Weyl asymptotics for the eigenvalues of dissipative operators and application to scattering

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Abstract. We study the eigenvalues of the generator of a contraction semigroup related to boundary problems for the wave equation with dissipative boundary conditions. These eigenvalues λ_j have negative real part and the corresponding solutions $u(t, x) = e^{\lambda_j t} f_j(x)$ have exponentially decreasing global energy as $t \to +\infty$. The existence of such eigenvalues is important for the scattering theory (the wave operators are not complete). We show that these eigenvalues are localised in a small neighbourhood of the negative real axis or in a set with bounded real part. For the eigenvalues close to the negative real axis we obtain a Weyl formula when the dissipation $\gamma(x)$ is such that $\gamma(x) > 1$. For strictly convex obstacles this Weyl formula describes the distribution of all eigenvalues. Moreover, for Maxwell's equations with dissipative boundary conditions we obtain Weyl formula for the eigenvalues close to the negative real axis under the assumption $\gamma(x) \neq 1$.