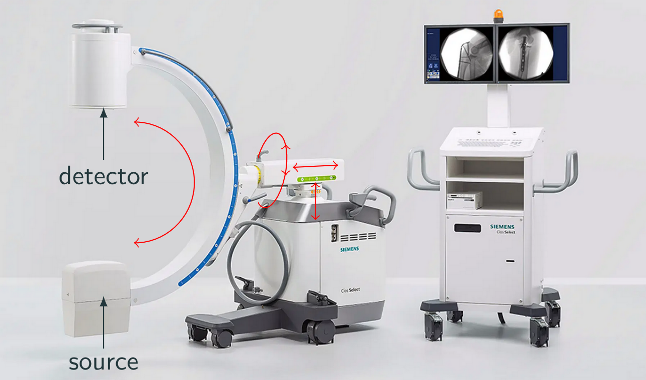
X-ray computed tomography with transverse truncation: focus on the DBP method

Nicolas Gindrier

Tomography Across the Scales, RICAM, Linz
21st October 2022

Scanner C-Arm

Context: medical X-ray tomography, image reconstruction

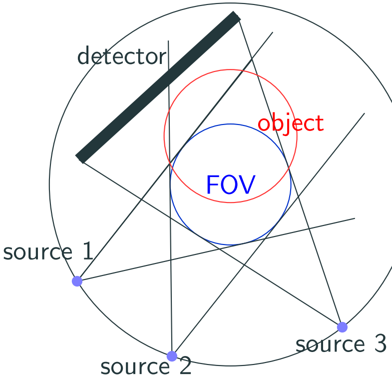


[Siemens image]

Motivation

In medical tomography, having reduced FOV can arise due to material and dose reduction issues.

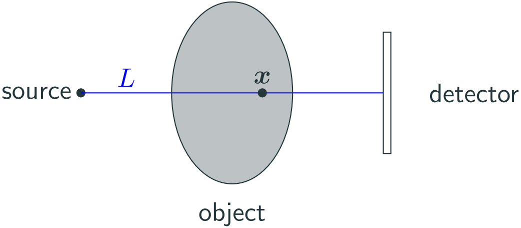
For a given (reduced) FOV, is it possible to have accurate reconstructions?

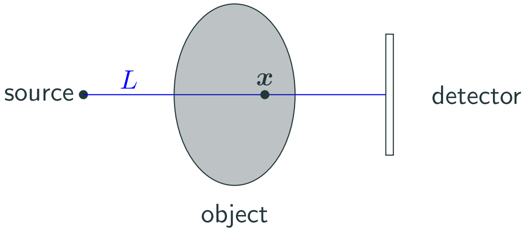


Plan

Radon transform

1.1 State of the art: 2D





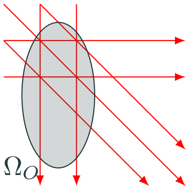
Radon transform:

$$\mathcal{R}f(L) = \int_{\boldsymbol{x} \in L} f(\boldsymbol{x}) \mathrm{d}L$$

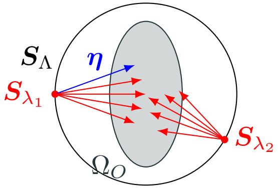
f: density function

Geometries

Parallel geometry



Fan-beam geometry (2D) or *cone-beam* (3D) for a circular trajectory



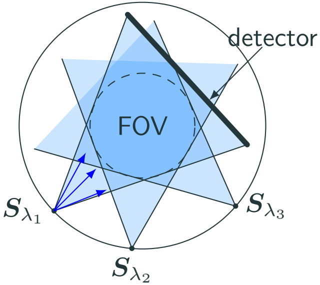
fan-beam/*cone-beam* projection
$g(\boldsymbol{S}_\lambda, \boldsymbol{\eta}) = \int_0^{+\infty} f(\boldsymbol{S}_\lambda + s\boldsymbol{\eta})\mathrm{d}s$

Ω_O: object (support)

$\boldsymbol{S}_\Lambda$: X-rays source *trajectory*

FOV

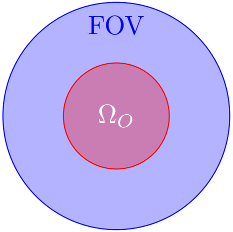
FOV in *fan-beam*/*cone-beam* geometry



In the whole 2D part, we consider a *complete circular trajectory*.

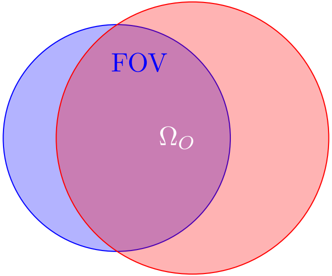
FOV and truncation

$\Omega_O \subset \mathrm{FOV}$



No truncation

$\Omega_O \not\subset \mathrm{FOV}$



Truncation

We are interested in reconstructions *with truncation*.

Types of reconstruction method

In tomography, especially CT, some methods exist to reconstruct:

- Analytical methods (FBP, DBP, etc)
- Iterative methods:
    - Algebraic methods (least squares, etc)
    - Statistical methods (ML-EM, etc)

Types of reconstruction method

In tomography, especially CT, some methods exist to reconstruct:

- Analytical methods (FBP, DBP, etc)
- Iterative methods:
  - Algebraic methods (least squares, etc)
  - Statistical methods (ML-EM, etc)

In X-rays tomography, deep learning methods can be used for some steps or for post processing, but not (yet) for the whole reconstruction.

We chose to investigate **analytical methods**, because we want to have **exact and stable reconstruction**. We want to determine **sufficient conditions of reconstruction**.

Reconstruction: FBP method

FBP (Filtered BackProjection) (1970s). For *fan-beam* geometry:

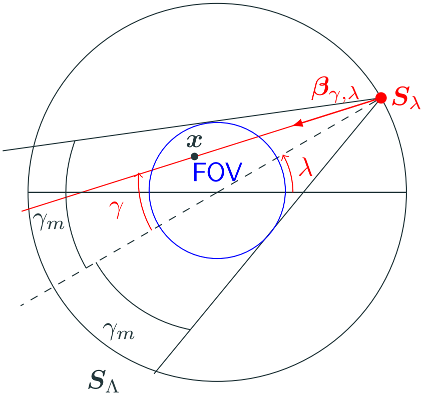
$$f(\boldsymbol{x}) = \frac{1}{2}\int_0^{2\pi} \frac{1}{\|\boldsymbol{x}-\boldsymbol{S}_\lambda\|^2}\int_{-\gamma_m}^{\gamma_m} g(\lambda, \boldsymbol{\beta}_{\gamma',\lambda})\widetilde{k_R}(\gamma-\gamma')R\cos\gamma'\,\mathrm{d}\gamma'\,\mathrm{d}\lambda$$

$$\gamma = \arg(\boldsymbol{S}_\lambda - \boldsymbol{x}) - \lambda$$

$$\widetilde{k_R}(\gamma) \stackrel{\text{def}}{=} \left(\frac{\gamma}{\sin\gamma}\right)^2 k_R(\gamma)$$

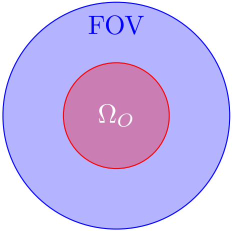
k_R ramp filter: $\mathcal{F}(k_R)(\rho) = |\rho|$

If FBP was local, we would only need the lines passing through $\boldsymbol{x}$.



