Overview: Piezoelectricity

Manfred Kaltenbacher

Department of Sensor Technology,
University Erlangen-Nuremberg

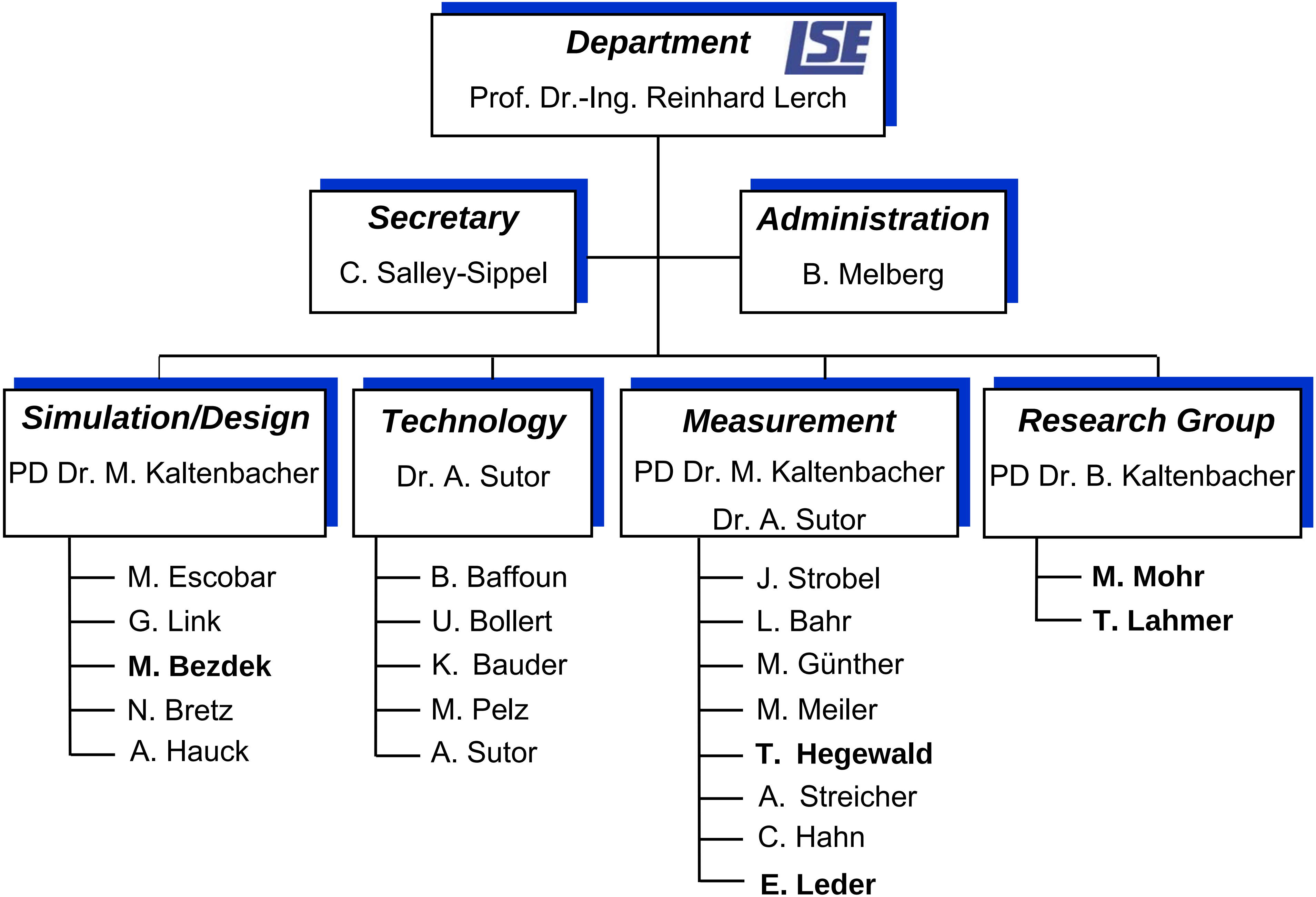
manfred@lse.eei.uni-erlangen.de
www.lse.uni-erlangen.de

Outline

- **Department of Sensor Technology**
- **Basics**
  - Piezoelectric effect
  - Piezoelectric equations
  - Piezoelectric materials
- **Finite Element Formulation**
  - Weak form
  - Spatial and time discretization
  - Solvers for the algebraic system of equations
- **Material parameter determination**
  - Experimental methods
  - Invers scheme

Location


Schleswig-Holstein
Mecklenburg-Vorpommern
Hamburg
Bremen
Niedersachsen
Berlin
Brandenburg
Sachsen-Anhalt
Nordrhein-Westfalen
Sachsen
Thüringen
Hessen
Rheinland-Pfalz
Saarland
Bayern
Baden-Württemberg
©intermap
Erlangen
Nuremberg
Bavaria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Overview: Piezoelectricity

- **Basics**
- Finite Element Formulation
- Material Parameter Determination

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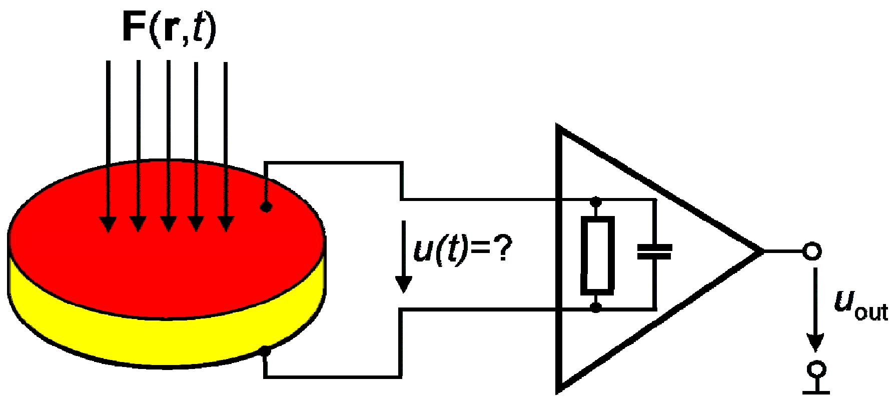
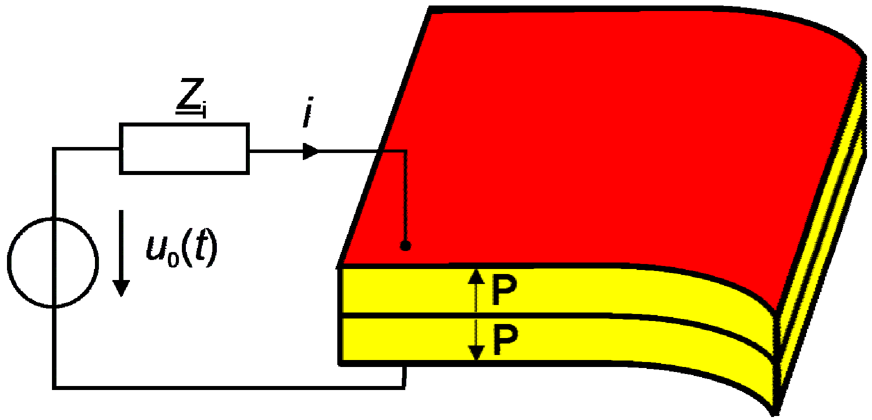
Piezoelectric Effect

- **Direct effect** (sensor)



- **Indirect effect** (actuator)



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Governing Equations

- **Constituitive equations**

$$[\boldsymbol{\sigma}] = [\mathbf{c}^E][\mathbf{S}] - [\mathbf{e}]^T\mathbf{E}$$

$$\mathbf{D} = [\mathbf{e}][\mathbf{S}] + [\boldsymbol{\varepsilon}^S]\mathbf{E}$$

$[\boldsymbol{\sigma}]$	...	stress tensor
$[\mathbf{S}]$	...	strain tensor
$\mathbf{D}$	...	electric induction
$\mathbf{E}$	...	electric field intensity
$[\mathbf{c}^E]$	...	tensor of elasticity moduli
$[\boldsymbol{\varepsilon}^S]$	...	tensor of dielectric constants
$[\mathbf{e}]$	...	tensor of piezoelectric moduli

- **Piezoelectric materials** can be subdivided in the following three categories
  - **Single crystals**, like quartz
  - **Piezoelectric ceramics** like barium titanate or lead zirconate
  - **Polymers** like PVDF (polyvinylidenfluoride)

