Dual-mixed *hp* finite element models in elasticity

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Properly constructed dual-mixed finite element models outperform the classical displacement-based models for several problems of elasticity and plasticity. Mixed finite elements can especially be advantageous when additional 'hidden' constraints are to be satisfied by the finite element spaces – the incompressibility constraint in elasticity and plasticity problems and the shear and membrane constraints appearing in plate and shell models when the thickness of the plate/shell approaches to zero are well-known examples. A significant advantage of mixed finite elements is that they are expected to be robust for not only p- but h-extension as well, i.e., they can be considered as real *hp* finite elements. In addition, dual-mixed variational formulations offer the possibility of approximating the stress or stress function space directly, resulting in better convergence rates and higher accuracy for the variable of primary interest in many engineering applications.

The talk presents dual-mixed *hp* finite element models for two-dimensional elasticity (membrane) problems and plate bending problems. The dimensional reduction for plates and the finite element formulations are based on Fraeijs de Veubeke's two-field dual-mixed variational principle in terms of non-symmetric stresses and rotations [1]. Local translational equilibrium and stress boundary conditions together with inter-element surface traction continuity are *a priori* satisfied using C^0-continuous approximations for first-order stress functions. Rotational equilibrium is enforced in a weak sense using the rotations as Lagrange multipliers. They are approximated independently on each element, neither continuity nor boundary conditions can be prescribed for them. Unmodified three-dimensional constitutive equations are employed in the dimensionally reduced plate model.

The numerical performance of the new *hp*-version quadrilateral membrane and plate bending elements will be demonstrated through examples. Since the dual-mixed formulation is a complementary energy-based approach, the $1/(1-2\nu)$ term, with ν being the Poisson ratio, is not present in the inverse constitutive equations applied (either for plane strain problems or in the three-dimensional case). This fact ensures that the formulation and the elements are free from incompressibility locking when ν is close to 0.5 [2, 3]. It will be shown that the rates of convergence in the energy norm and stress computations are practically independent of not only the Poisson ratio, but also of the thickness of the plate, i.e., the formulation and the plate element are free from shear locking as well [4]. It will be numerically justified that the locking-free properties mentioned exist independently of the polynomial degree of the elements, i.e., the stress computation is robust, reliable and accurate for not only higher order p-, but also for low order h-type approximations, even for extremely thin plates.

Acknowledgment. This work was supported in part by the Hungarian Scientific Research Fund through Grants No. OTKA T34358 and OTKA T49427.

References

[1] B. M. Fraeijs de Veubeke. A new variational principle for finite elastic displacements. *International Journal for Engineering Sciences*, 10:745–763, 1972.

[2] E. Bertóti. Dual-mixed *hp* finite element methods using first-order stress functions and rotations. *Computational Mechanics*, 26:39–51, 2000.

[3] E. Bertóti. Dual-mixed *p* and *hp* finite element methods for elastic membrane problems. *International Journal for Numerical Methods in Engineering*, 53:3–29, 2002.

[4] E. Bertóti. Stress-based and locking-free *hp* finite elements for thin elastic plates. In H. A. Mang, F. G. Rammerstorfer, and J. Eberhardsteiner, editors, *Proceedings of the Fifth World Congress on Computational Mechanics*, pages 1–10, Vienna, Austria, July 7-12, 2002. Vienna University of Technology, Austria.