On the origin of the Minnaert resonances

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It is well known that the presence, in a homogeneous acoustic medium, of a small inhomogeneity (of size ε) enjoying a high contrast of both its mass density and bulk modulus, may amplifiy the generated total fields. This amplification has been observed and quantified in the stationary regime, where it is more pronounced when the incident frequency is close to a specific value, depending on the capacitance of the perturbation and referred to as the *Minnaert frequency* ω_M. Such a resonant regime has tremendous applications in imaging, in the broad sense, and material sciences, to cite a few.

Here we explain the origin of this phenomenon: at first we show that the scattering of an incident wave of frequency ω is described by a self-adjoint ω-dependent Schrödinger operator with a singular potential supported at the inhomogeneity interface. Then, we consider its norm-resolvent limit in the low-frequency regime (corresponding in our setting to $\varepsilon \ll 1$). We prove that this limit is non-trivial – i.e., the frequency-dependent operator asymptotically differs from the Laplacian – if and only if $\omega = \omega_M$.

The limit operator describing the non-trivial scattering process is explicitly determined and belongs to the class of point perturbations of the Laplacian. When the frequency of the incident wave approaches ω_M, the scattering process undergoes a transition between an asymptotically trivial behaviour and a non-trivial one. At each frequency ω, we provide an explicit asymptotic expansion of the scattered field as $\varepsilon \to 0$. The resolvent-approach naturally yields a uniform-in-space control of the error terms, improving in this sense previous results.

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