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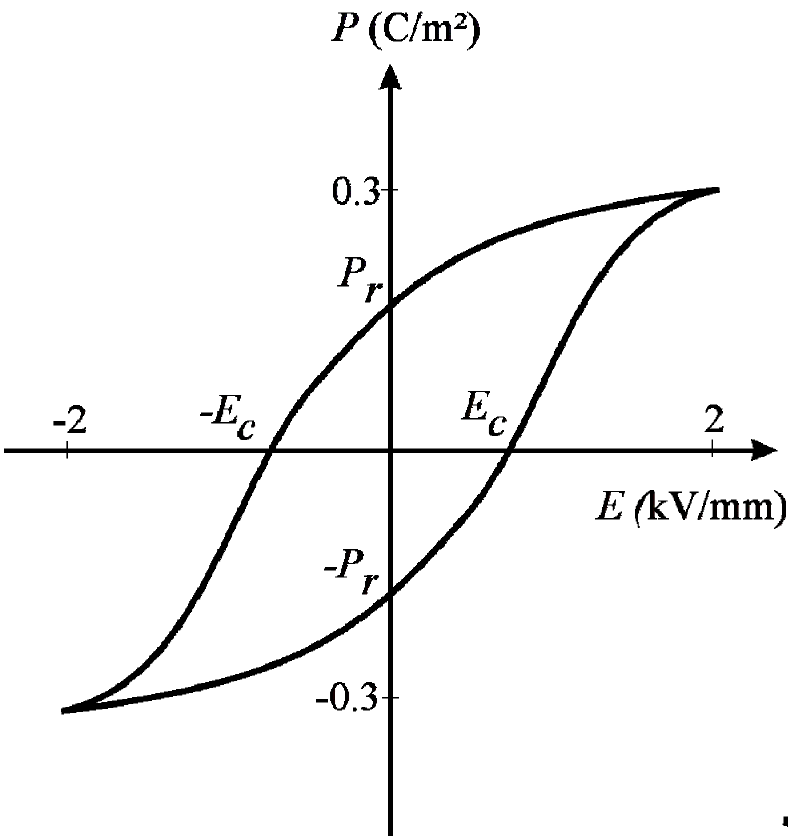
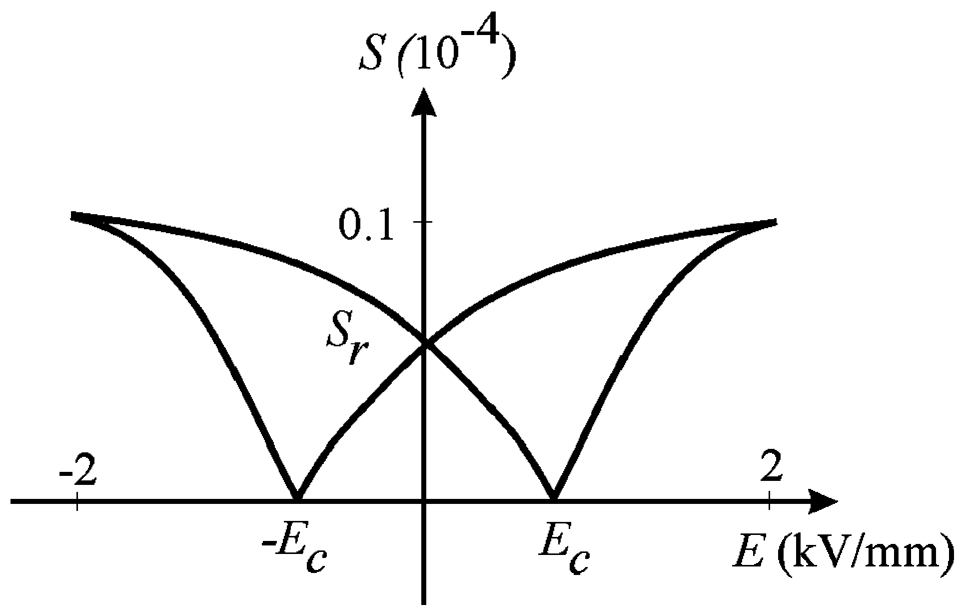
Tensors for 6mm Crystal Class

$$[\mathbf{c}^E] = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (c_{11}-c_{12})/2 \end{pmatrix}$$

$$[\mathrm{e}] = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{pmatrix} \qquad [\varepsilon^{S}] = \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}$$

Polarization / Strain versus Electric Field Intensity





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Overview: Piezoelectricity

- Basics
- **Finite Element Formulation**
- Material Parameter Determination

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Coupled Equations (I)

- **Mechanical field (Navier‘s equation)**

$$
\begin{aligned}
\mathrm{f}_{\mathrm{V}} + \mathcal{B}^T \boldsymbol{\sigma} &= \rho \ddot{\mathbf{u}} \\
\mathbf{S} &= \mathcal{B}\mathbf{u}
\end{aligned}
\qquad
\mathcal{B} = \begin{pmatrix}
\frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\
0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\
0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0
\end{pmatrix}^T
$$

$\boldsymbol{\sigma}$	...	stress tensor	$\mathbf{u}$	...	mechnaical displacement
$\mathbf{S}$	...	strain tensor	f_{V}	...	mechanical volume force

- **Electrostatic field** (Maxwell‘s equations)

$$
\begin{aligned}
\nabla \cdot \mathbf{D} &= 0 \\
\nabla \times \mathbf{E} &= 0 \Rightarrow \mathbf{E} = -\nabla V_{\mathrm{e}}
\end{aligned}
$$

$\mathbf{D}$	...	electric flux density	V_{e}	...	scalar electric potential
$\mathbf{E}$	...	electric field intensity			

Coupled Equations (II)

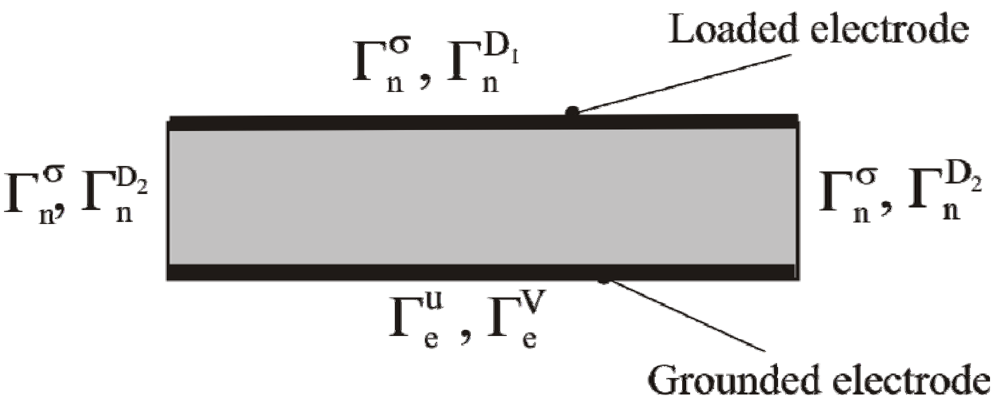
- **Constituitive equations**

$$
\begin{aligned}
[\boldsymbol{\sigma}] &= [\mathbf{c}^E][\mathbf{S}] - [\mathbf{e}]^T\mathbf{E} \\
\mathbf{D} &= [\mathbf{e}][\mathbf{S}] + [\varepsilon^S]\mathbf{E}
\end{aligned}
$$

- **Coupled equations**

$$
\begin{aligned}
\rho\ddot{\mathbf{u}} - \mathcal{B}^T\left([\mathbf{c}^E]\mathcal{B}\mathbf{u} + [\mathbf{e}]^T\nabla V_\mathrm{e}\right) &= \mathrm{f_V} \\
\nabla \cdot \left([\mathbf{e}]\mathcal{B}\mathbf{u} - [\varepsilon^S]\nabla V_\mathrm{e}\right) &= 0
\end{aligned}
$$

- **Boundary conditions**



$$
\begin{aligned}
\mathbf{u} &= 0 \quad \text{on } \Gamma_\mathrm{e}^u \\
V_\mathrm{e} &= 0 \quad \text{on } \Gamma_\mathrm{e}^V \\
\mathbf{n}\cdot[\boldsymbol{\sigma}] &= 0 \quad \text{on } \Gamma_\mathrm{n}^\sigma \\
\mathbf{n}\cdot\mathbf{D} &= -\frac{q_\mathrm{e}}{A} \quad \text{on } \Gamma_\mathrm{n}^{D_1} \\
\mathbf{n}\cdot\mathbf{D} &= 0 \quad \text{on } \Gamma_\mathrm{n}^{D_2}
\end{aligned}
$$

Finite Element Formulation (I)

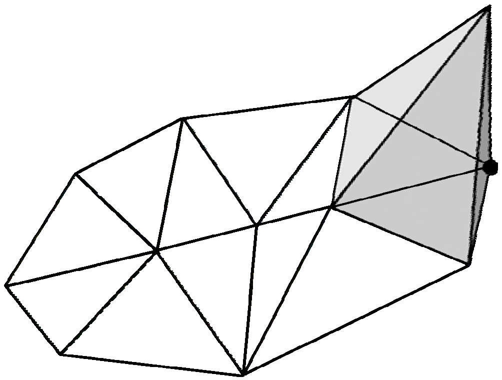
- **Weak form:** Find $(\mathbf{u} \in \mathbf{H}_0^1, V_\mathrm{e} \in H_0^1)$ such that

$$\int_\Omega \rho \mathbf{u}' \cdot \ddot{\mathbf{u}} \,\mathrm{d}\Omega + \int_\Omega (\mathcal{B}\mathbf{u}')^T [\mathbf{c}^E] \mathcal{B}\mathbf{u} \,\mathrm{d}\Omega + \int_\Omega (\mathcal{B}\mathbf{u}')^T [\mathrm{e}]^T \tilde{\mathcal{B}} V_\mathrm{e} \,\mathrm{d}\Omega = 0$$

$$\int_\Omega (\tilde{\mathcal{B}}\psi)^T \mathbf{e}(\mathcal{B}\mathbf{u}) \,\mathrm{d}\Omega - \int_\Omega (\tilde{\mathcal{B}}\psi)^T [\varepsilon^S] \tilde{\mathcal{B}} V_\mathrm{e} \,\mathrm{d}\Omega = -\int_{\Gamma_n^{D1}} \psi \frac{q_\mathrm{e}}{A} \,\mathrm{d}\Gamma$$

for any $(\mathbf{u}' \in \mathbf{H}_0^1, \psi \in H_0^1)$

- **Spatial approximation**



$$V_\mathrm{e} \approx V_\mathrm{e}^h = \sum_{a=1}^{n_\mathrm{n}} N_a V_{\mathrm{e}a}$$

$$\mathbf{u} \approx \mathbf{u}^h = \sum_{i=1}^{n_\mathrm{d}} \sum_{a=1}^{n_\mathrm{n}} N_a u_{ia} \mathbf{e}_i = \sum_{a=1}^{n_\mathrm{n}} \mathbf{N}_a \mathbf{u}_a \quad ; \quad \mathbf{N}_a = \begin{pmatrix} N_a & 0 & 0 \\ 0 & N_a & 0 \\ 0 & 0 & N_a \end{pmatrix}$$

