

Ellipsoidal and hyperbolic Radon transforms; microlocal properties and injectivity

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Abstract: We present novel microlocal and injectivity analyses of ellipsoid and hyperboloid Radon transforms. We introduce a new Radon transform, R , which defines the integrals of a compactly supported L^2 function, f , over ellipsoids and hyperboloids with centers on a smooth surface, S . R is shown to be a Fourier Integral Operator (FIO) and in our main theorem we prove that R satisfies the Bolker condition if and only if the support of f is not intersected by any plane tangent to S . We give examples where our theory can be applied. Focusing specifically on a cylindrical geometry of interest in Ultrasound Reflection Tomography (URT), we prove injectivity results and investigate the visible singularities. In addition, we present example reconstructions of image phantoms in two-dimensions, and validate our microlocal theory.

Keywords - ellipsoids, hyperboloids, Radon transforms, microlocal analysis, stability, injectivity