Sampling the X-ray transform on simple surfaces

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Abstract: Joint work with Plamen Stefanov (Purdue). On a Riemannian manifold-with-boundary, the geodesic X-ray transform maps a function to the collection of its integrals over all geodesics through the domain. The problem of inverting this transform arises in medical imaging (e.g., X-ray CT, SPECT) and seismology (e.g., linearized travel-time tomography). In the literature, it is now well-known that injectivity, stability (or mild instability) and inversion formulas hold at the continuous level, for example when (M, g) is a 'simple' surface. By 'simple' we mean (i) no infinite-length geodesic, (ii) no conjugate points, (iii) strictly geodesically convex boundary, arguably the most inversion-friendly case. The question of interest becomes to address numerical inversion from discrete data, where sampling issues arise.

In this talk, I will discuss the issue of proper discretizing and sampling related to geodesic X-ray transforms on simple surfaces, addressing the following questions: (a) Given a bandlimited function f, what are the minimal sampling rates needed on its X-ray transform $I_0 f$ for a faithful (=unaliased) recovery? (b) In the case where data is sampled below the expected requirements, can one predict the location, orientation and frequency of the artifacts generated?

The main tools to answer (a)-(b) are a combination of a reinterpretation of the classical Shannon-Nyquist theorem in semi-classical terms, as initiated by Plamen Stefanov in [1], and an accurate description of the canonical relation of the X-ray transform viewed as a (classical, then semi-classical) Fourier Integral Operator. Focusing on constant curvature surfaces as a first family of examples, we quantify the quality of a sampling scheme depending on geometric parameters of the surface (e.g., curvature and boundary curvature), and on the coordinate system used to represent the space of geodesics. Several (unaliased and aliased) examples will be given throughout.

Preprint available at https://arxiv.org/pdf/2110.05761.pdf

References: [1] P. Stefanov, Semiclassical sampling and discretization of certain linear inverse problems. SIAM J. Math. Anal., 52(6), 5554-5597, 2020.