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Finite Element Formulation (II)

- **Semidiscrete Galerkin formulation:**

$$\begin{pmatrix} \mathbf{M}_u & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{\underline{u}} \\ \ddot{\underline{V\mathrm{e}}} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_u & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\underline{u}} \\ \dot{\underline{V\mathrm{e}}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_u & \mathbf{K}_{uV} \\ \mathbf{K}_{uV}^T & -\mathbf{K}_V \end{pmatrix} \begin{pmatrix} \underline{u} \\ \underline{V\mathrm{e}} \end{pmatrix} = \begin{pmatrix} 0 \\ \underline{f}_q \end{pmatrix}$$

$\mathbf{M}_u$	...	mechanical mass matrix	$\mathbf{K}_{uV}$	...	coupling matrix
$\mathbf{C}_u$	...	mechanical damping matrix	$\mathbf{K}_V$	...	electric stiffness matrix
$\mathbf{K}_u$	...	mechanical stiffness matrix	$\underline{f}_q$	..	electric force vector
$\underline{u}$	...	mechanical displacement	$\underline{V\mathrm{e}}$	...	electric scalar potential

- Introduced damping via Rayleigh model:

$$\mathbf{C}_u = \alpha \mathbf{M}_u + \beta \mathbf{K}_u \qquad \alpha,\ \beta \ \dots \ \text{damping coefficients}$$

Finite Element Formulation (III)

- **Time discretization:** Newmark method

$$\underline{u}_{n+1} = \underline{u}_n + \Delta t\, \dot{\underline{u}}_n + \frac{\Delta t^2}{2}\Big((1-2\beta_{\mathrm{H}})\ddot{\underline{u}}_n + 2\beta_{\mathrm{H}}\ddot{\underline{u}}_{n+1}\Big)$$

$$\dot{\underline{u}}_{n+1} = \dot{\underline{u}}_n + \Delta t\Big((1-\gamma_{\mathrm{H}})\ddot{\underline{u}}_n + \gamma_{\mathrm{H}}\ddot{\underline{u}}_{n+1}\Big)$$

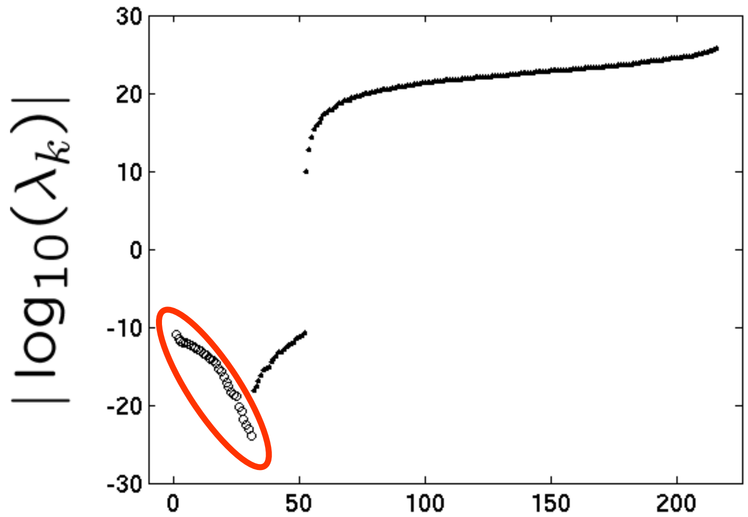
n ... time-step counter Δt ... time-step value
β_{H} .. integration parameter γ_{H} ... integration parameter

- **Choice of integration parameters:**
  - **Explicit:** Stability depends on mesh size, material parameters
    makes just sense for piezoelectic-structure interaction
    ⇒ explicit/implicit splitting
  - **Implicit:** $\beta_{\mathrm{H}} = 0.25,\ \gamma_{\mathrm{H}} = 0.5$
    ⇒ 2nd order accurate

Algebraic Solver (I)

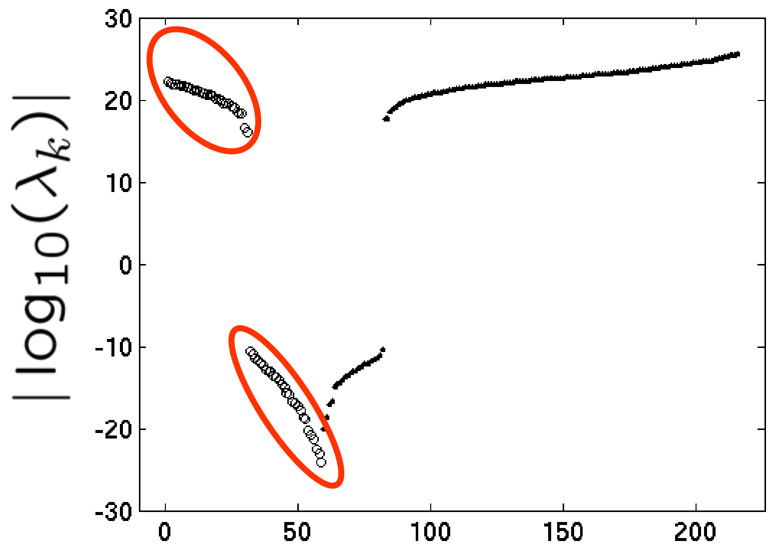
- **Static case**

$$\mathbf{A} = \begin{pmatrix} \mathbf{K}_u & \mathbf{K}_{uV} \\ \mathbf{K}_{uV}^T & -\mathbf{K}_V \end{pmatrix}$$



- **Harmonic case**

$$\mathbf{A} = \begin{pmatrix} \mathbf{K}_u - w^2\mathbf{M}_u & \mathbf{K}_{uV} \\ \mathbf{K}_{uV}^T & -\mathbf{K}_V \end{pmatrix}$$



Neagtive eigenvalues

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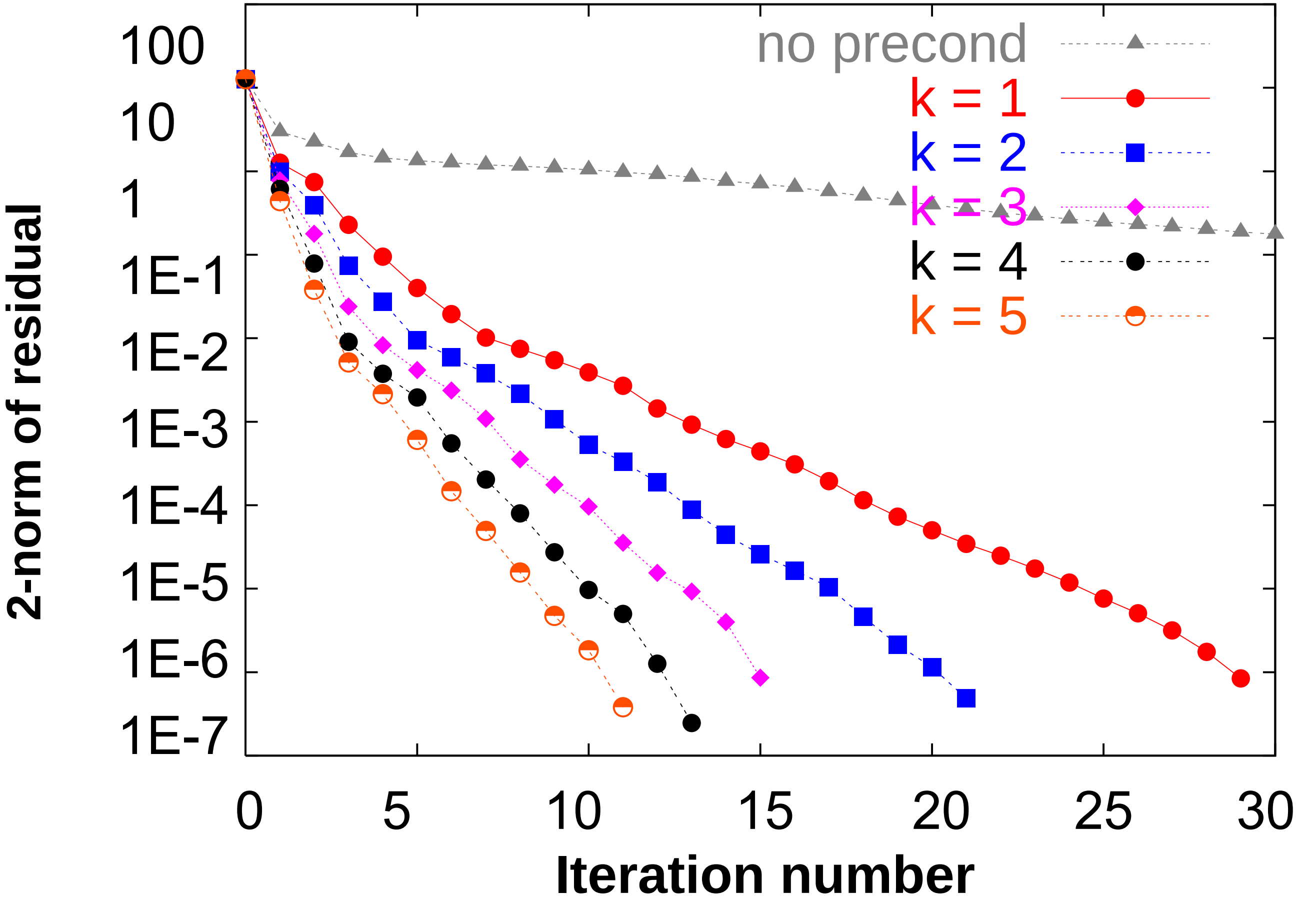
Algebraic Solver (II)

