrodes either short - circuited or left open
- $p_z(\mathbf{x},t) = \hat{p}(\mathbf{x})e^{i\omega t} = p_0 e^{i\omega t}, \, m^{*,eff} = 0$

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- FREE VIBRATIONS:

- $m^{*,f}=0$: Short - circuited electrodes or either n or m are even

$$\omega_{mn}=\alpha_{mn}\sqrt{\frac{D^{eff}}{P}}=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{a}\right)^2\right]\sqrt{\frac{D^{eff}}{P}}\,,\quad \hat{W}_{mn}(x,y)=\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right)$$

- $m^{*,f}\neq 0$: Electrodes left open and n and m odd

$$2A=\alpha_{mn}\frac{d}{D^{eff}+d}\int_A\left[\int_A\left(G_1^{\alpha_{mn}}(\mathbf{x},\xi)-G_2^{\alpha_{mn}}(\mathbf{x},\xi)\right)dA(\mathbf{x})\right]dA(\xi)\,,\ \hat{W}_{mn}(\xi)=\int_A\left(G_1^{\alpha_{mn}}(\mathbf{x},\xi)+G_2^{\alpha_{mn}}(\mathbf{x},\xi)\right)dA(\mathbf{x})$$

$$\int_A G_j^{\alpha_{mn}}(\mathbf{x},\xi)\,dA(\mathbf{x})=\sum_{k=1,3,\ldots}^{\infty}\sum_{l=1,3,\ldots}^{\infty}\frac{16}{kl\pi^2}\frac{1}{-(-1)^j\alpha_{mn}-\left[\left(\frac{k\pi}{a}\right)^2+\left(\frac{l\pi}{a}\right)^2\right]}\sin\left(\frac{k\pi}{a}\xi\right)\sin\left(\frac{l\pi}{a}\eta\right)$$

$$j=1,2\,,\quad 0\le x\le a\,,\quad 0\le y\le a$$

PART II: Numerical example

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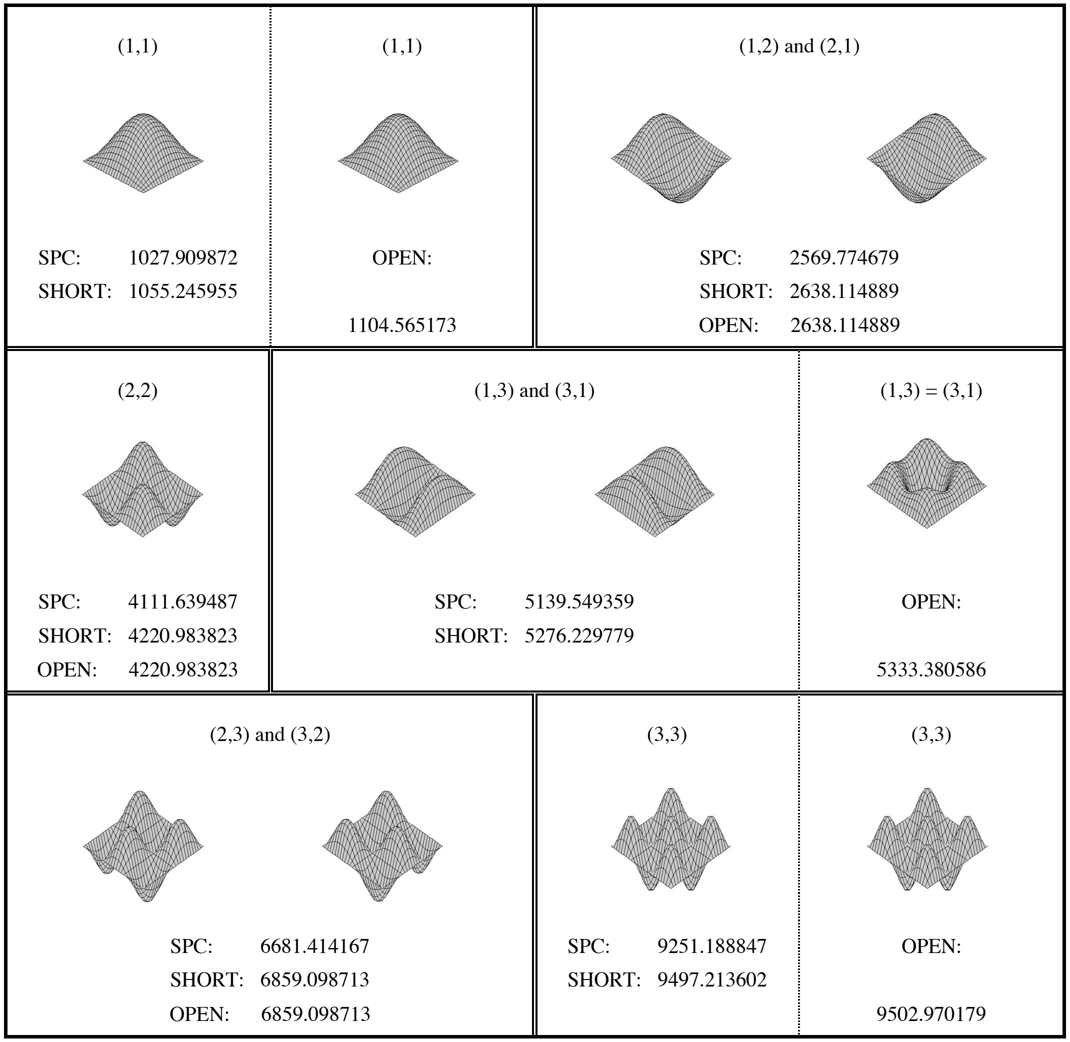

Natural frequencies [rad/s] and mode shapes of the square plate

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- FORCED VIBRATIONS:

- Sensor signal in upper layer: $y^u = -e z_m^u \left[\alpha \int_A \left(\hat{W}_2(\xi) - \hat{W}_1(\xi) \right) dA(\xi) - \frac{\hat{m}^{*,f}}{D^{eff}} A \right]$

- Dynamic magnification factor: $\hat{y} = 20 \log \left| y^u / y_{SPC}^{u(qs)} \right|$