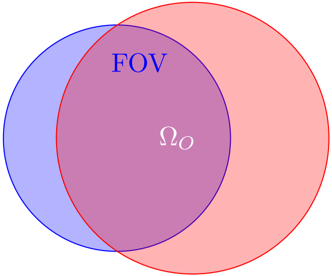
FBP and truncation

$\Omega_O \subset \mathrm{FOV}$



No truncation: we can apply FBP

$\Omega_O \not\subset \mathrm{FOV}$



Truncation: we cannot apply FBP

Introduction to DBP

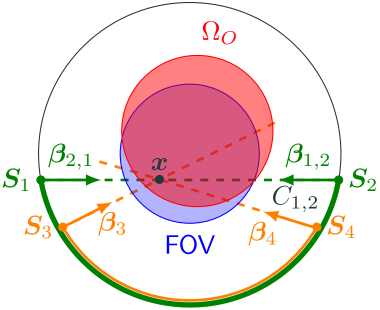
DBP: Differentiated Backprojection

- derivative instead of ramp filter
- but needs post processing of Hilbert transform

DBP: Differentiated Backprojection

- derivative instead of ramp filter
- but needs post processing of Hilbert transform

A *chord* $C_{i,j}$ is a line segment linking $\boldsymbol{S}_{\lambda_i}$ to $\boldsymbol{S}_{\lambda_j}$.



The DBP method allows the reconstruction of $\boldsymbol{x} \in C_{1,2}$.

Tools for DBP (Differentiated Backprojection, 2000s)

- (Directional) Hilbert transform:

$$\mathcal{H}_{\boldsymbol{\eta}} f(\boldsymbol{x}) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} \frac{f(\boldsymbol{x} - s\boldsymbol{\eta})}{\pi s} \mathrm{d}s$$

- (Directional) Hilbert transform:

$$\mathcal{H}_{\boldsymbol{\eta}} f(\boldsymbol{x}) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} \frac{f(\boldsymbol{x} - s\boldsymbol{\eta})}{\pi s} \mathrm{d}s$$

- Backprojection of the derivatives of projections (along the trajectory):

$$b_{i,j}(\boldsymbol{x}) \stackrel{\text{def}}{=} \int_{\lambda_i}^{\lambda_j} \frac{1}{\|\boldsymbol{x} - \boldsymbol{S}_\lambda\|} \left. \frac{\partial}{\partial \lambda} g(\lambda, \boldsymbol{\beta}) \right|_{\boldsymbol{\beta} = \frac{\boldsymbol{x} - \boldsymbol{S}_\lambda}{\|\boldsymbol{x} - \boldsymbol{S}_\lambda\|}} \mathrm{d}\lambda$$

Tools for DBP (Differentiated Backprojection, 2000s)

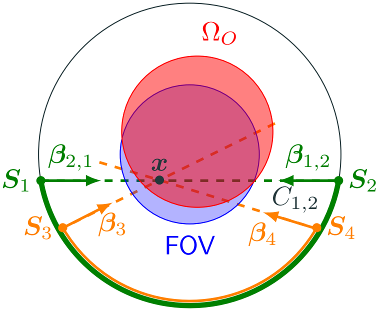
- (Directional) Hilbert transform:

$$\mathcal{H}_{\boldsymbol{\eta}} f(\boldsymbol{x}) \stackrel{\text{def}}{=} \int_{-\infty}^{+\infty} \frac{f(\boldsymbol{x} - s\boldsymbol{\eta})}{\pi s} \mathrm{d}s$$

- Backprojection of the derivatives of projections (along the trajectory):
$$b_{i,j}(\boldsymbol{x}) \stackrel{\text{def}}{=} \int_{\lambda_i}^{\lambda_j} \frac{1}{\|\boldsymbol{x} - \boldsymbol{S}_\lambda\|} \left. \frac{\partial}{\partial \lambda} g(\lambda, \boldsymbol{\beta}) \right|_{\boldsymbol{\beta} = \frac{\boldsymbol{x} - \boldsymbol{S}_\lambda}{\|\boldsymbol{x} - \boldsymbol{S}_\lambda\|}} \mathrm{d}\lambda$$
- Link between b and $\mathcal{H}$: $b_{i,j}(\boldsymbol{x}) = \pi(\mathcal{H}_{\boldsymbol{\beta}_j} f(\boldsymbol{x}) - \mathcal{H}_{\boldsymbol{\beta}_i} f(\boldsymbol{x}))$

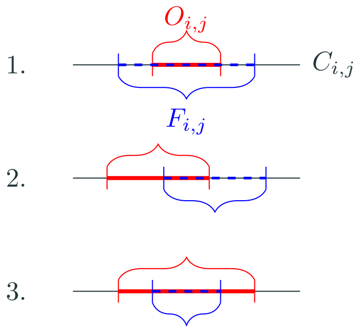
For $\boldsymbol{x}$ on a chord $C_{1,2}$, the equation $b_{i,j}(\boldsymbol{x}) = \pi(\mathcal{H}_{\boldsymbol{\beta}_j} f(\boldsymbol{x}) - \mathcal{H}_{\boldsymbol{\beta}_i} f(\boldsymbol{x}))$ becomes:

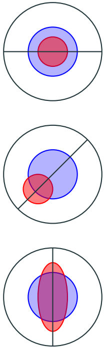
$$b_{1,2}(\boldsymbol{x}) = 2\pi\mathcal{H}_{\boldsymbol{\beta}_{1,2}} f(\boldsymbol{x})$$



Now we want to invert $\mathcal{H}f(\boldsymbol{x})$

Inversion of the Hilbert transform

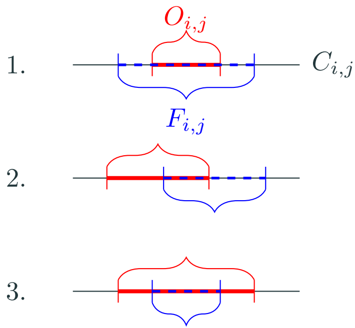


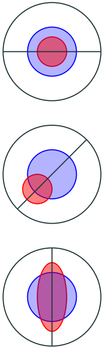


[1]F. Noo, R. Clackdoyle, and J. D. Pack. A two-step Hilbert transform method for 2D image reconstruction. Physics in Medicine and Biology, 49(17) :3903–3923, 2004

[2]M. Defrise, F. Noo, R. Clackdoyle, and H. Kudo. Truncated Hilbert transform and image reconstruction from limited tomographic data, 2006

Inversion of the Hilbert transform



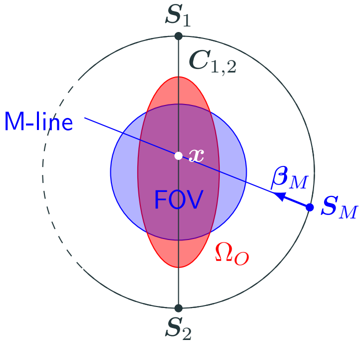


1. analytical inversion of the Hilbert transform (in the direction of $C_{i,j}$)[1]
2. numerical inversion [2]
3. no possible inversion in the direction of $C_{i,j}$...we must use another method

