

Microlocal analysis of internal waves in 2D aquaria

Speaker: Semyon Dyatlov, MIT

Abstract: For a bounded smooth planar domain Ω , we study the forced evolution problem for the 4th order PDE

$$(\partial_t^2 \Delta + \partial_{x_2}^2)u(t, x) = f(x) \cos(\lambda t), \quad t \geq 0, \quad x \in \Omega \quad (1)$$

with homogeneous initial conditions and Dirichlet boundary conditions on $\partial\Omega$. This is motivated by concentration of fluid velocity on attractors for stratified fluids in effectively 2-dimensional aquaria, first observed experimentally in 1997.

The behavior of solutions to (1) is intimately tied to the *chess billiard* map on the boundary $\partial\Omega$, which depends on the forcing frequency λ . Under the natural assumption that the chess billiard b has the Morse–Smale property, we show that as $t \rightarrow \infty$ the singular part of the solution u concentrates on the attractive cycle of b . The proof combines various tools from microlocal analysis, scattering theory, and hyperbolic dynamics. Joint work with Jian Wang and Maciej Zworski.