- Solution is:

$$\hat{W}_j(\xi) = -\frac{\hat{m}^{*,f}}{2D^{eff}} \int_A G_j^\alpha(\mathbf{x},\xi)\, dA(\mathbf{x}) + \int_A (-1)^j \frac{\alpha}{2\mu} \hat{p}(\mathbf{x}) G_j^\alpha(\mathbf{x},\xi)\, dA(\mathbf{x})$$

$$\text{with} \quad \hat{m}^{*,f} \left(1 - \frac{\alpha}{2A} \frac{d}{D^{eff} + d} \int_A \left[\int_A \left(G_1^\alpha(\mathbf{x},\xi) - G_2^\alpha(\mathbf{x},\xi) \right) dA(\mathbf{x}) \right] dA(\xi) \right) =$$

$$= \frac{1}{2} \frac{d}{D^{eff} + d} \frac{1}{A} \left(\int_A \left[\int_A \hat{p}(\mathbf{x}) \left(G_1^\alpha(\mathbf{x},\xi) + G_2^\alpha(\mathbf{x},\xi) \right) dA(\mathbf{x}) \right] dA(\xi) \right)$$

PART II:
Numerical example

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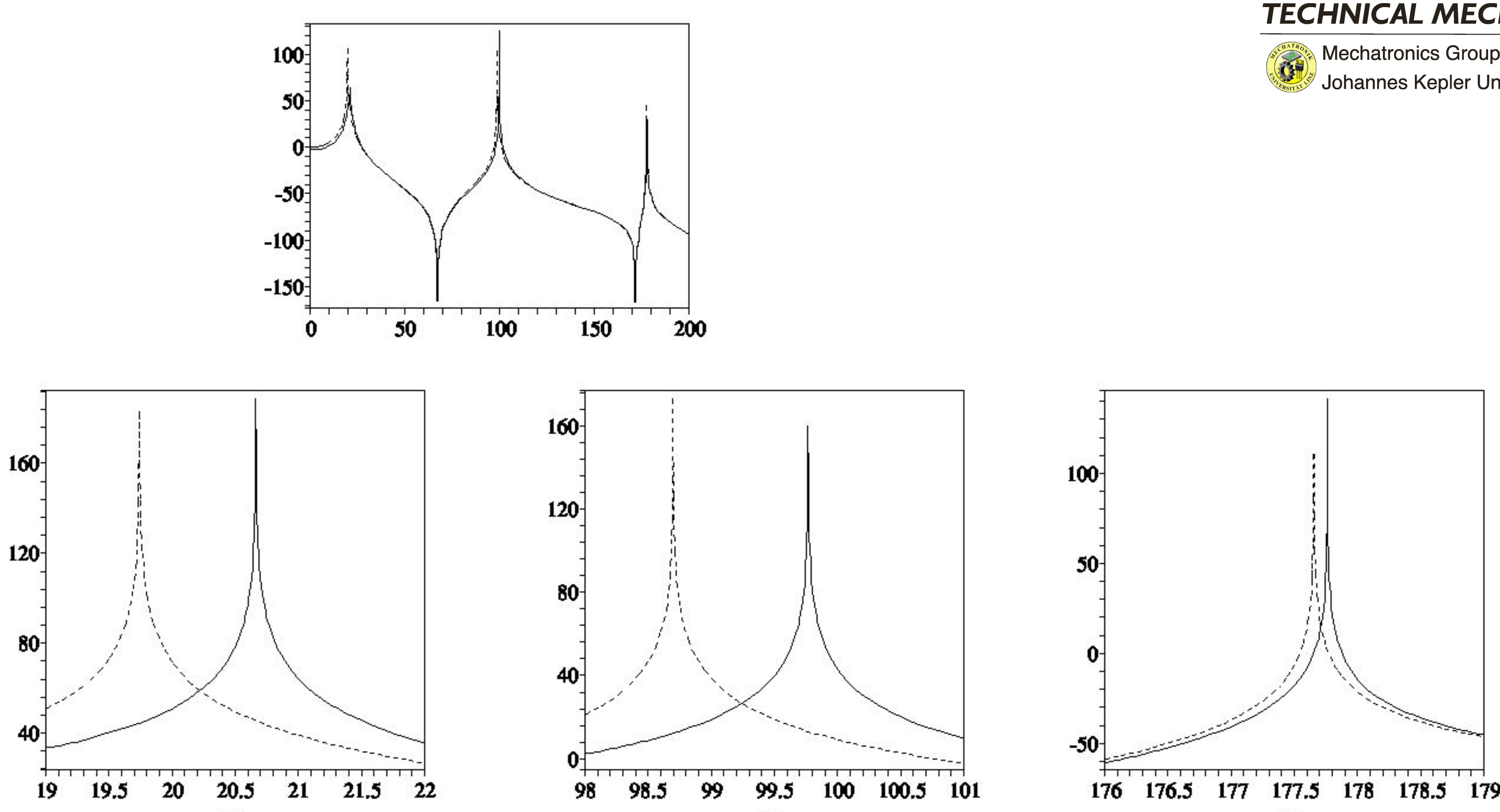

Dynamic magnification factor for different ranges of α

PART II: Numerical example

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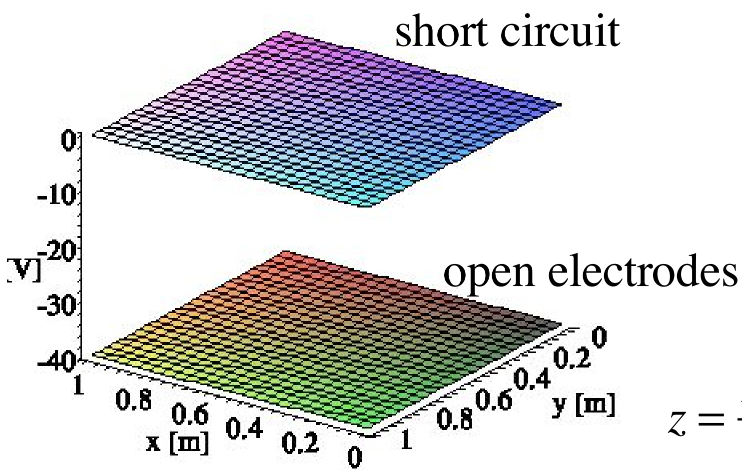
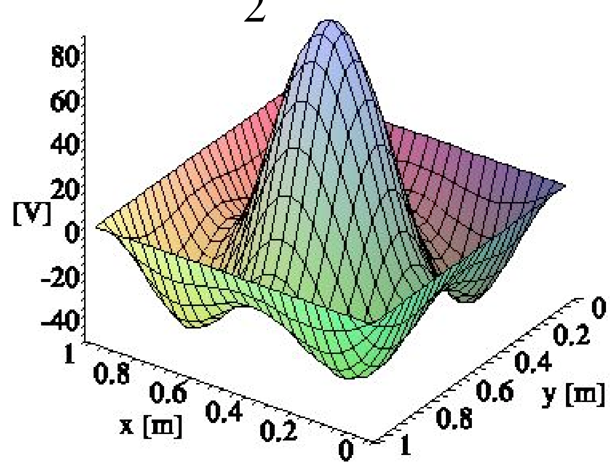
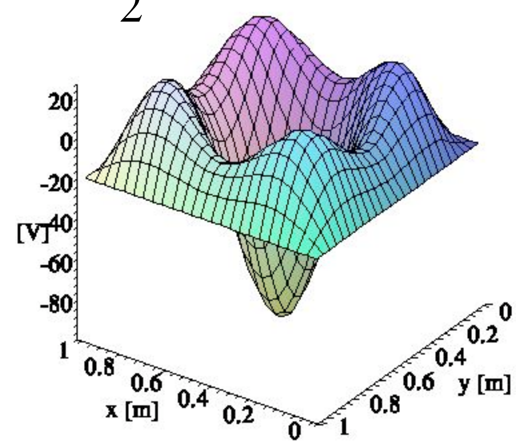


