"*TBA*"

Thomas Bloom University of Cambridge

Abstract

To follow

"*On Romanov's constant and related problems*"

Christian Elsholtz TU Graz

Abstract

We report on joint work with Jan-Christoph Schlage-Puchta (1), and with Florian Luca and Stefan Planitzer (2).

1) We show that the lower density of integers representable as a sum of a prime and a power of two is at least 0.107, improving the previous record of 0.0936275 by Habsieger and Sivak-Fischler.

We also prove that the set of integers with exactly one representation of the form $p + 2^k$ has positive density. Previous results of this kind needed "at most 15" in place of "exactly one". To achieve this result we introduce a new method. In particular we make use of uneven distribution of sums of a power of two and a reduced residue class.

2) We study Romanov type problems by replacing the logarithmic sequence $\{2^k : k \in \mathbb{N}\}$ with other sequences with logarithmic counting functions. In particular, for $n = p + 2^{2^k} + m!$ and $n = p + 2^{2^k} + 2^q$ where $m, k \in \mathbb{N}$ and p, q are primes we observe that there exist infinite arithmetic progressions not containing any number of these types, respectively. Also, the proportion of integers not of the form $p + 2^{2^k} + m!$ is larger than $\frac{3}{4}$. The proportion of integers not of the form $p + 2^{2^k} + 2^q$ is at least $\frac{2}{3}$.

"*Subsets of Cayley graphs that induce many edges*"

Oliver Janzer University of Cambridge

Abstract

Let G be a regular graph of degree d and let $A \subset V(G)$. Say that A is *η-closed* if the average degree of the subgraph induced by A is at least ηd. This says that if we choose a random vertex $x \in A$ and a random neighbour y of x, then the probability that $y \in A$ is at least η. In recent joint work with Tim Gowers, we were aiming to obtain a qualitative description of closed subsets of the Cayley graph Γ whose vertex set is $\mathbb{F}_2^{n_1} \otimes \cdots \otimes \mathbb{F}_2^{n_d}$ with two vertices joined by an edge if their difference is of the form $u_1 \otimes \cdots \otimes u_d$. For the matrix case (that is, when $d = 2$), such a description was obtained by Khot, Minzer and Safra, a breakthrough that completed the proof of the 2-to-2 conjecture. We have formulated a conjecture for higher dimensions, and proved it in an important special case. Also, we have identified a statement about η-closed sets in Cayley graphs on arbitrary finite Abelian groups that implies the conjecture and can be considered as a "highly asymmetric Balog-Szemerédi-Gowers theorem" when it holds. However, this statement is not true for an arbitrary Cayley graph. It remains to decide whether the statement can be proved for the Cayley graph Γ.

"*The additive energy of Ahlfors-David regular sets*"

Brendan Murphy University of Bristol

Abstract

An Ahlfors-David regular set is a fractal set with the "right" amount of mass in each interval. Examples include Cantor sets and limit sets of Fuchsian groups. These sets cannot contain long arithmetic progressions; Dyatlov and Zahl combined this property with a multi-scale analysis to prove that AD regular sets cannot have maximal additive energy. We improve Dyatlov and Zahl's result, obtaining near-optimal dependence on the "regularity constant" of the AD regular set.

Though this sounds like a problem in fractal geometry, we immediately discretize the problem and use methods employed by Bourgain and Bourgain and Chang from their work on the discretized sum-product problem and the weak Erdos-Szemeredi conjecture.

"*Progression-free sets and rank of matrices*"
Péter Pál Pach Budapest University of Technology and Economics

Abstract

In this talk we will discuss lower and upper bounds for the size of k-AP-free subsets of $\mathbb{Z}_m^n$, that is, for $r_k(\mathbb{Z}_m^n)$, in certain cases. Specifically, we will discuss some recent lower bounds given by Elsholtz and myself. In the case $m = 4, k = 3$ we present a construction which gives the tight answer up to $n \leq 5$ and point out some connections with coding theory. We will also mention some open questions (and some partial answers) about related linear algebraic problems.

"*Intersection patterns in the plane*"
Zuzana Patakova IST, Vienna

Abstract

After Elekes had shown a connection between the Szemerédi-Trotter incidence bound and the sum-product problem, the role of planar intersection patterns in additive combinatorics became apparent. So far, most sum-product estimates are underlaid with geometric methods. In order to get some useful applications, the intersection patterns of the considered objects cannot be too wild.