[1] F. Noo, R. Clackdoyle, and J. D. Pack. A two-step Hilbert transform method for 2D image reconstruction. Physics in Medicine and Biology, 49(17) :3903–3923, 2004

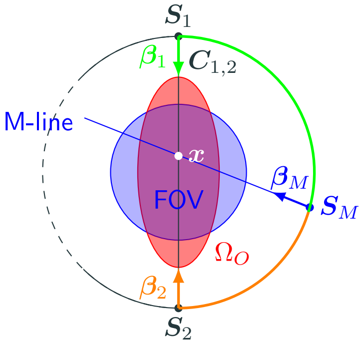
[2] M. Defrise, F. Noo, R. Clackdoyle, and H. Kudo. Truncated Hilbert transform and image reconstruction from limited tomographic data, 2006

M-lines method

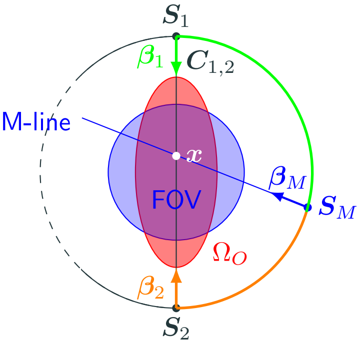


‡J. D. Pack, F. Noo, and R. Clackdoyle. Cone-beam reconstruction using the back- projection of locally filtered projections. IEEE Transactions on Medical Imaging, 24(1) :70–85, 2005

M-lines method



M-lines method

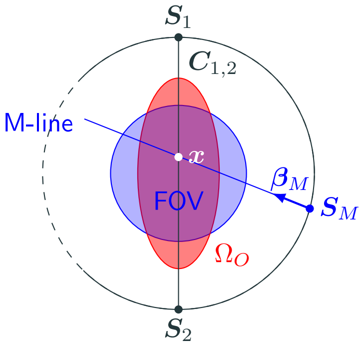


$b_{1,M}(\boldsymbol{x}) = \pi(\mathcal{H}_{\boldsymbol{\beta}_M} f(\boldsymbol{x}) - \mathcal{H}_{\boldsymbol{\beta}_1} f(\boldsymbol{x}))$

$b_{2,M}(\boldsymbol{x}) = \pi(\mathcal{H}_{\boldsymbol{\beta}_M} f(\boldsymbol{x}) - \mathcal{H}_{\boldsymbol{\beta}_2} f(\boldsymbol{x}))$

$\Rightarrow b_{1,M}(\boldsymbol{x}) + b_{2,M}(\boldsymbol{x}) = 2\mathcal{H}_{\boldsymbol{\beta}_M} f(\boldsymbol{x})$

M-lines method

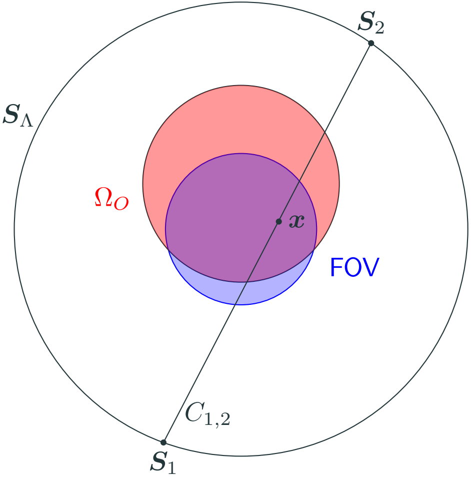


Inversion of the Hilbert transform in the direction of β_M instead of $C_{1,2}$.

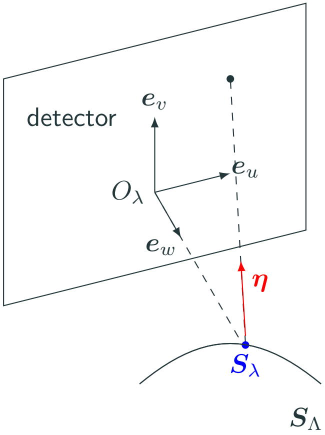
§J. D. Pack, F. Noo, and R. Clackdoyle. Cone-beam reconstruction using the back- projection of locally filtered projections. IEEE Transactions on Medical Imaging, 24(1) :70–85, 2005

Conclusion for 2D DBP

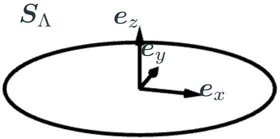
For a circular trajectory, if a part of the FOV is outside Ω_O, then the 2D DBP can always be used.



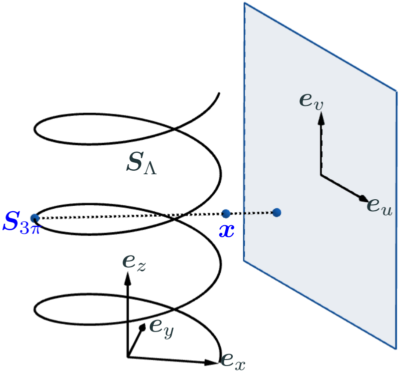
1.3 State of the art: 3D



Cone-beam projections $g(\lambda, \boldsymbol{\eta}) \stackrel{\text{def}}{=} \int_0^{+\infty} f(\boldsymbol{S}_\lambda + l\boldsymbol{\eta})\mathrm{d}l$



Circular trajectory



Helical trajectory

Tuy and Finch results

Tuy condition [5] : *Without truncation*, we have stable reconstruction of an object in the *convex hull* $\Omega_{\boldsymbol{S}_\Lambda}$ of a continuous, simply-connected, and bounded trajectory $\boldsymbol{S}_\Lambda$.

[5] Tuy, Heang K. 1983. "Inversion Formula For Cone-Beam Reconstruction." SIAM Journal on Applied Mathematics 43 (3): 546–552

[6] Finch, David V. 1985. "Cone Beam Reconstruction with Sources on a Curve". SIAM Journal on Applied Mathematics 45 (4): 665–673

Tuy and Finch results

Tuy condition [5] : *Without truncation*, we have stable reconstruction of an object in the *convex hull* $\Omega_{\boldsymbol{S}_\Lambda}$ of a continuous, simply-connected, and bounded trajectory $\boldsymbol{S}_\Lambda$.

Finch result [6]: Tuy condition is necessary (there is no stable reconstruction outside $\Omega_{\boldsymbol{S}_\Lambda}$).

[5] Tuy, Heang K. 1983. "Inversion Formula For Cone-Beam Reconstruction." SIAM Journal on Applied Mathematics 43 (3): 546–552

[6] Finch, David V. 1985. "Cone Beam Reconstruction with Sources on a Curve". SIAM Journal on Applied Mathematics 45 (4): 665–673

Tuy and Finch results

Tuy condition [5] : *Without truncation*, we have stable reconstruction of an object in the *convex hull* $\Omega_{\boldsymbol{S}_\Lambda}$ of a continuous, simply-connected, and bounded trajectory $\boldsymbol{S}_\Lambda$.

Finch result [6]: Tuy condition is necessary (there is no stable reconstruction outside $\Omega_{\boldsymbol{S}_\Lambda}$).