- **GMRES-solver** (General Minimal Residual)
- **ILDL(k)-preconditioner** (Incomplete LDL-decomposition with fill-in level k)



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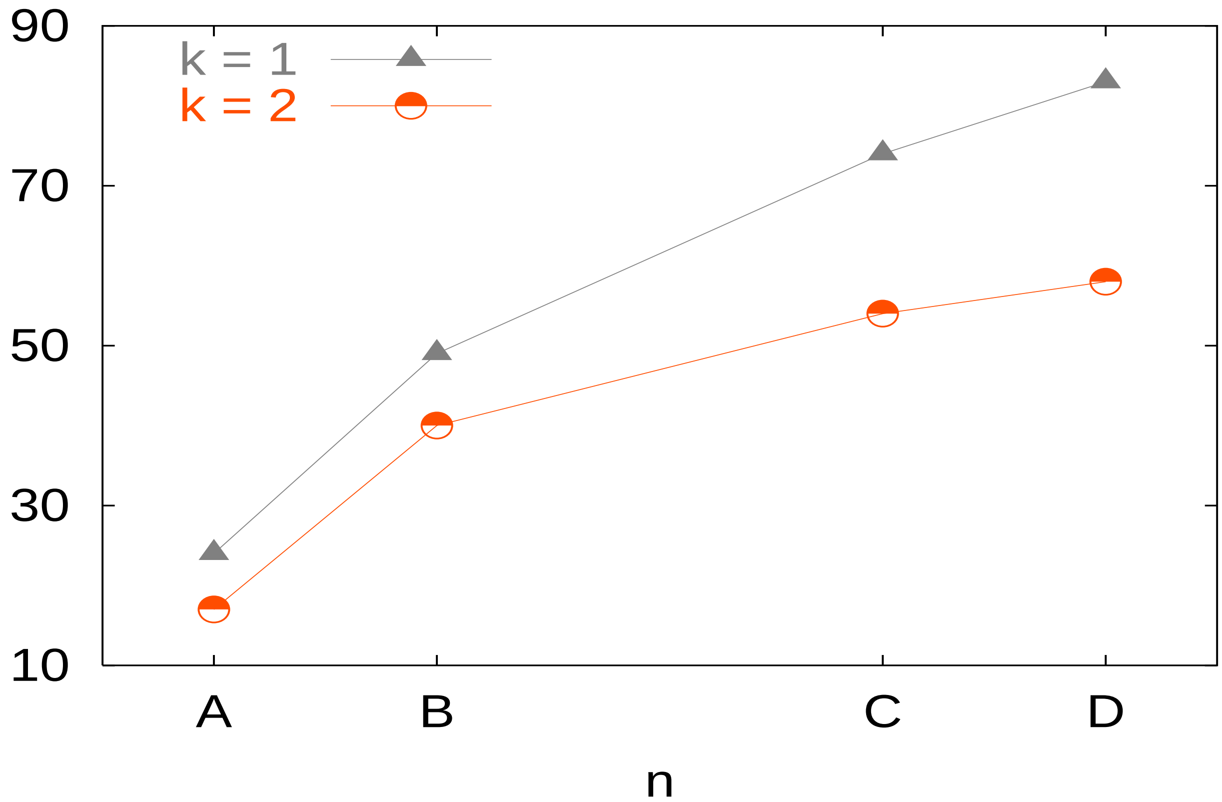
Algebraic Solver (III)

- **Dependency of iteration number on problem size**



A: 36.519 B: 77.696 C: 300.243 D: 618.403

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- Basics
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- **Material Parameter Determination**

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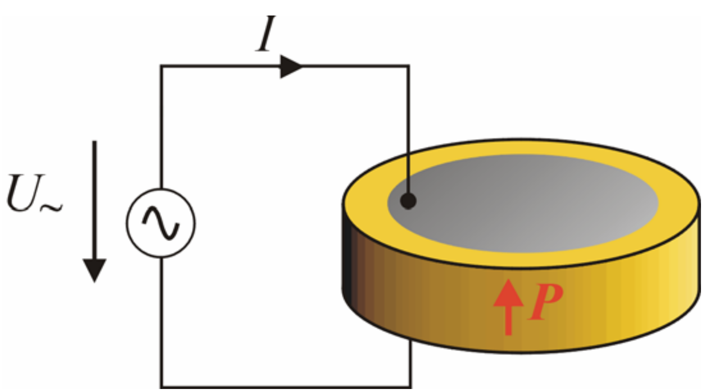
State of the Art

- **Test samples** with **special geometries: (IEEE Standard)**

  Simplification to the one-dimensional case, direct relation between resonance frequencies and coefficients

- Example: **thickness resonantor**



thickness << radius

$$\frac{k_t^2}{1-k_t^2} = \frac{e_{33}^2}{c_{33}^E \varepsilon_{33}^S}$$

$$k_t^2 = \frac{\pi}{2}\frac{f_s}{f_p}\tan\left(\frac{\pi}{2}\frac{f_p - f_s}{f_p}\right)$$

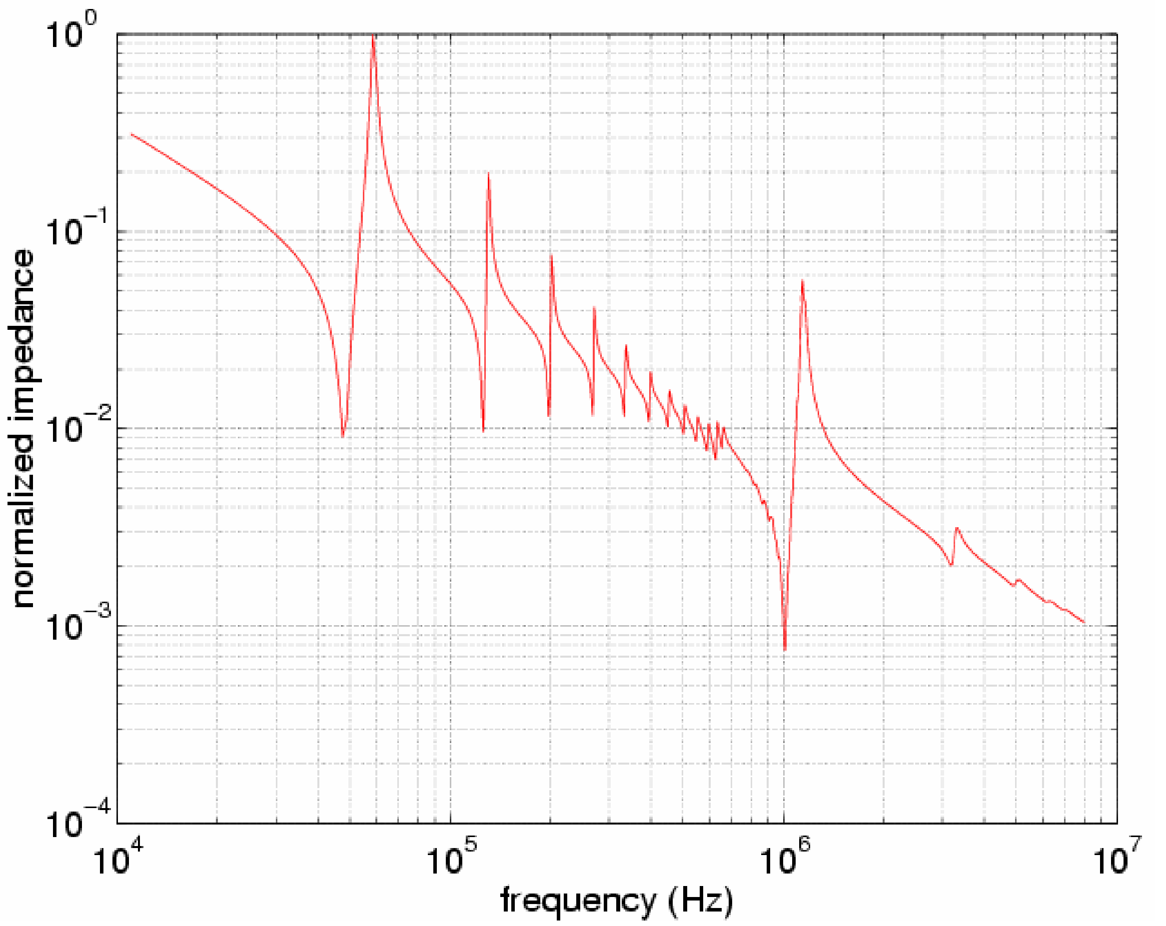
Identification of Material Parameters by Simulation of the Full System