$z = z_0$

$z = \frac{z_0}{2};$ short circuit



$z = \frac{z_0}{2};$ open electrodes



Electric potential distribution for $\alpha = 99.2$

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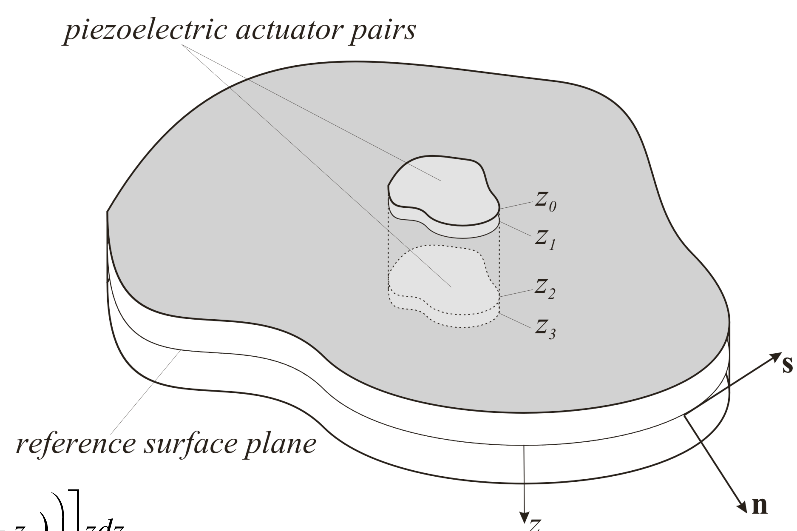
$$A: \quad div(div\mathbf{M}) + p_z = P\ddot{w}_0$$

$$C: \quad \left[div\mathbf{M}\cdot\mathbf{n} + \nabla(\mathbf{Mn}\cdot\mathbf{s})\cdot\mathbf{s} - \bar{q}\right]\delta w_0 = 0\,, \quad (\mathbf{Mn}\cdot\mathbf{n} - \bar{m})(\nabla\delta w_0\cdot\mathbf{n}) = 0$$

$$P: \quad (\mathbf{Mn}\cdot\mathbf{s})\delta w_0\Big]_{P^+}^{P^-} = 0$$

with patch: $\mathbf{M} = \mathbf{D}^{eff} : \boldsymbol{\kappa} - \mathbf{M}^{*,eff}\,, \quad P = P_0 + \Delta P$

without patch: $\mathbf{M} = \mathbf{D}_0 : \boldsymbol{\kappa}\,, \quad P = P_0$



$$P_0 = \int_{z_0}^{z_1} \rho^{substrate} dz\,, \quad \Delta P = 2\int_{z_0}^{z_1} \rho^{patch} dz$$

$$\mathbf{M}^{*,eff} = \int_{z_0}^{z_1} \bar{\mathbf{e}}\frac{V^u}{h_p} z dz + \int_{z_2}^{z_3} \bar{\mathbf{e}}\frac{V^l}{h_p} z dz = \int_{z_0}^{z_1} \bar{\mathbf{e}}\frac{2V^u}{h_p} z dz\,, \quad \mathbf{D}_0 = \int_{z_1}^{z_2} \mathbf{C}^{substrate} z^2 dz$$

$$\Delta\mathbf{D} = \int_{z_0}^{z_1}\left[\mathbf{C}^{patch} z + \frac{\bar{\mathbf{e}}\otimes\bar{\mathbf{e}}}{\eta}\left(z - \frac{1}{2}(z_0 + z_1)\right)\right] z dz + \int_{z_2}^{z_3}\left[\mathbf{C}^{patch} z + \frac{\bar{\mathbf{e}}\otimes\bar{\mathbf{e}}}{\eta}\left(z - \frac{1}{2}(z_2 + z_3)\right)\right] z dz$$

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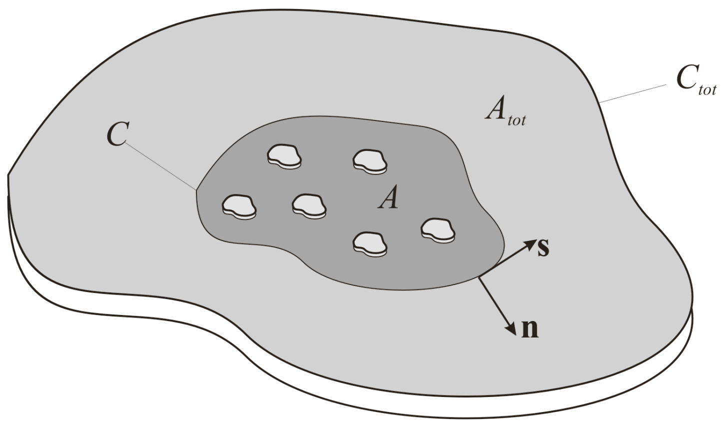