Example: for a circular trajectory, the only exact reconstruction is in the circle plane

[5] Tuy, Heang K. 1983. "Inversion Formula For Cone-Beam Reconstruction." SIAM Journal on Applied Mathematics 43 (3): 546–552

[6] Finch, David V. 1985. "Cone Beam Reconstruction with Sources on a Curve". SIAM Journal on Applied Mathematics 45 (4): 665–673

Closed trajectories with a circular basis and FOV

$$\boldsymbol{S}_\Lambda = \{R\cos\lambda, R\sin\lambda, Z(\lambda) | \lambda \in \Lambda\}$$

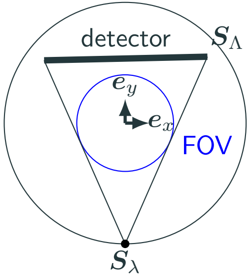
$R > 0$, $Z(\lambda)$ function with period 2π

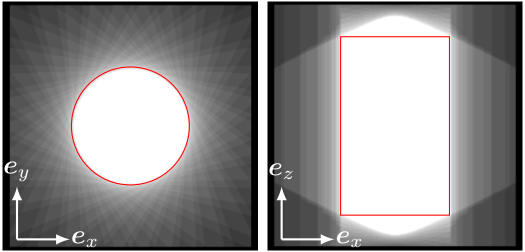
Closed trajectories with a circular basis and FOV

$$\boldsymbol{S}_\Lambda = \{R\cos\lambda, R\sin\lambda, Z(\lambda) | \lambda \in \Lambda\}$$

$R > 0$, $Z(\lambda)$ function with period 2π

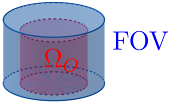
For these trajectories, we can define a cylindrical FOV (with a rigid source-detector assembly).



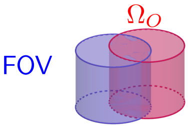


Ω_O is a cylinder

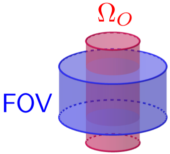
No truncation (detector big enough)



Transverse truncation



Axial truncation

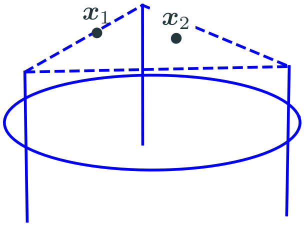


Some methods of cone-beam reconstruction

1. "Historical" methods (1980-90, Tuy, Grangeat/Smith...): no possible truncation
2. Methods of "filtering lines on a detector" (≈2000, Katsevich...): possible axial truncation
3. DBP (≈2000, Pan, Sidky, Noo, Pack...): various possible truncation

Subtlety of the 3D DBP

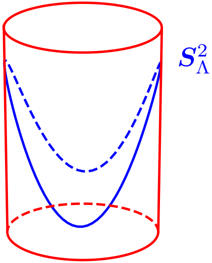
In 3D, a point of the convex envelope of a trajectory, even a closed one, might belong to only one chord, or even none! The 3D DBP is more "subtle" than in 2D

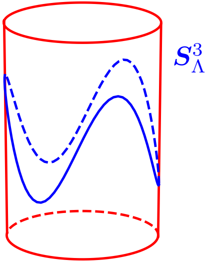


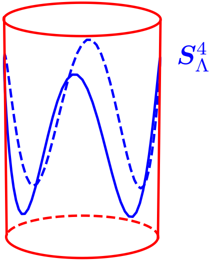
$\boldsymbol{x}_1$ only intersected by one chord, no chord for $\boldsymbol{x}_2$

1.4 State of the art: introduction to n-sin trajectories

$$\boldsymbol{S}_{\Lambda}^{n} = \{(R\cos\lambda, R\sin\lambda, H\cos(n\lambda))^T | \lambda \in \Lambda = [0, 2\pi[\}, n \in \mathbb{N}, n > 1, (H, R) \in \mathbb{R}_{+}^{2} \quad (1)$$





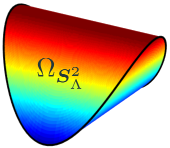


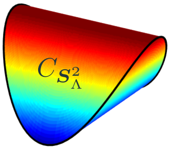
Known results about 2-sin trajectory

To perform reconstruction with DBP, we must know convex hull and location of chords.

$C_{\boldsymbol{S}^n_\Lambda}$: union of chords for the trajectory $\boldsymbol{S}^n_\Lambda$

$C_{\boldsymbol{S}^2_\Lambda} = \Omega_{\boldsymbol{S}^2_\Lambda}$ [7]: the convex hull of the 2-sin trajectory is the set of chords

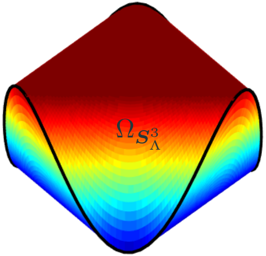


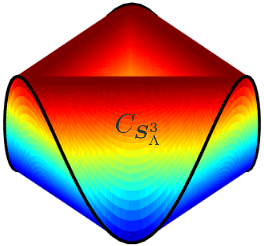


[7] J. D. Pack, F. Noo, and H. Kudo. Investigation of saddle trajectories for cardiac CT imaging in cone-beam geometry, 2004

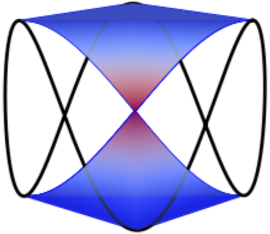
Contributions

Geometrical contributions: 3-sin trajectory

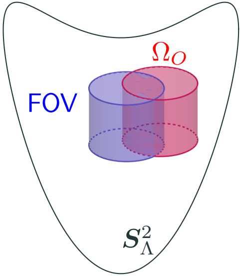




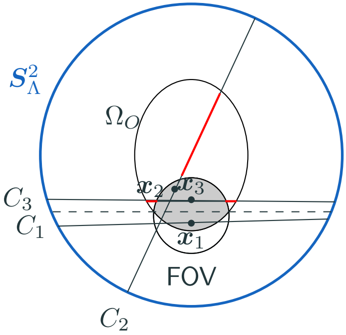
$$N_{\boldsymbol{S}^3_\Lambda} = \Omega_{\boldsymbol{S}^3_\Lambda} \setminus C_{\boldsymbol{S}^3_\Lambda}$$



Configuration: *only transverse* truncation and $\boldsymbol{x} \in \mathrm{FOV} \cap \Omega_{\boldsymbol{S}^2_\Lambda} \cap \Omega_O$



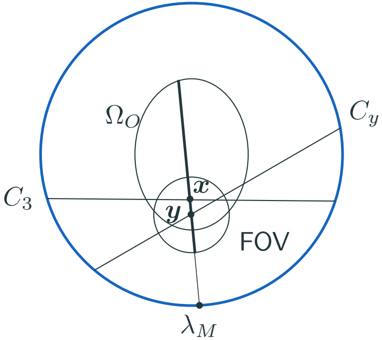
Use of DBP for the 2-sin trajectory

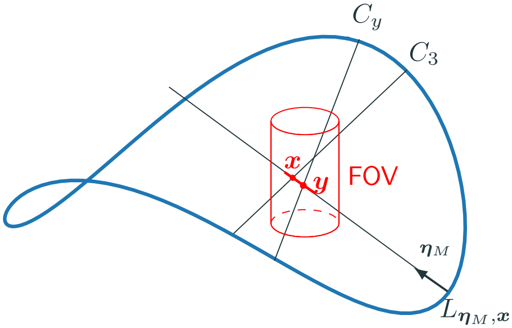


2-sin trajectory from above

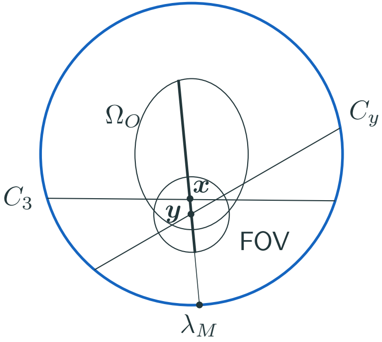
- C_1: possible reconstruction (explicit formula)
- C_2: possible reconstruction (iterative method)
- C_3: impossible reconstruction only with the chord...but we can use the M-lines method

