

## Quaternions in Kinematics

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**Abstract:** Quaternions are generally known as a substitute for roll, pitch and yaw angles of a rigid body that avoids gimbal lock, the singularity associated with alignment of the roll and yaw axes. What is perhaps less known is the algebra of quaternions encodes the geometry of a spherical triangle, specifically the triangle formed by the rotation axes of two rotation matrices and their product. This geometric relationship extends to spatial displacements where dual quaternions encode the geometry of a spatial triangle. This relationship can be made explicit using Clifford Algebra exponentials yielding a convenient formulation for the Kinematic Synthesis of robotic systems. It also opens the door to double quaternions, also called biquaternions, which provide a bi-invariant distance between spatial displacements, with an error that depends on the size of the hypersphere selected to approximate three dimensional space. Furthermore, double quaternions provide a Bezier interpolation of spatial displacements that mirrors the well-known Bezier interpolation of rotations using quaternions.