$$C:\ \begin{cases} div\mathbf{M}\cdot\mathbf{n}+\nabla(\mathbf{Mn}\cdot\mathbf{s})\cdot\mathbf{s}=\bar{q}\,, \quad \mathbf{Mn}\cdot\mathbf{n}=\bar{m} \\ w_0=\bar{w}_0\,, \quad \mathbf{Mn}\cdot\mathbf{n}=\bar{m} \\ div\mathbf{M}\cdot\mathbf{n}+\nabla(\mathbf{Mn}\cdot\mathbf{s})\cdot\mathbf{s}=\bar{q}\,, \quad \nabla w_0\cdot\mathbf{n}=\bar{\psi} \\ w_0=\bar{w}_0\,, \quad \nabla w_0\cdot\mathbf{n}=\bar{\psi} \end{cases}$$

$$\int_0^t\int_A\left[\mathbf{M}^p+\mathbf{M}^{*,eff}\right](\mathbf{x},t-\tau):\boldsymbol{\kappa}^d(\mathbf{x},t)\,dA\,dt=$$

$$\int_0^t\left[\int_A p_z^d(\mathbf{x},t-\tau)\tilde{w}_0(\mathbf{x},t)\,dA+\int_{C_q}\bar{q}^d(\mathbf{x},t-\tau)\tilde{w}_0(\mathbf{x},t)\,dC-\int_{C_m}\bar{m}^d(\mathbf{x},t-\tau)\left(\nabla\tilde{w}_0(\mathbf{x},t)\cdot\mathbf{n}\right)dC\right]dt$$

If $\mathbf{M}^p+\mathbf{M}^*=\mathbf{0}$, then $w_0=\breve{w}_0$.

PART III: Exact solution

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$$A: \quad div(div\mathbf{M}^p) + \tilde{p}_z = 0$$

$$C: \begin{cases} div\mathbf{M}^p \cdot \mathbf{n} + \nabla\left(\mathbf{M}^p\mathbf{n}\cdot\mathbf{s}\right)\cdot\mathbf{s} = \tilde{q}\,, \quad \mathbf{M}^p\mathbf{n}\cdot\mathbf{n} = \tilde{m} \\ \mathbf{M}^p\mathbf{n}\cdot\mathbf{n} = \tilde{m} \\ div\mathbf{M}^p \cdot \mathbf{n} + \nabla\left(\mathbf{M}^p\mathbf{n}\cdot\mathbf{s}\right)\cdot\mathbf{s} = \tilde{q} \\ \text{no boundary conditions to be satisfied} \end{cases}$$

$$A: \quad \tilde{p}_z = p_z - \bar{P}\ddot{\breve{w}}_0 + div\left(div\breve{\mathbf{M}}\right), \quad \breve{\mathbf{M}} = \bar{\mathbf{D}} : \breve{\boldsymbol{\kappa}}\,, \quad \breve{\boldsymbol{\kappa}} = -\frac{1}{2}\left[\nabla\nabla\breve{w}_0 + \left(\nabla\nabla\breve{w}_0\right)^T\right]$$

$$C: \quad \tilde{q} = \bar{q} - div\breve{\mathbf{M}}\cdot\mathbf{n} - \nabla\left(\breve{\mathbf{M}}\mathbf{n}\cdot\mathbf{s}\right)\cdot\mathbf{s}\,, \quad \tilde{m} = \bar{m} - \breve{\mathbf{M}}\mathbf{n}\cdot\mathbf{n}$$

$$A: \quad \nabla\nabla\breve{w}_0 = \mathbf{0}\,, \quad C: \begin{cases} \text{no conditions to be satisfied} \\ \breve{w}_0 = \bar{w}_0 \\ \nabla\breve{w}_0\cdot\mathbf{n} = \bar{\psi} \\ \breve{w}_0 = \bar{w}_0\,, \quad \nabla\breve{w}_0\cdot\mathbf{n} = \bar{\psi} \end{cases}$$

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- $i = 1,\ldots,n$ patches within A
- Each patch is laocated within A_i^{sub} with $A_i^{sub} \subseteq A_i$ and $\bigcap_{i=1}^{n} A_i = A$
- Piezoelectric actuation at patch i: $\mathbf{M}_i^{*,eff}(t)$

$$\int_0^t \int_A \left[\mathbf{M}^p + \mathbf{M}^{*,eff}\right](\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA\, dt =$$

$$= \int_0^t \left[\sum_{i=1}^{n} \left(\mathbf{M}_i^{*,eff}(t-\tau) : \int_{A_i^{sub}} \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right) + \int_A \mathbf{M}^p(\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right] dt$$

PART III: Approximate solution; direct method

$$\int_0^t \int_A \left[\mathbf{M}^p + \mathbf{M}^{*,eff}\right](\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA\, dt =$$

$$= \int_0^t \left[\sum_{i=1}^{n} \left(\mathbf{M}_i^{*,eff}(t-\tau) : \int_{A_i^{sub}} \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right) + \int_A \mathbf{M}^p(\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right] dt$$

$$\mathbf{M}_i^{*,eff}(t) = -\frac{1}{A_i^{sub}} \int_{A_i} \mathbf{M}^p(\mathbf{x}, t)\, dA$$

- High number of piezoelectric patches possible
- Easy to use

$$\int_0^t \int_A \left[\mathbf{M}^p + \mathbf{M}^{*,eff}\right](\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA\, dt =$$

$$= \int_0^t \left[\sum_{i=1}^n \left(\mathbf{M}_i^{*,eff}(t-\tau) : \int_{A_i^{sub}} \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right) + \int_A \mathbf{M}^p(\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right] dt$$

- Special dummy loading cases are considered; for each case

$$\int_0^t \sum_{i=1}^n \left(\mathbf{M}_i^{*,eff}(t-\tau) : \int_{A_i^{sub}} \boldsymbol{\kappa}^{d,j}(\mathbf{x}, t)\, dA \right) dt = -\int_0^t \int_A \mathbf{M}^p(\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^{d,j}(\mathbf{x}, t)\, dA\, dt$$

- n dummy loading cases required

- Mechanical interpretation of quality of approximate solution

$$\begin{cases} \bar{m}^{d,j} = 0\,, \quad \bar{q}^{d,j} = 0\,, \quad p_z^{d,j}(\mathbf{x}, t) = f(t) \\ \text{then} \quad \int_0^t f(t-\tau) \int_A \tilde{w}_0(\mathbf{x}, t)\, dA\, dt = 0 \end{cases}$$

$$\int_0^t \int_A \left[\mathbf{M}^p + \mathbf{M}^{*,eff}\right](\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA\, dt =$$

$$= \int_0^t \left[\sum_{i=1}^{n} \left(\mathbf{M}_i^{*,eff}(t-\tau) : \int_{A_i^{sub}} \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right) + \int_A \mathbf{M}^p(\mathbf{x}, t-\tau) : \boldsymbol{\kappa}^d(\mathbf{x}, t)\, dA \right] dt$$