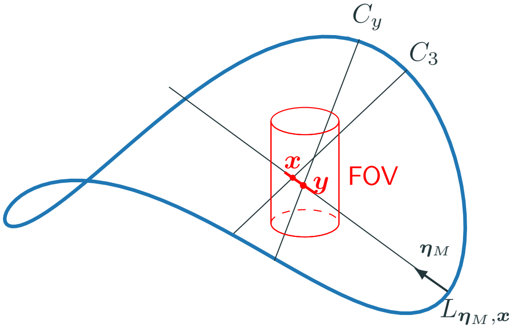
M-lines for 2-sin





M-lines for 2-sin

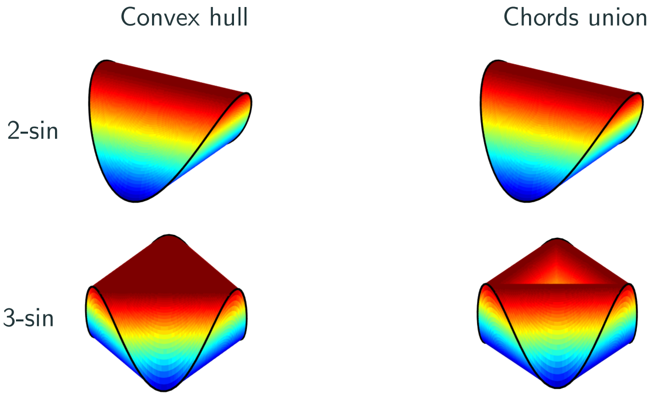




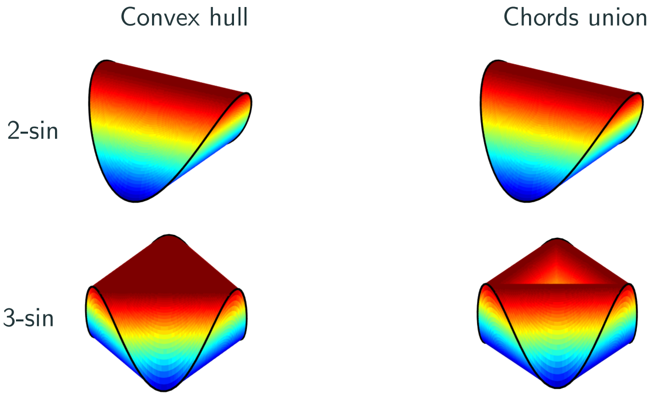
Therefore we can use DBP for the 2-sin trajectory with transverse truncation.

Convex hulls and set of chords

3.2 Tomographic contributions: "good" results for 3-sin

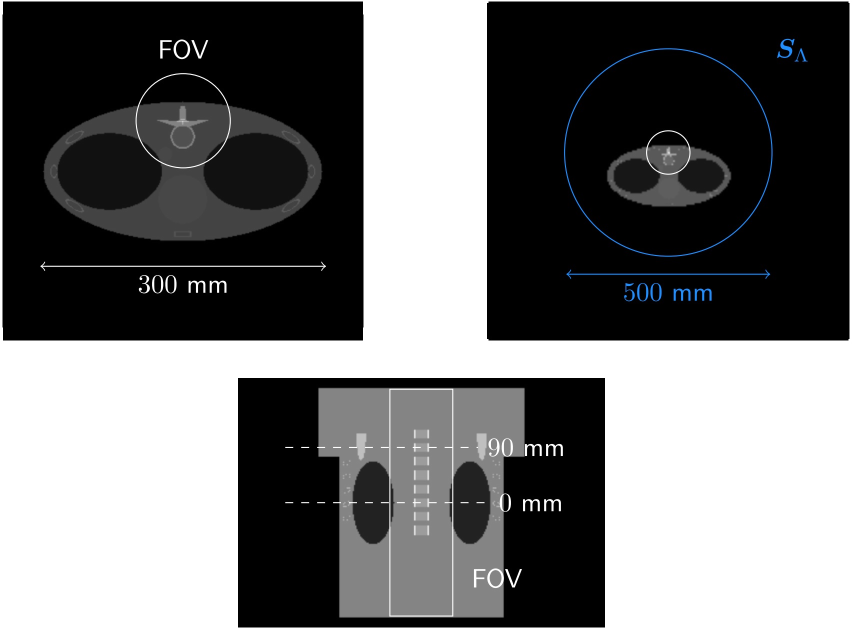


Convex hulls and set of chords

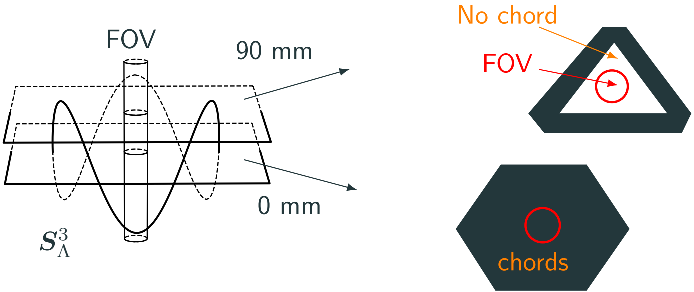


For the 3-sin trajectory some chords are "missing". With a centered FOV it is impossible to use DBP for all points

Simulations: the Forbild thorax phantom



Simulations: both sections



Reconstruction characteristics

We perform an *iterative reconstruction*:

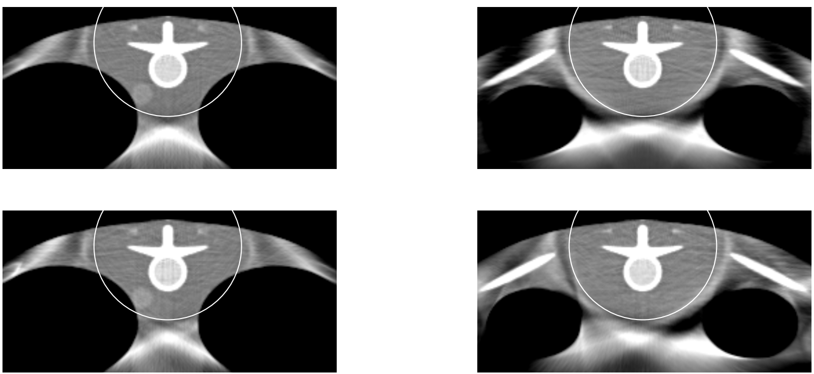
- least squares method with conjugate gradient (minimizing $\|(Rf - p)\|_2^2 + \gamma \|\nabla f\|_2^2$)
- $\gamma = 100$
- image size: 380 x 152 x 382
- 120 iterations
- 200 source positions
- $R = 250$ mm, $H = 100$ mm