Find **material tensors**

$$\mathbf{c}^E, \mathbf{e}, \ \varepsilon^S$$

from measured impedance

$$Z(\omega) = \frac{\hat{\phi}(\omega)|_{\Gamma_e}}{\imath\omega\hat{q}^e(\omega)}$$



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Equipment for characterization of linear and nonlinear material properties

Impedance / gain-phase analyzer

linear **nonlinear**

electrical
impedance
pulse response

mechanical
displacement
force

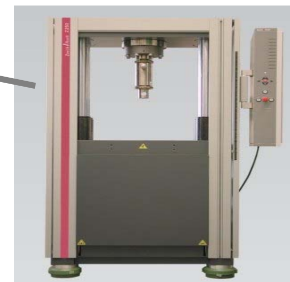
acoustical
pulse echoe

measurements

500W CW RF-amplifier

differential fibre interferometer

100kN materials testing machine



Combined measurement methods for accurate piezoelectric material parameters

Ritec SNAP: two 5kW-pulse amplifiers with receivers

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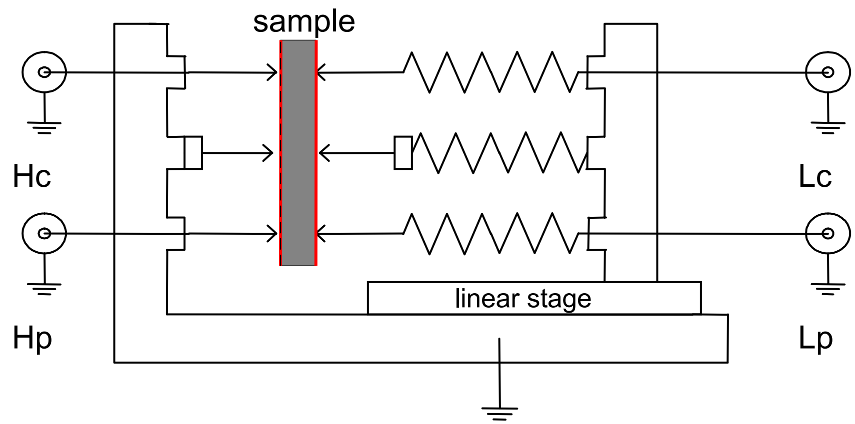
Measurement setup

Automated setup for impedance measurement

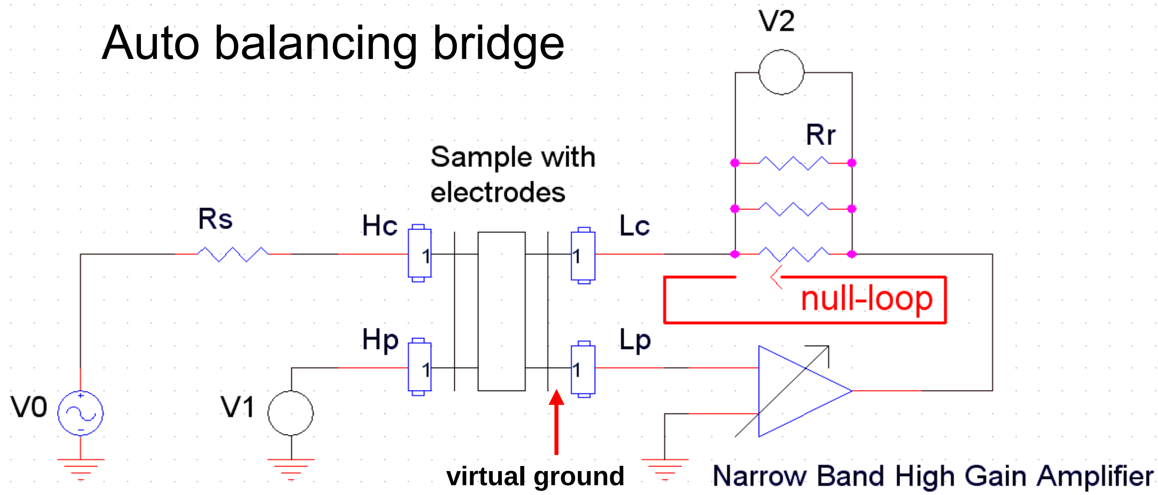
Contacting phase

- X-positioning of the spring contact pair Lc-Lp for contact force
- Testing contact resistance with constant current



Measuring sequence

- Contacting phase for the short/open circuit
- Measurement for calibration
- Contacting phase for the piezo sample
- Material measurement (V0=1Veff)



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Material configuration

Bulk samples

– PZT disc (radial and thickness mode)

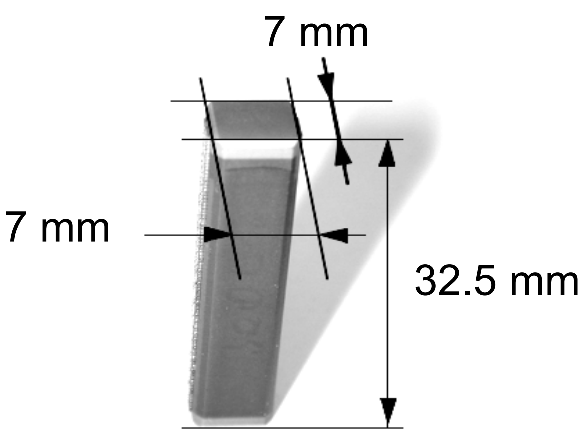
thickness: 0.43 mm, radius: 6.35 mm

– PZT strip (transversal mode)

thickness: 0.43 mm, length: 11.03 mm, width: 2.44 mm

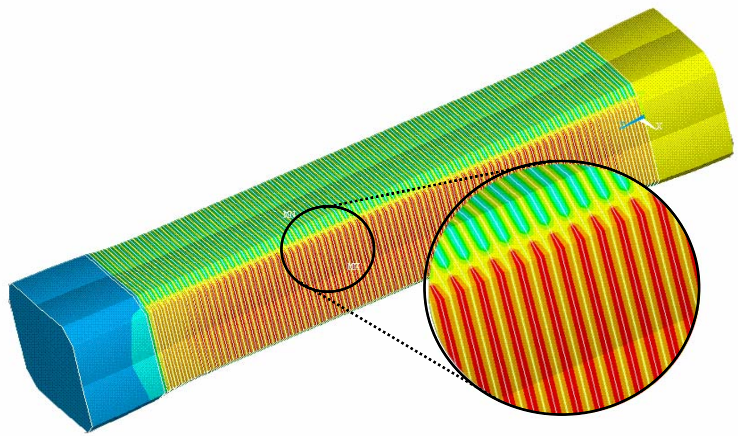
not within the IEEE range because width > 5 * thickness is not fulfilled