- Statically admissible moment and self-moment seperable:

$$\mathbf{M}^p(\mathbf{x}, t) = \sum_{j=1}^{m} \mathbf{M}_j^p(\mathbf{x}) f_j(t), \quad \mathbf{M}^{*,eff}(\mathbf{x}, t) = \sum_{j=1}^{m} \mathbf{M}_j^{*,eff}(\mathbf{x}) g_j(t)$$

- $\mathbf{M}_j^p(\mathbf{x})$'s account for space wise distributions of: $\tilde{p}_z, \tilde{m}, \tilde{q}$

- Patch self-moment are taken as: $\mathbf{M}_i^{*,eff}(t) = \sum_{j=1}^{m} \mathbf{M}_{ij}^{*,eff} g_j(t)$ with: $\mathbf{M}_{ij}^{*,eff} = -\frac{1}{A_i^{sub}} \int_{A_i} \mathbf{M}_j^p(\mathbf{x})\, dA$

- m equations for m unknown time variations $g_j(t)$

$$\int_0^t g_j(t-\tau)\sum_{i=1}^{n}\left(\mathbf{M}_{ij}^{*,eff} : \int_{A_i^{sub}} \boldsymbol{\kappa}^{d,j}(\mathbf{x},t)dA\right)dt = -\int_0^t\int_A \mathbf{M}^p(\mathbf{x},t-\tau) : \boldsymbol{\kappa}^{d,j}(\mathbf{x},t)dAdt$$

by using m dummy loading cases:

$$p_z^{d,j}(\mathbf{x},t) = \tilde{p}_z(\mathbf{x},t)\,, \quad \bar{m}^{d,j}(\mathbf{x},t) = \tilde{m}(\mathbf{x},t)\,, \quad \bar{q}^{d,j}(\mathbf{x},t) = \tilde{q}(\mathbf{x},t)$$

- High number of piezoelectric patches possible

- Mechanical interpretation of quality of approximate solution

PART III: Numerical example

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- Clamped circular Plate: $R_{tot} = 0.2m$ and $h = 3mm$
- Natural frequencies of aluminium plate: $f_1 = 182.0\,Hz, f_2 = 708.7\,Hz, f_3 = 1588\,Hz$
- Control deflection within $r \le R = 3cm$ to be $w_0(r,t) = \bar{w}_0 f(t)$
- Statically admissible moment: $r \le R: \quad div(div\mathbf{M}^p) + p_z - \bar{\rho}^{(0)}\bar{w}_0\ddot{f}(t) = 0, \quad r = R: \quad \mathbf{M}^p\mathbf{n}\cdot\mathbf{n} = \bar{m}$
- Force loading: $r \le R: \quad p_z(r,t) = p_0 e^{i\omega t}, \quad p_0 = 100\,N/m^2, \quad \omega = 2\pi f = 2\pi\left(100, 500, 1000\right)s^{-1}$
- Statically admissible moment: $r \le R: \quad \Delta M^p + p_0 + \omega^2\bar{\rho}^{(0)}\bar{w}_0 = 0, \quad r = R: \quad M^p = \bar{m}$

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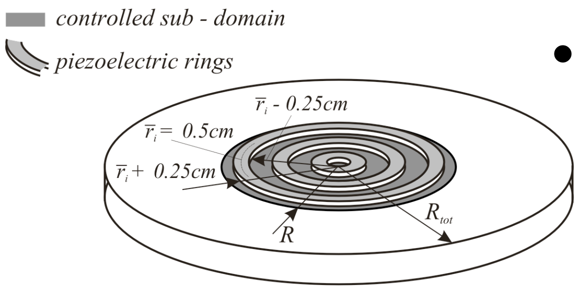

- 3 PZT-5A concentric patches: $h_p = 0.5mm$

  $r \in \left[\bar{r}_i \pm 0.25cm\right], \quad \bar{r}_i = \left(0.5, 1.5, 2.5\right)cm$

- Direct method: $M_i^{*,eff} \int_{r_i-0.0025}^{r_i+0.0025} 2\pi r dr = -\int_{r_i-0.005}^{r_i+0.005} M^p(r) 2\pi r dr$

- Indirect method: $\sum_{i=1}^{3} \left(M_i^{*,eff} \int_{r_i-0.0025}^{r_i+0.0025} \Delta w_0^{d,j}(r) 2\pi r dr \right) + \int_0^R M^p(r) \Delta w_0^{d,j}(r) 2\pi r dr = 0$

- Dummy loadings: $p_z^{d,1}(r,t) = 1e^{i\omega t}$, $p_z^{d,2}(r,t) = (\bar{P} / P_0)e^{i\omega t}$, $\bar{m}^{d,2} = 1e^{i\omega t}$

then $\int_0^R \tilde{w}_0(r) 2\pi r dr = 0$ and $\int_0^R \frac{\bar{P}}{P_0} \tilde{w}_0(r) 2\pi r dr = 0$ and $\left[\frac{\partial \tilde{w}_0(r)}{\partial r}\right]_{r=R} = 0.$