There are several parameters measuring the complexity of the intersection pattern of a given set system: Helly number, fractional Helly number, VC-dimension etc. However, for a given family, the values of these parameters are usually not obvious. In contrary, topological parameters can be determined rather easily, e.g. number of connected components of the sets, number of 1-dim holes, etc. We are going to show how these parameters relate.

"*Three-term progression free sets in* $\mathbb{Z}_8^n$"
Cosmin Pohoata Caltech

Abstract

Let G be a finite group, and let $r_3(G)$ represent the size of the largest subset of G without non-trivial three-term progressions. In a recent breakthrough, Croot, Lev and Pach proved that $r_3(\mathbb{Z}_4^n) \leqslant (3.611)^n$ using the polynomial method. For general finite abelian groups $G \cong \mathbb{Z}_{m_1} \bigoplus \mathbb{Z}_{m_k}$, where $m_1, \ldots, m_n$ denote positive integers such that $m_1 | \ldots | m_n$, their result also implies a bound of the form $r_3(G) \leqslant (0.903)^{\mathrm{rk}_4(G)}|G|$, with $\mathrm{rk}_4(G)$ representing the number of indices $i \in \{1, \ldots, n\}$ with m_i divisible by 4. In particular, $r_3(\mathbb{Z}_8^n) \leqslant (7.222)^n$. In this talk, I will discuss an improved bound for $\mathbb{Z}_8^n$. This is joint work with Fedor Petrov.

"*Small sumsets in* $\mathbb{R}$"
Anne de Roton Université de Lorraine, Nancy

Abstract

We describe the structure of subsets A and B of real numbers such that the sumset $A + B$ has small measure. We first prove a continuous version of Freiman's 3k-4 theorem as generalized by Grynkiewicz, this result gives some information on the structure of A, B and $A + B$ when $\lambda(A + B) < \lambda(A) + \lambda(B) + \min(\lambda(A), \lambda(B))$. We also use a result on small sumsets on the circle to describe sets A of real numbers such that $\lambda(A + A) < (3 + c)\lambda(A)$ for a small constant c. This last result is joint work with Pablo Candela.

"*On growth in groups* $SL_2(\mathbb{F}_p)$ *and* $Aff(\mathbb{F}_p)$*.*"
Misha Rudnev University of Bristol

Abstract

This joint work with Shkredov came about as an attempt to understand the relationship between growth in matrix groups of small dimension and sum-product type geometric incidence bounds. After the groundbreaking paper of Helfgott, which did use them, the two phenomena have often been mentioned side-by-side. indeed, one sure does add and multiply scalars when multiplying matrices. It seems, it all depends on the level of questions asked about groups. Roughly, if the question is just about cardinality growth, one may do without

counting solutions of equations with variables in a discrete set, but prying and poking further, say asking for energy bounds, will inevitably lead to it.

For the affine group, whose elements are $(a,b) \in \mathbb{F}_p^* \times \mathbb{F}_p$, with semidirect product multiplication, one can merely use the sharp Sznyi theorem on the number of directions, determined by a plane point set and get a very good (in comparison to prior results) bound on the structure of a slowly growing set of affine transformations $A \leq p$ (otherwise things get better) with $|AAA \leq K|A|$. Either A is contained in a one-dimensional subgroup (a non-vertical line through $(1,0)$), or its projection on the a-axis has size most K^3.

For $SL_2(\mathbb{F}_p)$, without using sum-product type estimates but rather much coarser Larsen-Pink type dimensional inequalities (which in the $SL_2(\mathbb{F}_p)$ case are very easy) we get an improvement of the growth exponent lower bound $\frac{1}{1512}$, due to Kowalski (for a generator set A, bigger in size than an absolute constant) to $\frac{1}{21}$. A known, but not necessarily least upper bound is approximately .3. Admittedly, we achieve the improvement just by locally improving and streamlining Kowalski's proof, more or less on its every step.

