An attack on the new variations of flexible closed polyhedral surfaces

Speaker: Zeyuan He

Abstract: The history of flexible closed polyhedral surfaces is familiar to our community. Inspired by the self-intersected octahedral surfaces (Bricard, 1897), Connelly (1977) constructed the first example of flexible closed polyhedral surface using special geometric insights. On top of this initial investigation, Steffen (1978) gave a simple, and the most famous variation, which is called the Steffen's polyhedron (see https://en.wikipedia.org/wiki/Steffen's_polyhedron). Subsequently, numerous studies have explored general principles governing these structures. In particular, any continuous change in the configuration of a polyhedral surface must preserve the mean curvature integral (Alexander, 1985; Almgren and Rivin, 1998); volume (the Bellows Theorem; Sabitov, 1995) and Dehn invariant (Gaifullin and Ignashchenko; 2018). There must be a cycle of edges whose lengths sum up to zero once suitably weighted by 1 and -1 (Gallet et al, 2021; 2022).

From an engineering perspective, I interpret a flexible closed polyhedral surface as a more constrained origami structure, which sparks my curiosity about the new variations and its form-finding ability. In this talk I will provide an overview of the existing variations; elucidate their geometric properties and flexibility; and report some new variations. All demonstrations will be visually engaging and accessible to audiences across diverse disciplines. I will show that a key challenge lies in eliminating self-intersections, underscoring the need for further theoretical advancements.