PART III:
Comparison direct and indirect method

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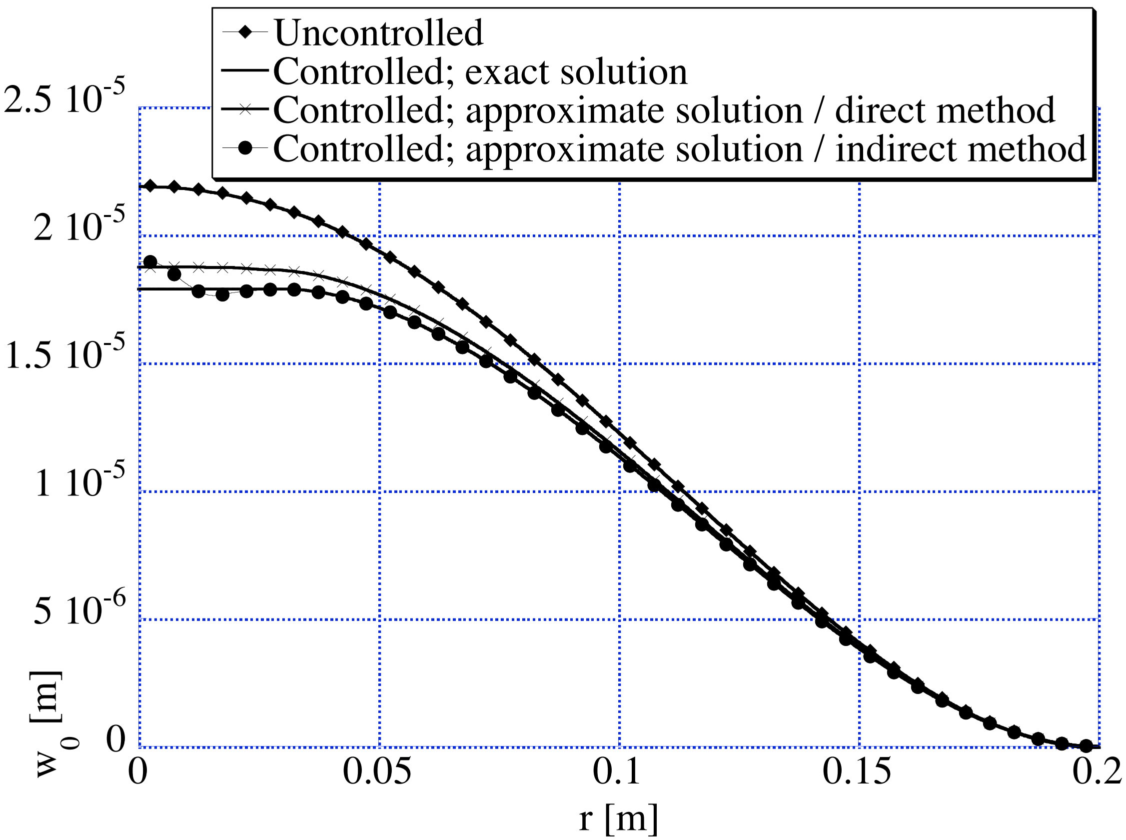

$$\omega = 2\pi 100 s^{-1}$$

PART III:
Comparison direct and indirect method

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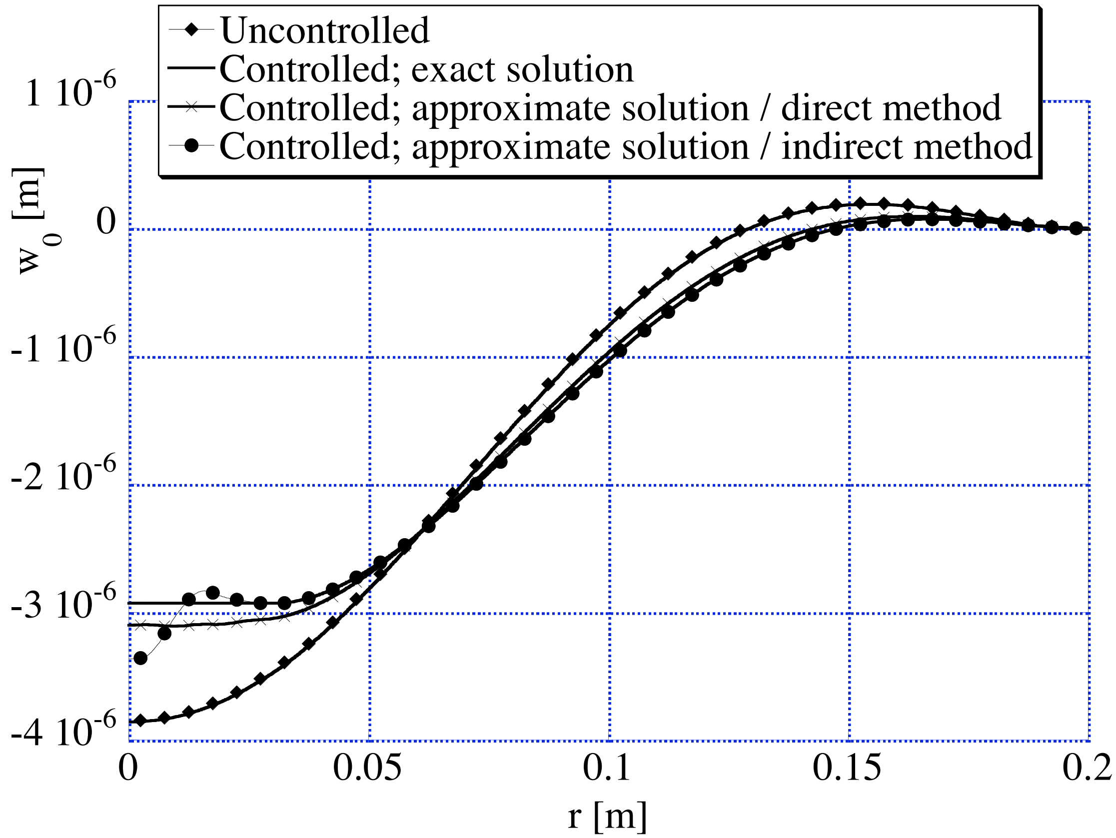
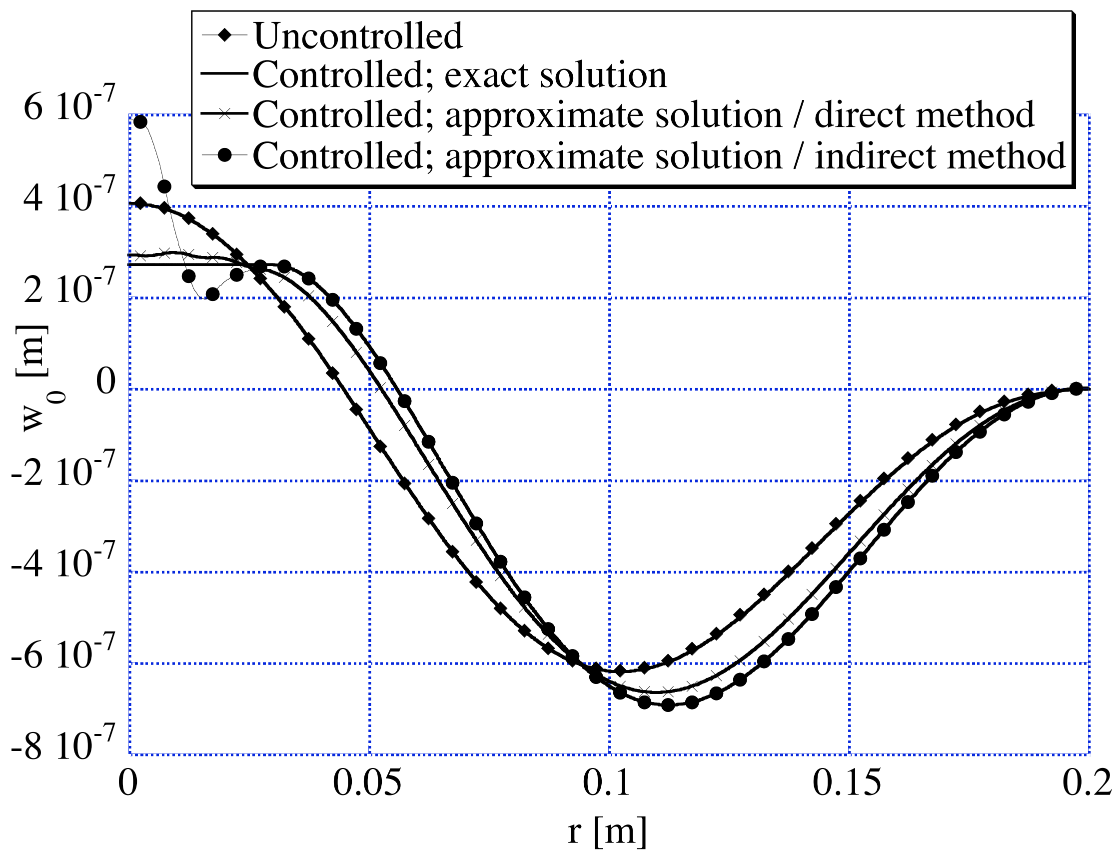