Reconstruction results

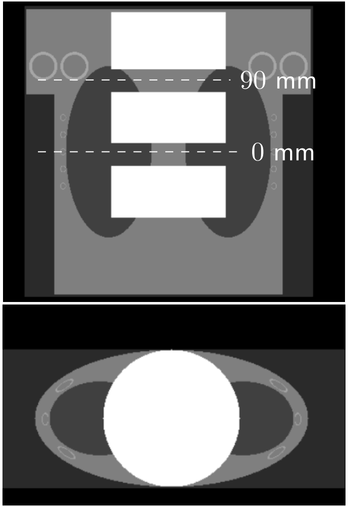
$z = 0$ mm | $z = 90$ mm

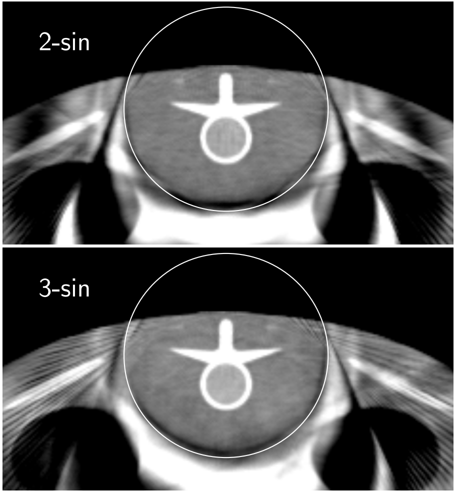
2-sin

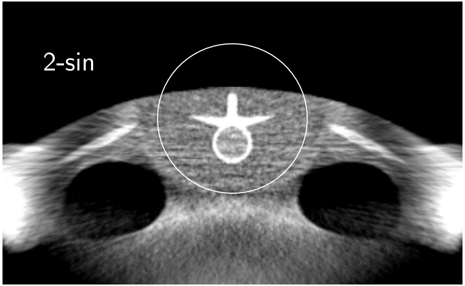
3-sin

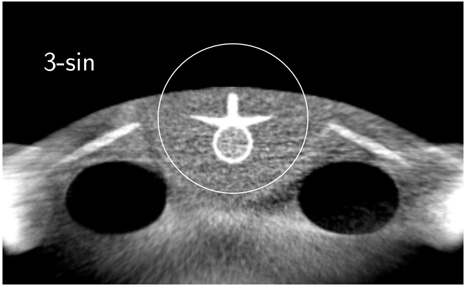


No (apparent) difference between 2-sin and 3-sin!


90 mm
0 mm


2-sin
3-sin





Projections with Poisson noise added

New questions

- Can we consider more truncation for the 2-sin trajectory?
- How can we justify the results for the 3-sin?

New questions

- Can we consider more truncation for the 2-sin trajectory? *Yes, this was presented at MIC 2020: sufficient conditions for a reconstruction with axial AND transverse truncation*[8]
- How to justify the results for the 3-sin? *Partially, a restricted configuration has been presented at Fully3D 2021*[9]

[8] N. Gindrier, R. Clackdoyle, S. Rit, and L. Desbat. Sufficient field-of-view for the M-line method in cone-beam CT. In 2020 IEEE Nuclear Science Symposium and Medical Imaging Conference (NSS/MIC), Boston (virtual), United States, 2020

[9] N. Gindrier, L. Desbat, and R. Clackdoyle. CB reconstruction for the 3-sin trajectory with transverse truncation. In 6th Virtual International Meeting on Fully 3D Image Reconstruction in Radiology and Nuclear Medicine, Leuven, 2021

Conclusion

- **DBP** is the predominant method to manage **transverse truncation**...
- ...but to reconstruct a point by this method, it must be on a **chord**.
- Any point x of the **convex hull** of the **2-sin trajectory** is crossed by a chord: x can be often reconstructed by DBP, despite important truncation

Conclusion

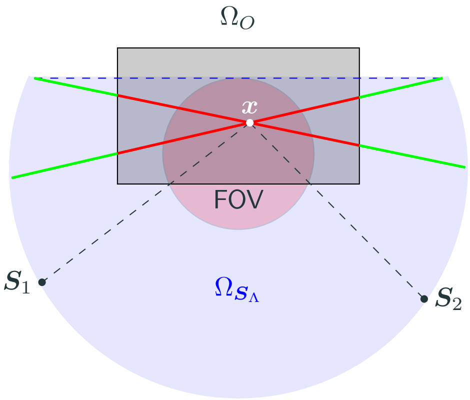
- **DBP** is the predominant method to manage **transverse truncation**...
- ...but to reconstruct a point by this method, it must be on a **chord**.
- Any point x of the **convex hull** of the **2-sin trajectory** is crossed by a chord: x can be often reconstructed by DBP, despite important truncation
- Some points of the convex hull of the **3-sin trajectory** are not intersected by a chord, yet reconstructions with transverse truncation seem possible
- It is possible to justify it for some restricted configurations...
- ...but the general case remains unsolved

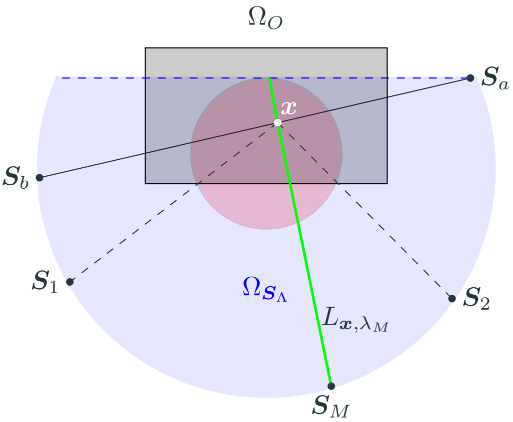
Conclusion

- **DBP** is the predominant method to manage **transverse truncation**...
- ...but to reconstruct a point by this method, it must be on a **chord**.
- Any point x of the **convex hull** of the **2-sin trajectory** is crossed by a chord: x can be often reconstructed by DBP, despite important truncation
- Some points of the convex hull of the **3-sin trajectory** are not intersected by a chord, yet reconstructions with transverse truncation seem possible
- It is possible to justify it for some restricted configurations...
- ...but the general case remains unsolved

THANK YOU. TIME FOR QUESTIONS

Annexes : utilité des lignes M en 2D

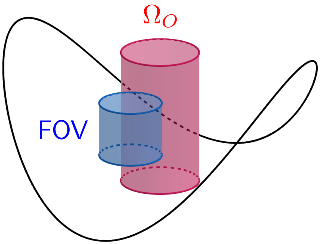




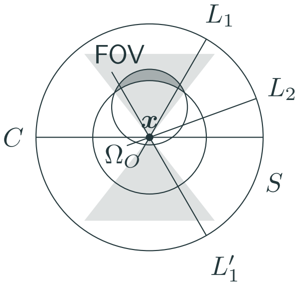
Troncations axiales et transverses avec la trajectoire 2-sin : MIC

