Phase-Field Relaxation of Topology Optimization with Local Stress Constraints with an Optimal KKT-Solver

Martin Burger * Roman Stainko †

October 31, 2005

Abstract

We introduce a new relaxation scheme for structural topology optimization problems with local stress constraints based on a phase-field method. The starting point of the relaxation is a reformulation of the material problem involving linear and 0–1 constraints only. The 0–1 constraints are then relaxed and approximated by a Cahn-Hilliard type penalty in the objective functional, which yields convergence of minimizers to 0–1 designs as the penalty parameter decreases to zero. A major advantage of this kind of relaxation opposed to standard approaches is a uniform constraint qualification that is satisfied for any positive value of the penalization parameter.

The relaxation scheme yields a large-scale optimization problem with a high number of linear inequality constraints. We discretize the problem by finite elements and solve the arising finite-dimensional programming problems by a primal-dual interior- point method. Numerical experiments for problems with stress constraints based on different criteria indicate the success and robustness of the new approach.

For a potential speed-up we consider the interior-point related KKT system. Here we construct an optimal solver by using a multi-grid approach with a multiplicative Schwarz-type smoother.

*Institut für Industriemathematik, Johannes Kepler Universität, Altenbergerstr. 69, A 4040 Linz, Austria. e-mail: martin.burger@jku.at.

†Spezialforschungsbereich SFB F 013 Numerical and Symbolic Scientific Computing. Altenbergerstr. 69, A 4040 Linz, Austria. e-mail: roman.stainko@jku.at.