

# Spectral rigidity and neural lens rigidity of terrestrial planets

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**Abstract:** We consider the determination of the interior of terrestrial planets from a geometrical viewpoint. First, we present spectral rigidity for spherically symmetric manifolds with boundary and interior interfaces determined by discontinuities in the metric under certain conditions. Rather than a single metric, we allow two distinct metrics in between the interfaces enabling the consideration of two wave types, like P- and S-polarized waves in isotropic elastic solids. Terrestrial planets in our solar system are approximately spherically symmetric and support toroidal and spheroidal modes. Discontinuities typically correspond with phase transitions in their interiors. Our rigidity result applies to such planets as we ensure that our conditions are satisfied in generally accepted models in the presence of a fluid outer core. The method of proof relies on a trace formula, relating the spectrum of the manifold with boundary to its length spectrum. The geodesics in such a manifold get reflected and transmitted when they hit an interface, creating a complex geometry for the analysis. In addition, we allow such geodesics to hit an interface at certain critical angles where a scattered ray can graze or “glide” along an interface. We allow so-called anti-Herglotz jumps in the wave speeds and trapped rays that never interact with the boundary of the manifold. Second, we study the recovery of an isotropic metric of a “planetary” manifold from lens data. We address the question what can be recovered even without uniqueness. We present an approach following an implicit neural representation where the metric is parametrized by a deep neural network, thus introducing a deep manifold, that admits exact spatial derivatives needed to compute geodesics.

Research in collaboration with I. Dokmanić, J. Ilmavirta, V. Katsnelson, K. Kothari, H. Nguyen and G. Uhlmann.