Contexte: Trajectoire *2-sin* avec troncations *axiales ET transverses*

Objectif: Donner des *conditions suffisantes* pour appliquer la méthode DBP à cette configuration



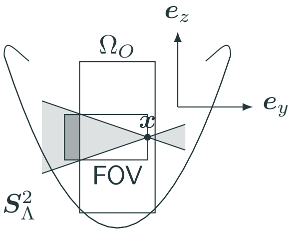
Rappels DBP et lignes M

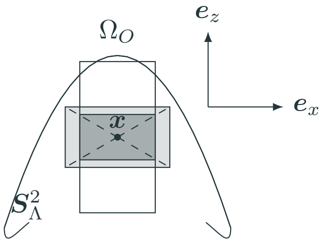


Corde C: reconstruction impossible (de $\boldsymbol{x}$)

Ligne M L_2: reconstruction impossible (L_2 n'intersecte pas la région gris foncé $\mathrm{FOV} \setminus \Omega_O$)

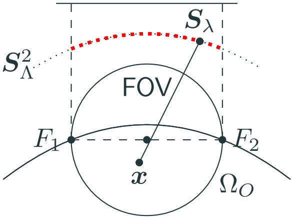
Lignes M L_1 et L'_1: reconstruction possible

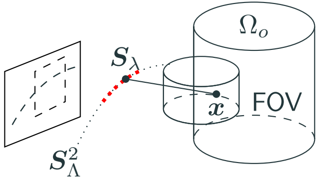




Un exemple de FOV non utilisable pour la méthode des lignes M, parce que les (deux) cônes de lignes (gris clair) de x qui intersectent $\mathrm{FOV} \setminus \Omega_O$ (gris foncé) n'intersectent pas latrajectoire 2-sin S^2_Λ.

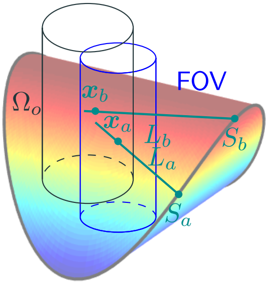
Condition suffisante de type 1

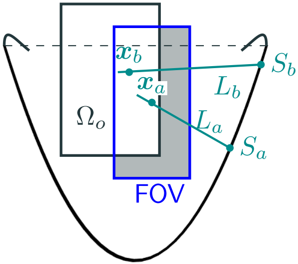




FOV suffisant de type 1 : la projection du *FOV semi-circulaire* doit intersecter la trajectoire de source

Condition suffisante de type 2

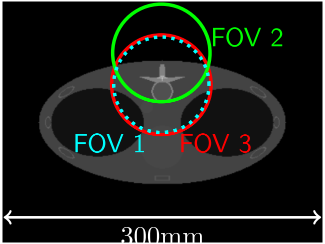


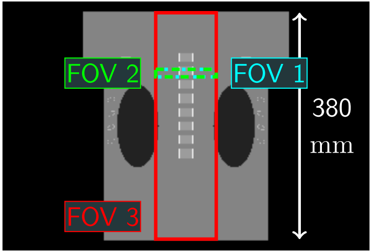


FOV suffisant de type 2 : conditions concernant la hauteur du FOV

Le FOV doit dépasser l'enveloppe convexe ou l'objet des deux côtés.

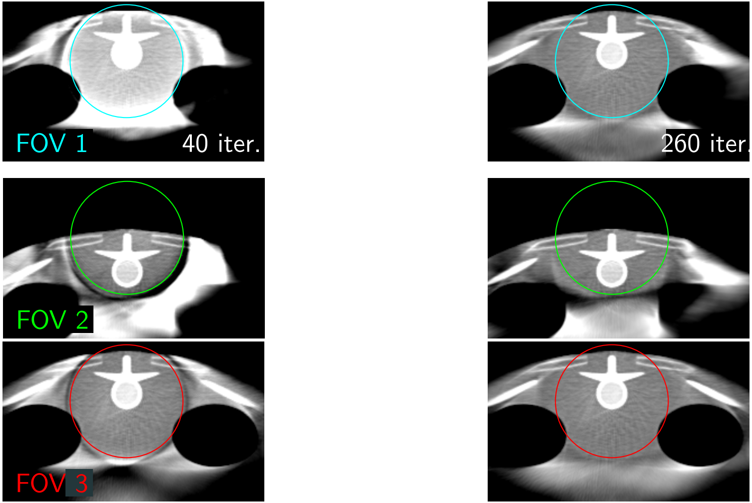
FOV	R_F	H_F	C_F	suffisant	dim_det
1		18	(0,30,78)	not	92×69
2	50	18	(0,30,78)	type 1	92×437
3		380	(0,67,-190)	type 2	92×69





3 FOV avec le fantôme Forbild thorax

Simulations et résultats



Les 3 FOV donnent des bonnes reconstructions mais les FOV non suffisant ont une convergence plus lente.

3-sin trajectory and transverse truncation

Context: As we have seen, the 3-sin trajectory with transverse truncation is not suitable for the DBP method, but “exact” reconstructions seems to be possible.

Goal: Provide a configuration using a method (including DBP) to do such reconstructions

[10] F. Noo, A. Wunderlich, L. Günter, and H. Kudo. On the problem of axial data truncation in the reverse helix geometry. In 10th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, pages 90–93, 2009

Context: As we have seen, the 3-sin trajectory with transverse truncation is not suitable for the DBP method, but "exact" reconstructions seems to be possible.

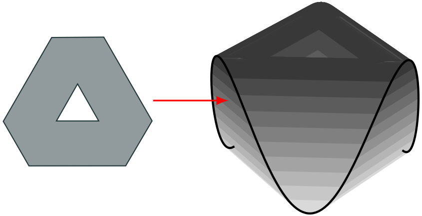
Goal: Provide a configuration using a method (including DBP) to do such reconstructions

Reconstruction method in 4 steps (inspired by [10] for another trajectory with axial truncation) :

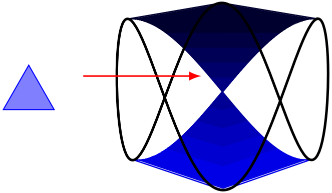
1. *Reconstruction* of $\Omega_{\mathrm{DBP}} \subseteq \mathrm{FOV} \cap \Omega_O \cap C_{\boldsymbol{S}^3_\Lambda}$ with the DBP method
2. *Reprojection* of reconstructed points
3. *Subtraction of reprojections* from original conebeam data, which gives a new configuration with a smaller object, $\Omega_O \setminus \Omega_{\mathrm{DBP}} = \Omega_{\mathrm{in}} \cup \Omega_{\mathrm{out}}$, with $\Omega_{\mathrm{out}} \stackrel{\text{def}}{=} \Omega_O \setminus (\Omega_{\mathrm{DBP}} \cup \Omega_{\mathrm{in}})$
4. Reconstruction of Ω_{in} with one of various methods for conebeam reconstruction for non-truncated projections

[10] F. Noo, A. Wunderlich, L. Günter, and H. Kudo. On the problem of axial data truncation in the reverse helix geometry. In 10th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, pages 90–93, 2009