Device sample: stack actuator of a CR-injector application



layers:
200

material:
the same as the bulk samples



Finite element model

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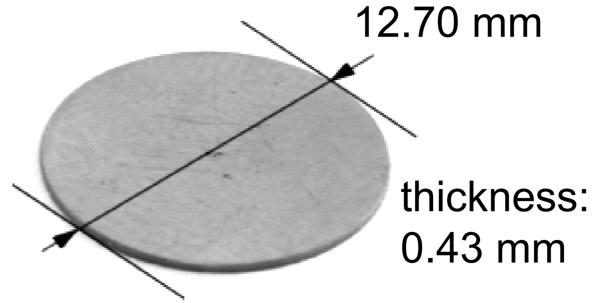
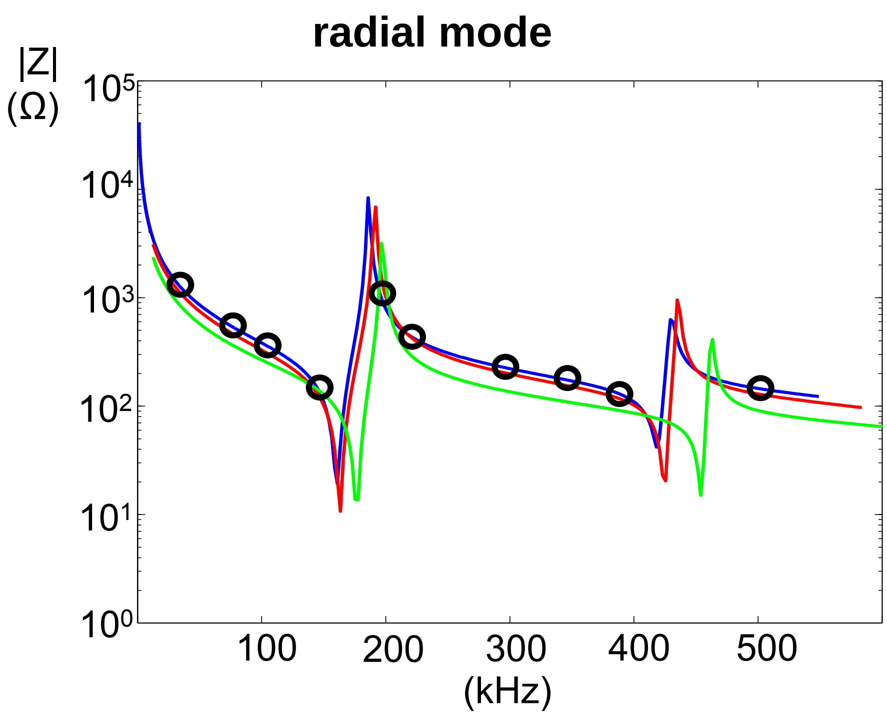
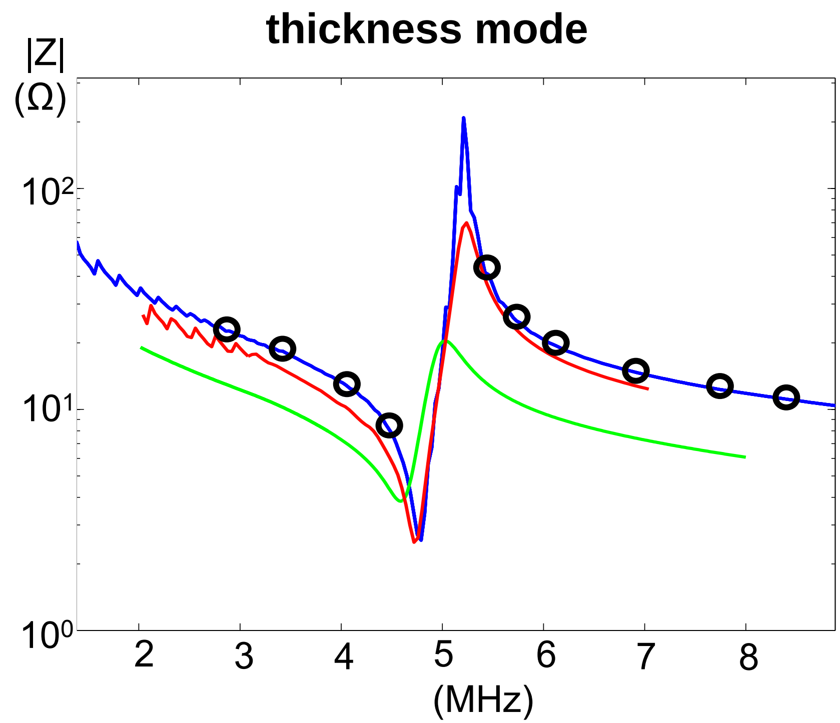
Results (1)



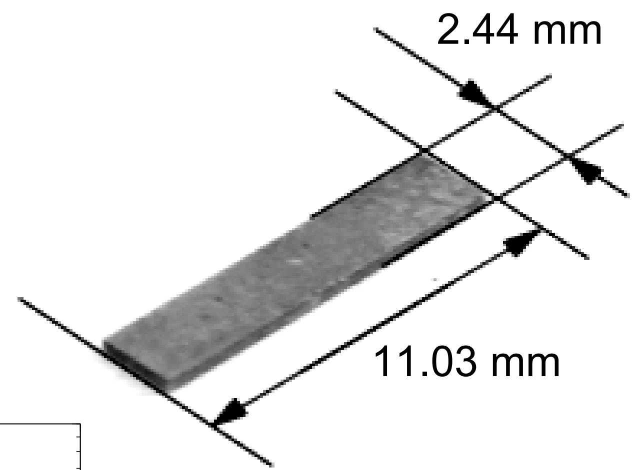
PZT disc – fitted impedance curves





- measurement (blue line)
- forward simulation with fitted material parameters (red line)
- forward simulation with IEEE material parameters (green line)
- data for the vector of measurements $\hat{y}$ (circles)

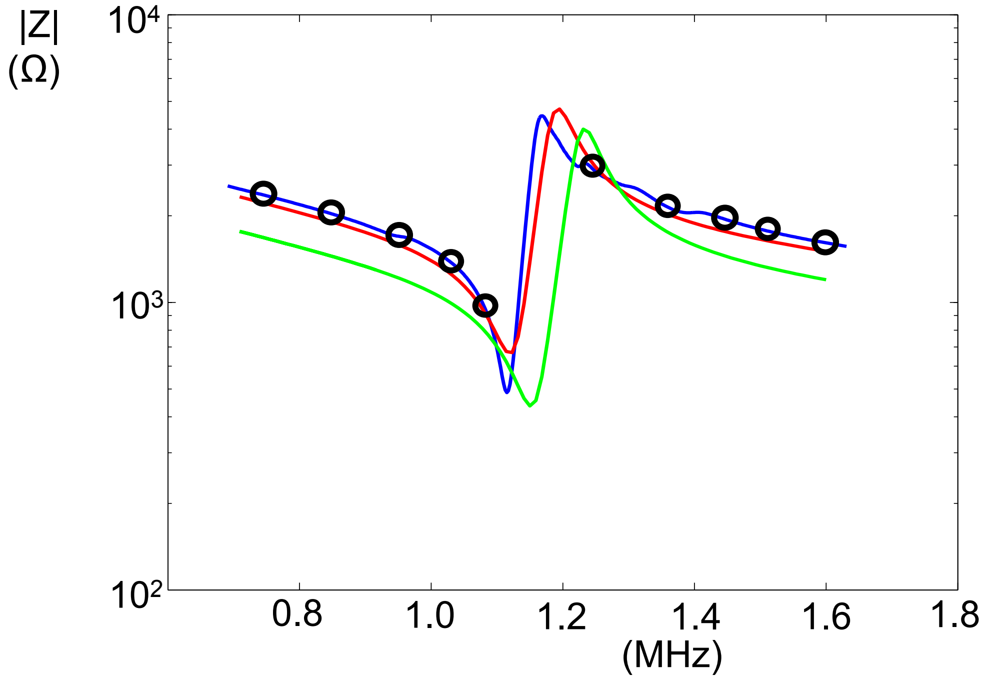
Results (2)



thickness: 0.43 mm

PZT strip – fitted impedance curves

transversal mode



- measurement
- forward simulation with fitted material parameters
- forward simulation with IEEE material parameters
- data for the vector of measurements $\hat{y}$

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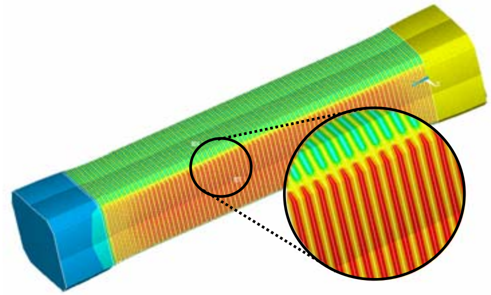
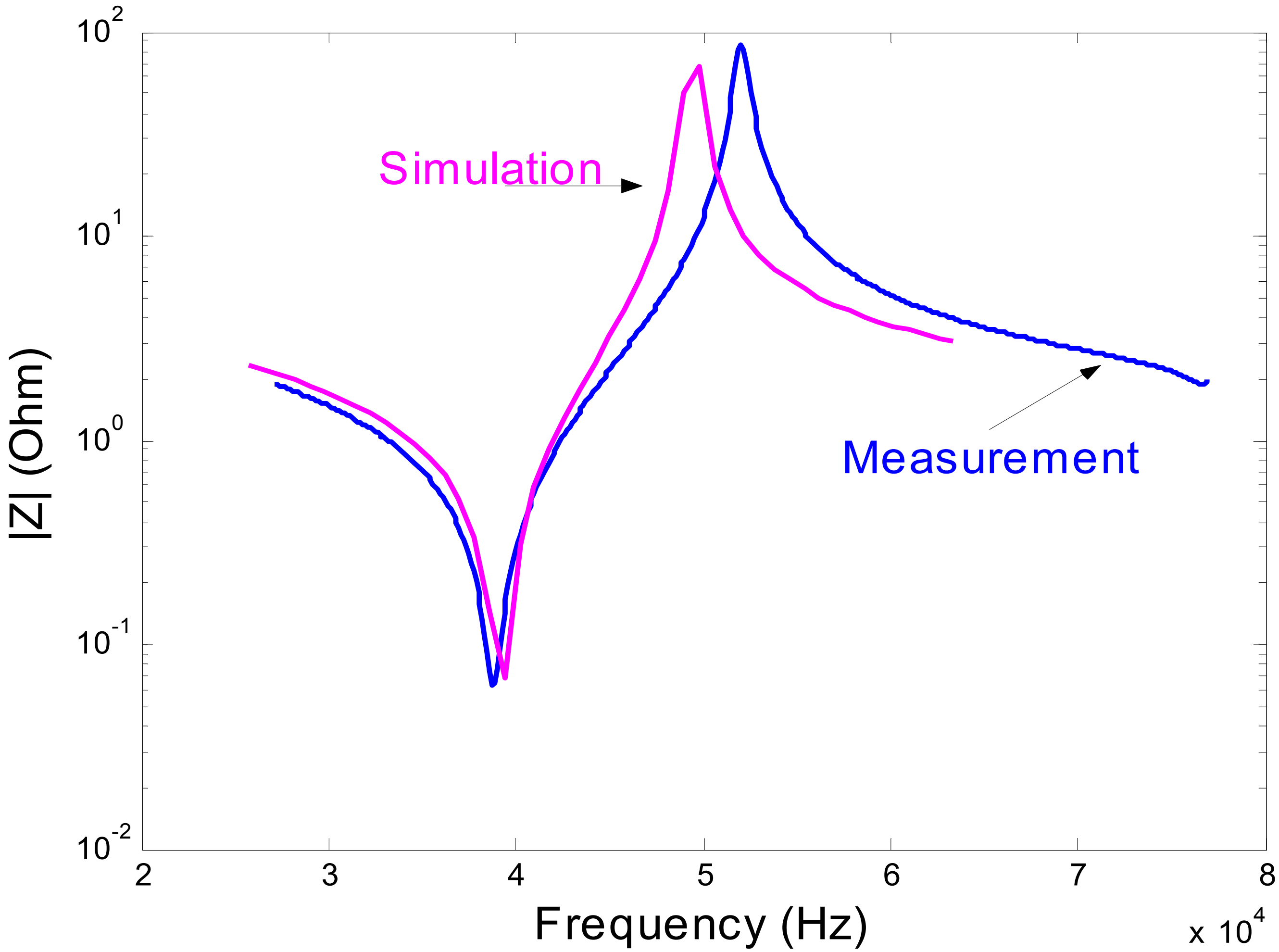
Results (3)