$$\omega = 2\pi 500 s^{-1}$$

PART III:
Comparison direct and indirect method



$$\omega = 2\pi 1000 s^{-1}$$

PART III: Combined method

- 6 PZT-5A concentric patches: $h_p = 0.5mm$

$$r \in \left[\bar{r}_i \pm 0.125cm\right], \quad \bar{r}_i = \left(0.25, 0.75, 1.25, 1.75, 2.25, 2.75\right)cm$$

- Combined method: $M_{ij}^{*,eff} \int_{r_i-0.00125}^{r_i+0.00125} 2\pi r dr = -\int_{r_i-0.0025}^{r_i+0.0025} M_j^P(r) 2\pi r dr$

$$M_1^P \leftrightarrow p_0\,, \quad M_2^P \leftrightarrow \bar{P}/P_0\,, \quad M_3^P \leftrightarrow \bar{m} \text{ at } r = R$$

- Self-moment at each concentric ring: $M_i^{*,eff} e^{i\omega t} = \sum_{j=1}^{3} M_{ij}^{*,eff} K_j e^{i\omega t}$

$$\sum_{i=1}^{6}\left(\sum_{j=1}^{3} M_{ij}^{*,eff} K_j\right) \int_{r_i-0.00125}^{r_i+0.00125} \Delta w_0^{d,k}(r) 2\pi r dr = -\int_0^R M^p(r) \Delta w_0^{d,k}(r) 2\pi r dr$$

- Dummy loadings: $p_z^{d,1}(r,t) = 1/(2\pi\bar{r})e^{i\omega t}\,, \quad p_z^{d,2}(r,t) = (\bar{P}/P_0)e^{i\omega t}\,, \quad \bar{m}^{d,2} = 1e^{i\omega t}$

$$\text{then } \int_0^R \tilde{w}_0(r) r/\bar{r} dr = 0 \text{ and } \int_0^R \frac{\bar{P}}{P_0}\tilde{w}_0(r) 2\pi r dr = 0 \text{ and } \left[\frac{\partial \tilde{w}_0(r)}{\partial r}\right]_{r=R} = 0.$$

PART III: Combined method

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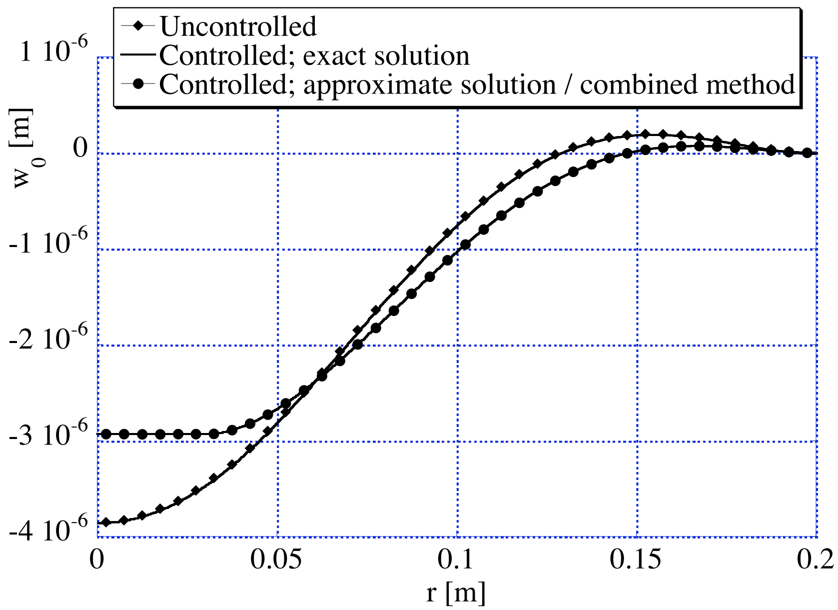