"*Induced Doubling*"

Imre Ruzsa Rényi Institute, Budapest

Abstract

We try to understand the structure of those configurations whose presence makes a sumset large. Let U be a finite set in $\mathbb{Z}^d$. We define its *induced doubling* as

$$\alpha(U) = \inf_{A \supset U} \frac{|A+A|}{|A|}.$$

This is closely connected with the dimension. For instance, if U is a simplex, then $A \supset U$ means that A is proper d-dimensional, and then $|A+A| \geq (d+1)|A| - d(d+1)/2$, whence $\alpha(U) = 1 + d/2$. If $U = \{0,1\}^d$, the discrete cube, then $2^{d/2} \leq \alpha(U) \leq (3/2)^d$. The general behaviour of this quantity is rather mysterious, we have many questions and conjectures and few results.

Joint work in (slow) progress with Máté Matolcsi, George Shakan and Dmitrii Zhelezov

"*The Uniformity Conjecture and the Sum-product Phenomenon*"

Jozsef Solymosi University of British Columbia

Abstract

The sum-product phenomenon states that, for most of the $F(x,y)$ polynomials no matter how do we select two sets of numbers A and B, where $|A| = |B| = n$, the range of $F(A,B)$ will be much larger than n. We will see that assuming a major conjecture in arithmetic geometry, the Uniformity Conjecture of Bombieri and Lang, one can improve some of the classical results in this area.

"*Ordinary hyperplanes and hyperspheres*"

Konrad Swanepoel London School of Economics

Abstract

Let S be a set of n points in real d-dimensional space such that any d of the points span a hyperplane. We say that a hyperplane is ordinary if it intersects exactly $d+1$ points of S. We show that for n sufficiently large depending on d, if S has only $O(n^{d-1})$ ordinary hyperplanes, then all but $O(1)$ points of S lie on a hyperplane or an elliptic normal curve or an acnodal rational normal curve. As a consequence, we determine the minimum number of ordinary hyperplanes for all sufficiently large n, thus proving a conjecture of Ball and Monserrat. We have analogous results for ordinary hyperspheres. This is joint work with Aaron Lin.

"*Sets with small k-fold sumsets*"

Katarzyna Taczała Adam Mickiewicz University, Poznan

Abstract

In 2014 Eberhard, Green and Manners proved a structural result about the sets with a doubling constant less than 4. We generalize this theorem to the sets with small k-fold sumset. We also show how this result can be applied to solve a problem from additive Ramsey theory. This is joint work with Tomasz Schoen.

"*The largest projective cube-free subsets of Z_{2^n}*"
Adam Zsolt Wagner ETH Zurich

Abstract

What is the largest subset of Z_{2^n} that doesn't contain a projective d-cube? In the Boolean lattice, Sperner's, Erdos's, Kleitman's and Samotij's theorems state that families that do not contain many chains must have a very specific layered structure. We show that if instead of Z_2^n we work in Z_{2^n}, analogous statements hold if one replaces the word k-chain by projective cube of dimension 2^{k-1}. The largest d-cube-free subset of Z_{2^n}, if d is not a power of two, exhibits a much more interesting behaviour. (Joint work with Jason Long)

"*Pair correlations and additive combinatorics*"
Aled Walker University of Cambridge

Abstract

Let A be a set of natural numbers. The metric poissonian property, which concerns the pair correlations of dilates of A modulo 1, was first introduced to pure mathematics by Rudnick and Sarnak in 1998. But recently it was discovered that techniques from additive combinatorics, particularly relating to the additive energy of A, could be used to develop a greater understanding of this property. In this talk I will give an overview of these new developments, and describe some recent work (joint with Thomas Bloom) that demonstrates the usefulness of sum-product estimates in this arena.

"*The size of $A(A+1)$*"
Audie Warren RICAM, Linz

Abstract

It was recently proved by Rudnev, Shakan, and Shkredov that in arbitrary fields, $|AA| + |A+A| \gtrsim |A|^{11/9}$. In this talk we outline how the method they used can be adapted to show that also $|A(A+1)| \gtrsim |A|^{11/9}$, by use of some alternative lemmas on multiplicative energy in terms of products of shifts.

"*The height function and the few products many sums conjecture*"
Dmitry Zhelezov Rényi Institute, Budapest

Abstract

We discuss classical bounds for the heights of solutions to $x + y = 1$ with x, y lying in a multiplicative subgroup of bounded rank. Well known to number theorists, the methods developed to prove such bounds seem to be overlooked by the arithmetic combinatorics community. We further discuss how similar ideas may help attacking the few products many sums conjecture (also known as the weak Erdős-Szemerédi conjecture).