Stack actuator (based on results of raw material)



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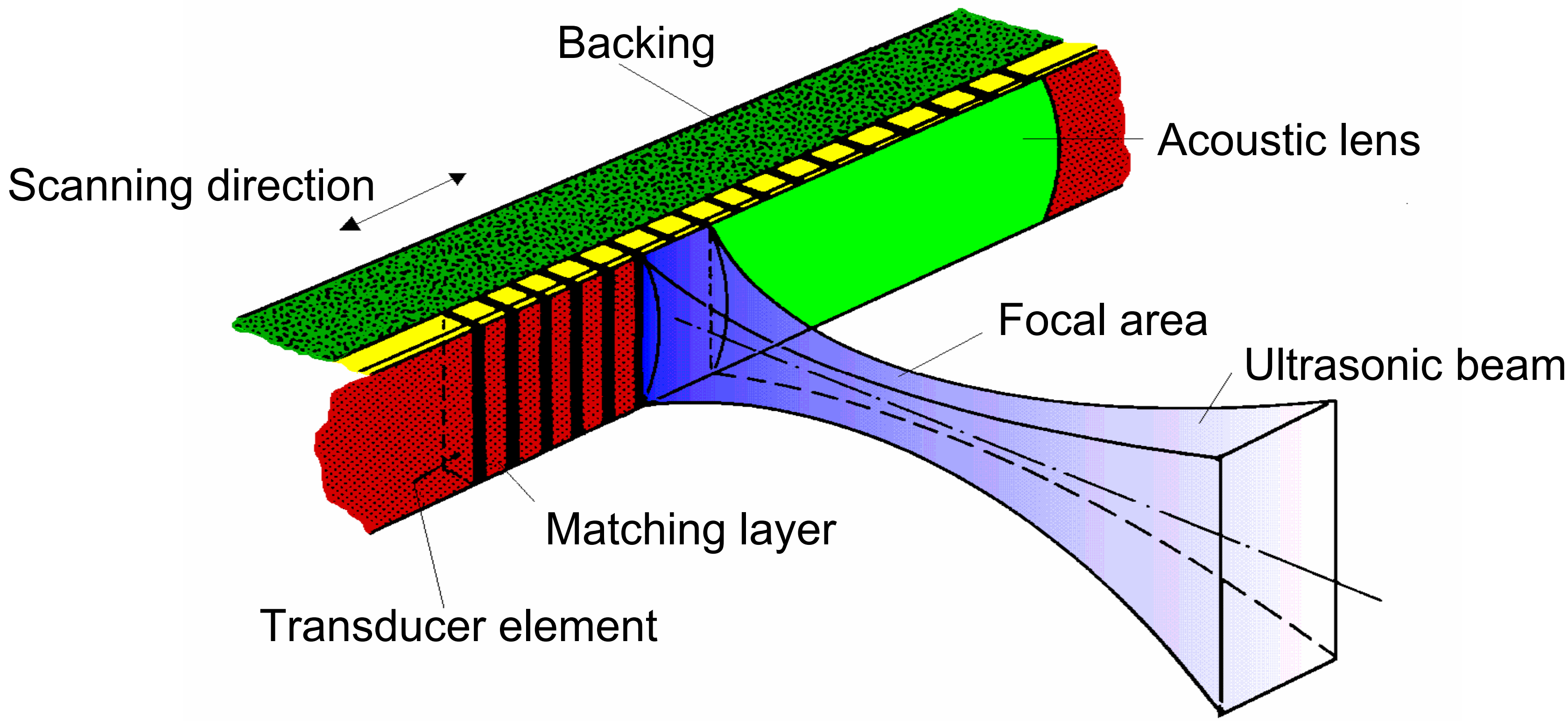
Applications

- Medical diagnostic, nondestructive testing



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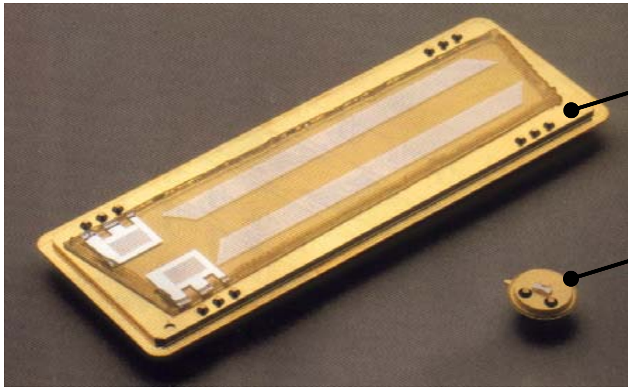
Applications

- Surface acoustic wave devices (mobile phones, television)



Puls compression filter (43 MHz)

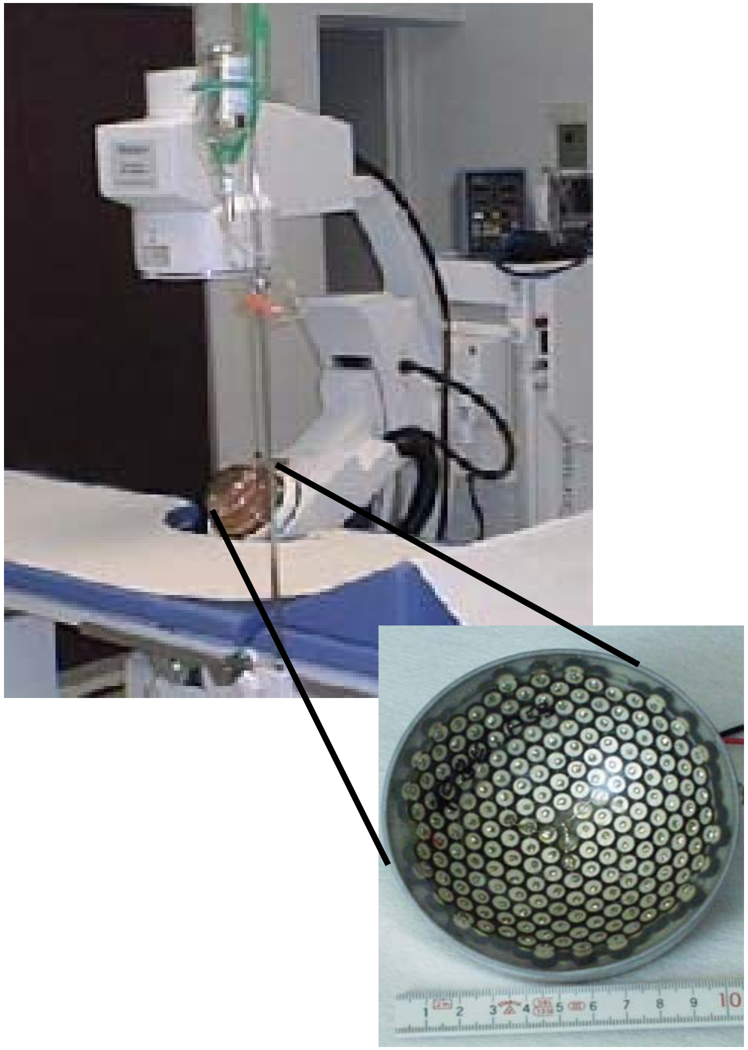
Resonator (1 GHz)

