ANALYSIS OF A FINITE PML APPROXIMATION FOR THE THREE DIMENSIONAL TIME-HARMONIC MAXWELL AND ACOUSTIC SCATTERING PROBLEMS

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Abstract. We consider the approximation of the frequency domain three dimensional Maxwell scattering problem using a truncated domain perfectly matched layer (PML) (cf. [1] and [2]). We also treat the time-harmonic PML approximation to the acoustic scattering problem. Following [3], a transitional layer based on spherical geometry is defined which results in a constant coefficient problem outside the transition. A truncated (computational) domain is then defined which covers the transition region. The truncated domain need only have a minimally smooth outer boundary (e.g., Lipschitz continuous). We consider the truncated PML problem which results when a perfectly conducting boundary condition is imposed on the outer boundary of the truncated domain. The existence and uniqueness of solutions to the truncated PML problem will be shown provided that the truncated domain is sufficiently large, e.g., contains a sphere of radius R_t. We also show exponential (in the parameter R_t) convergence of the truncated PML solution to the solution of the original scattering problem inside the transition layer.

Our results are important in that they are the first which show that the truncated PML problem can be posed on a domain with non-smooth outer boundary. This allows the use of approximation based on polygonal meshes. In addition, even though the transition coefficients depend on spherical geometry, they can be made arbitrarily smooth and hence the resulting problems are amenable to numerical quadrature. Approximation schemes based on our analysis are the focus of future research.

References

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This work was supported in part by the National Science Foundation through grant No. 0311902.