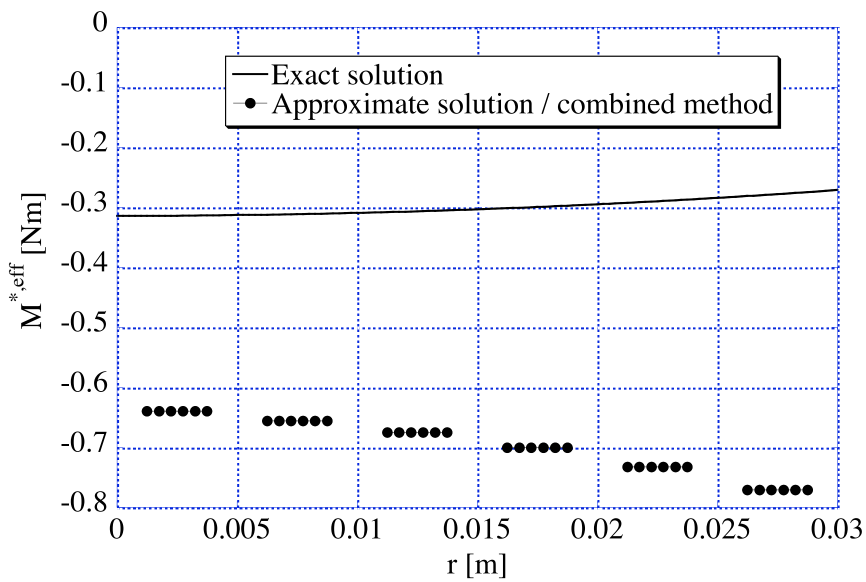

$$\omega = 2\pi 500 s^{-1}$$

- $0.16m = R_{sub} \le r \le R_{tot} = 0.2m$: 4 PZT-5A concentric patches
  $r \in \left[\bar{r}_i \pm 0.25cm\right], \quad \bar{r}_i = \left(0.165, 0.175, 0.185, 0.195\right)m$

$$R_{sub} \le r \le R_{tot}: \quad \Delta M^p + p_0 = 0\,, \quad r = R_{sub}: \quad M^p = \bar{m} \quad and \quad \frac{\partial M^p}{\partial r} = \bar{q}$$

- Combined method: $M_{ij}^{*,eff} \int\limits_{r_i-0.0025}^{r_i+0.0025} 2\pi r dr = -\int\limits_{r_i-0.005}^{r_i+0.005} M_j^P(r) 2\pi r dr$

  $M_1^P \leftrightarrow p_0\,, \quad M_2^P \leftrightarrow \bar{q}$ at $r = R\,, \quad M_3^P \leftrightarrow \bar{m}$ at $r = R$

- Self-moment at each concentric ring: $M_i^{*,eff} e^{i\omega t} = \sum_{j=1}^{3} M_{ij}^{*,eff} K_j e^{i\omega t}$

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$$\sum_{i=1}^{4}\left(\sum_{j=1}^{3} M_{ij}^{*,eff} K_j\right)\int_{r_i-0.0025}^{r_i+0.0025} \Delta w_0^{d,k}(r)2\pi r dr = -\int_{R_{sub}}^{R_{tot}} M^p(r)\Delta w_0^{d,k}(r)2\pi r dr$$

- Dummy loadings: $p_z^{d,1}(r,t)=1/(2\pi\bar{r})e^{i\omega t}$, $\bar{q}^{d,2}=1e^{i\omega t}$, $\bar{m}^{d,2}=1e^{i\omega t}$

then $\int_{R_{sub}}^{R_{tot}} \tilde{w}_0(r)\frac{2\pi r}{2\pi\bar{r}}dr = 0$ and $\left[\tilde{w}_0(r)\right]_{r=R_{sub}} = 0$ and $\left[\frac{\partial\tilde{w}_0(r)}{\partial r}\right]_{r=R} = 0.$

PART III: Control of alternative subdomain

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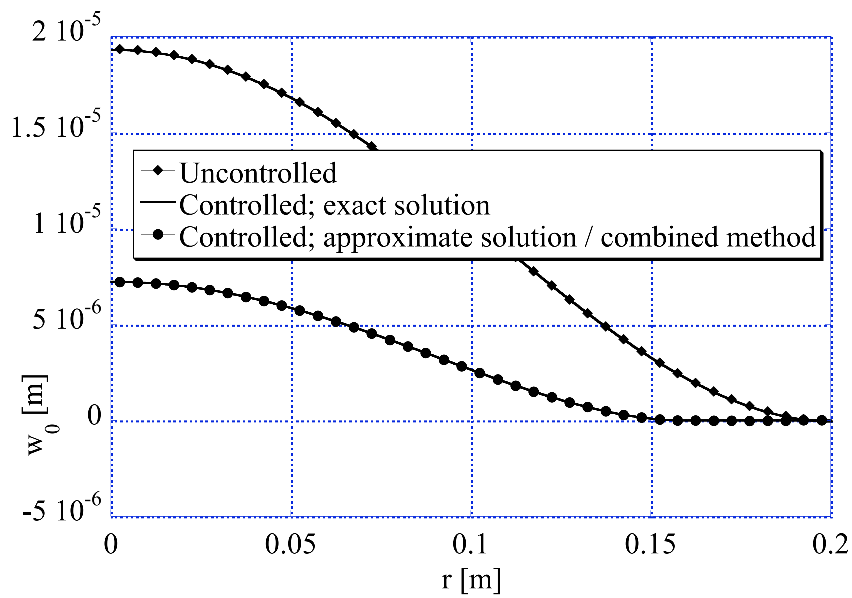

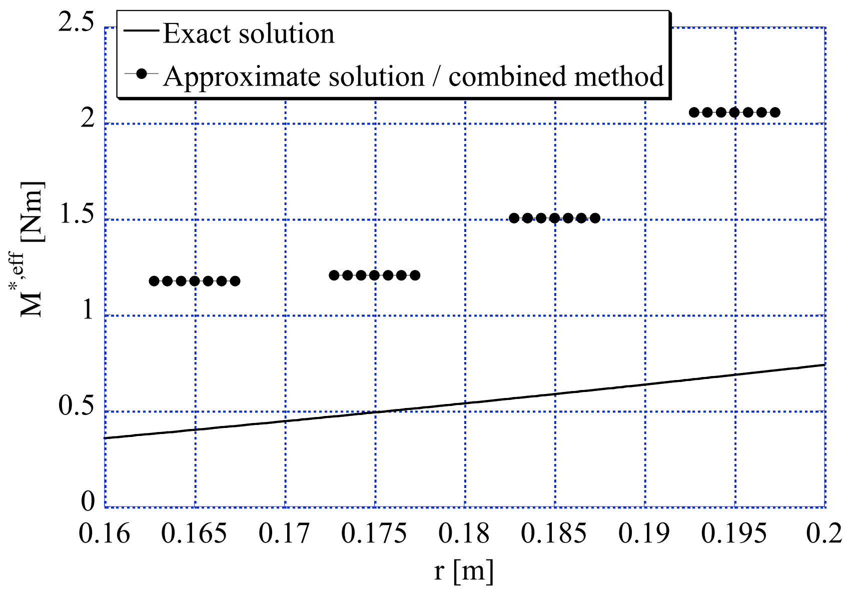

$$\omega = 2\pi 100 s^{-1}$$