Notations

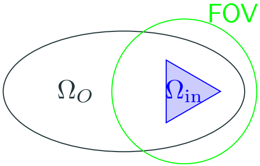


$C_{\boldsymbol{S}^3_\Lambda}$

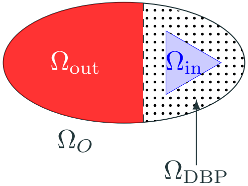


$N_{\boldsymbol{S}^3_\Lambda}$

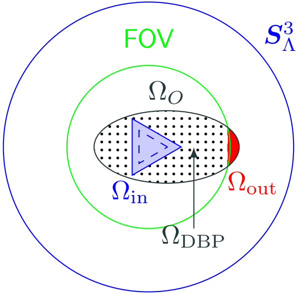
$$\Omega_{\text{in}} \stackrel{\text{def}}{=} \Omega_O \cap N_{\boldsymbol{S}^3_\Lambda} \cap \text{FOV}$$

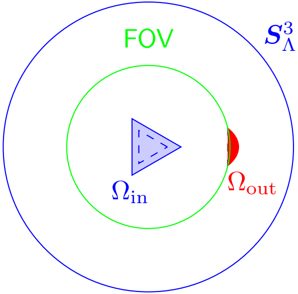


$$\Omega_{\text{out}} \stackrel{\text{def}}{=} \Omega_O \setminus (\Omega_{\text{DBP}} \cup \Omega_{\text{in}})$$



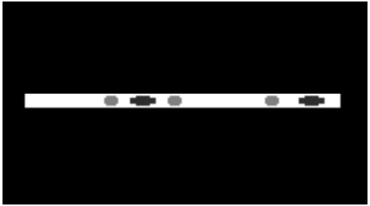
Proposed configuration



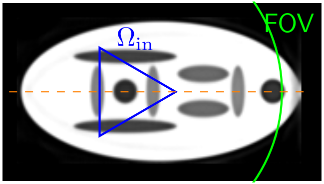


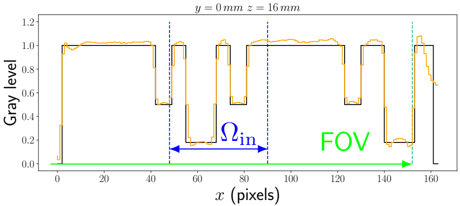
Firstly, we reconstruct Ω_{DBP}, then we reproject and subtract, so we have Ω_{in} and Ω_{out}. Then we reconstruct Ω_{in}.

Simulations



The phantom used and its reconstruction (60 iterations with the method of least squares with conjugate gradient in a volume of $162 \times 82 \times 8$ voxels).





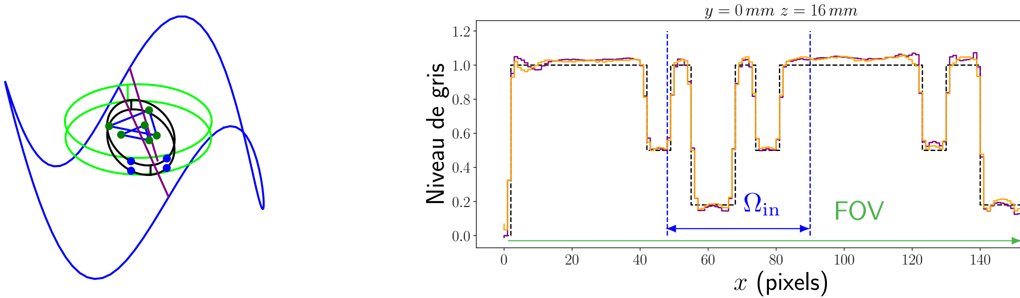
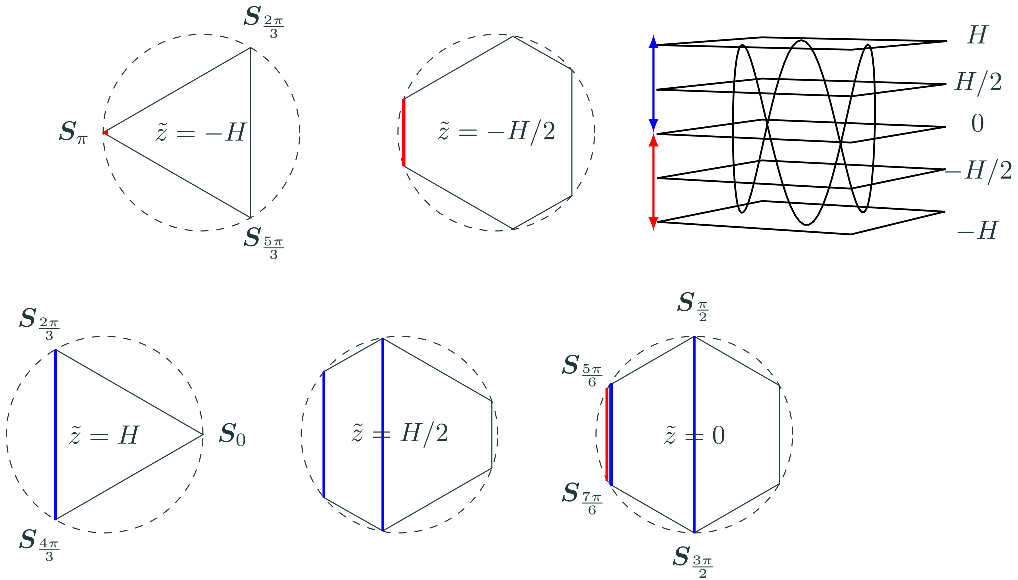


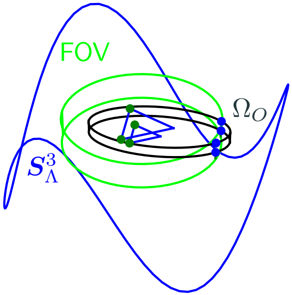
Figure 1: Gauche : Une configuration avec deux lignes contaminées. Droite : Un profil de reconstruction $y = 0$ mm et $z = 16$ mm, la ligne orange représente la reconstruction tronquée sans lignes contaminées et la ligne violette représente la reconstruction tronquée avec lignes contaminées.

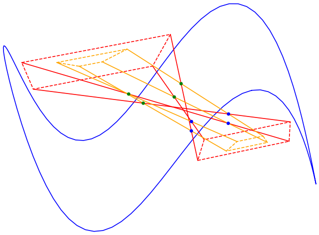
Union des cordes de S^3_Λ

On engendre une surface avec certaines cordes en faisant varier $\tilde{z}$.



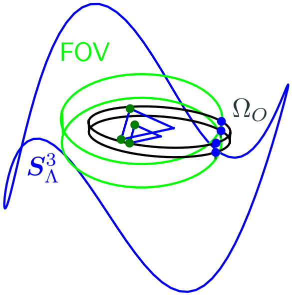
Vérification des lignes contaminées

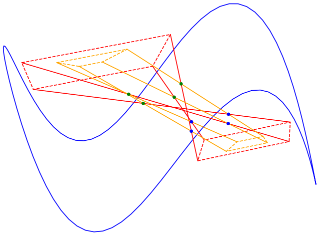




Tracé des lignes intersectant à la fois $\Omega_{\rm in}$ et $\Omega_{\rm out}$

Vérification des lignes contaminées





Tracé des lignes intersectant à la fois Ω_{in} et Ω_{out}

Aucune de ces lignes n'intersecte la trajectoire : pas de ligne contaminée