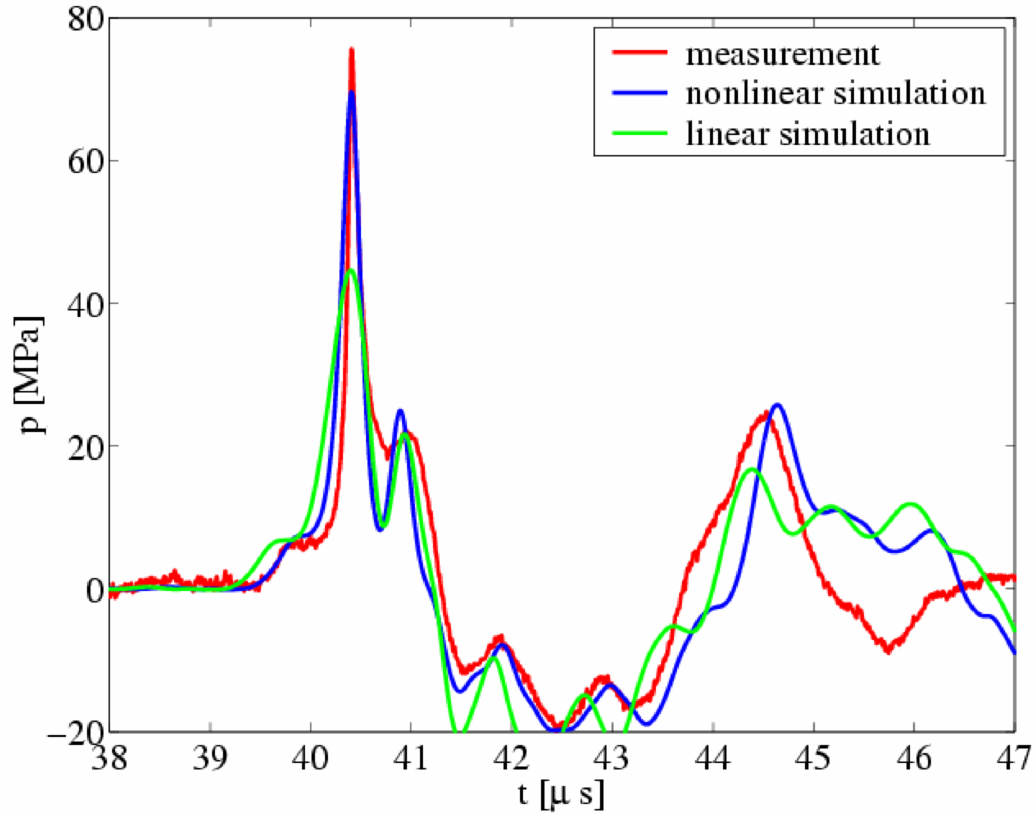
Origin: Siemens

Sensor for wheel pressure

Applications

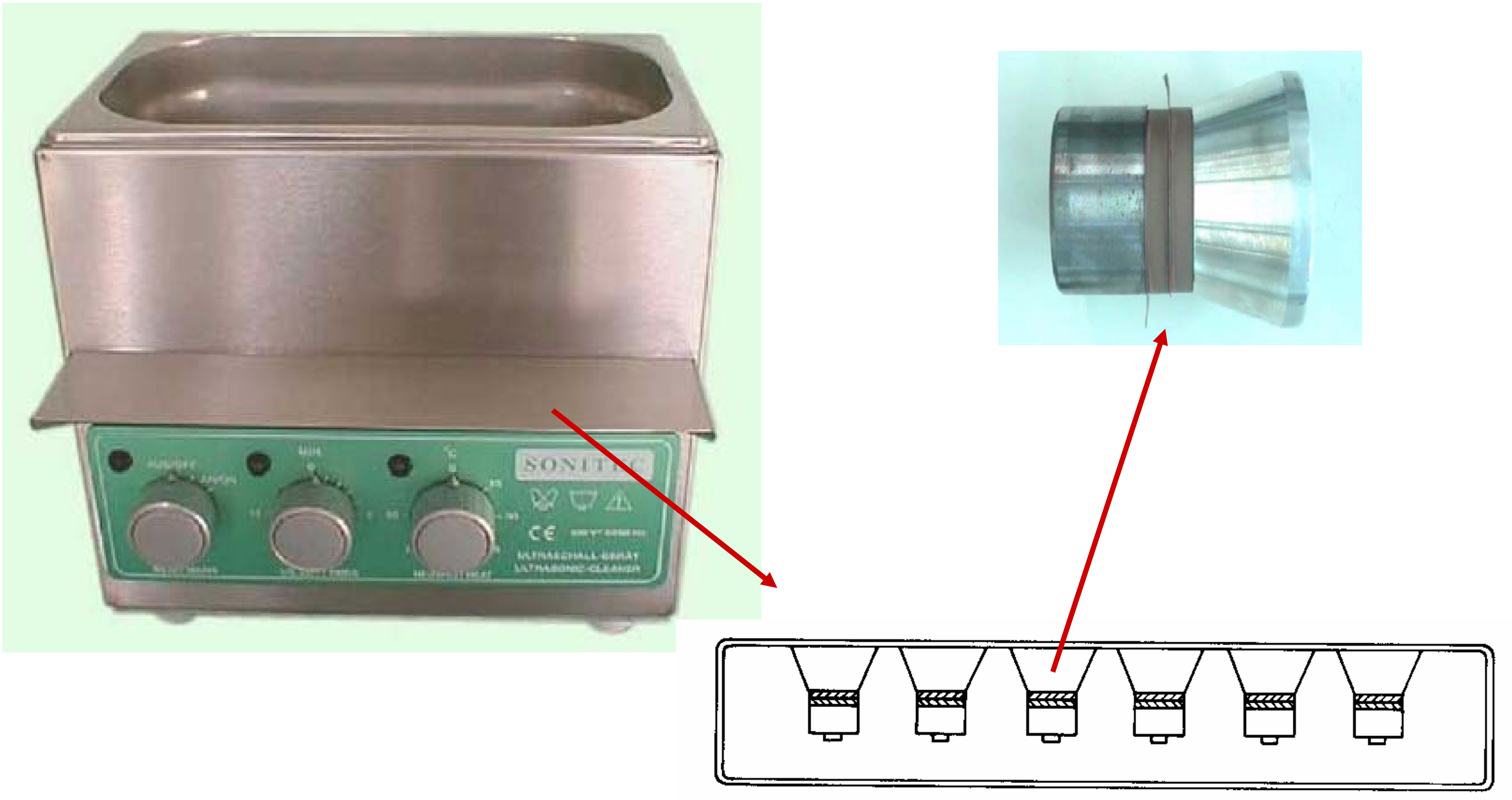
- High intensity ultrasound: e.g. lithotripsy





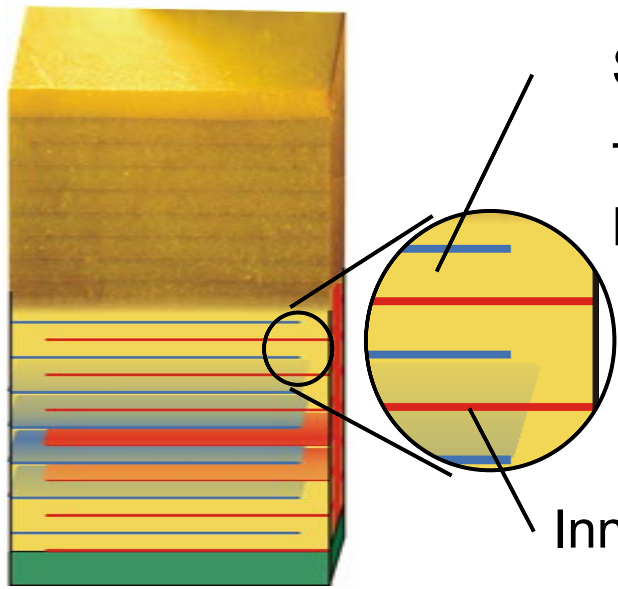
Applications

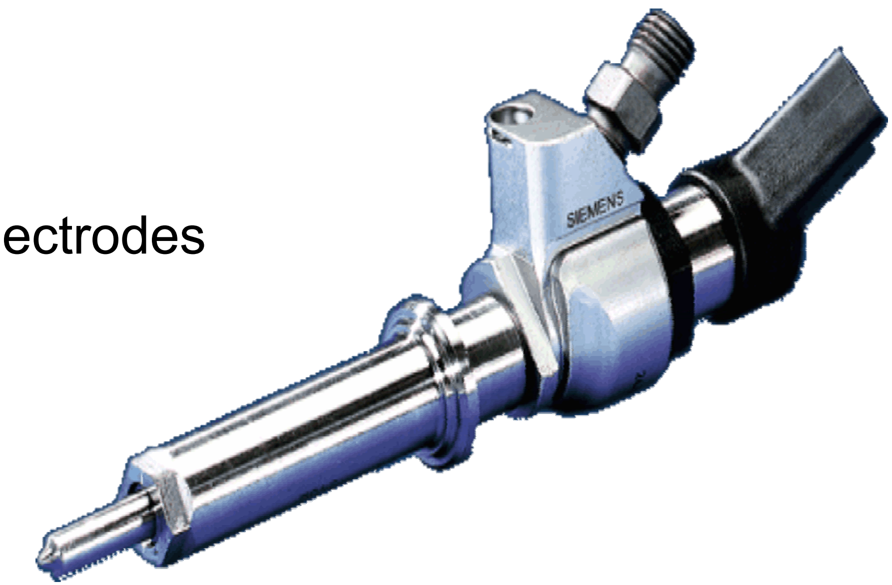
- High intensity ultrasound: e.g. cleaning



Applications

- Stack actuators (common-rail injection systems)





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Thank you for your attention

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