Filtered schemes for Hamilton-Jacobi equations

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joint work with Adam Oberman

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Numerical methods for Hamilton-Jacobi equations
in optimal control and related fields
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McGill
1. Hamilton-Jacobi equations
2. Filtered Schemes
3. Numerical schemes
   - Schemes for the Eikonal equation
   - Schemes for Hamilton-Jacobi equations
4. Explicit formulas
   - Eikonal equation
   - HJ equations
5. Results
6. Conclusions
Contents

1 Hamilton-Jacobi equations

2 Filtered Schemes

3 Numerical schemes
   • Schemes for the Eikonal equation
   • Schemes for Hamilton-Jacobi equations

4 Explicit formulas
   • Eikonal equation
   • HJ equations

5 Results

6 Conclusions
Consider the Hamilton-Jacobi (HJ) equation

\[
\begin{cases}
    H(x, \nabla u) = f(x), & x \in \Omega \setminus \Gamma, \\
    u(x) = g(x), & x \in \Gamma \subset \Omega,
\end{cases}
\]

where

- $\nabla u$ is the gradient of the function $u$,
- $\Omega$ is a bounded open set,
- $\Gamma$ is a subset of $\Omega$,
- $H$ is a nonlinear Lipschitz continuous function.
In particular, we will focus on the Eikonal equation

\[
\begin{cases}
|\nabla u(x)| = f(x), & \text{for } x \text{ outside } \Gamma, \\
u(x) = g(x), & \text{for } x \text{ on } \Gamma,
\end{cases}
\]

where \( f > 0 \) and \( \Gamma \) is closed, bounded set.

**Fact:** Solutions are usually piecewise smooth and globally Lipschitz.
Related work

- Semi-Lagrangian schemes (Falconi, Ferretti ’02 and Cristiani, Falconi ’07)
- Central schemes (Lin, Tadmor ’00)
- ENO and WENO (Osher, Shu ’91)
- Compact upwind second order scheme (Benamou, Luo, Zhao ’03)
- Abgrall blended scheme (Abgrall ’09)
Contents

1 Hamilton-Jacobi equations

2 Filtered Schemes

3 Numerical schemes
   • Schemes for the Eikonal equation
   • Schemes for Hamilton-Jacobi equations

4 Explicit formulas
   • Eikonal equation
   • HJ equations

5 Results

6 Conclusions
In the one-dimensional case we are interested in solving $|u_x| = f(x)$. The \textit{monotone} (upwind) scheme is given by

$$|u_x^h|^M = \max \left\{ -\frac{u(x + h) - u(x)}{h}, \frac{u(x) - u(x - h)}{h}, 0 \right\}$$

$$= \max \left\{ -D^+ u(x), D^- u(x), 0 \right\}$$

This scheme is also \textit{consistent} and \textit{stable} and so, by the Barles and Souganidis theory, it converges to the unique viscosity solution.

\textbf{Fact}: Monotone schemes are provably convergent but only first order accurate.
We could consider the second order accurate centered scheme

\[ |u_h^{xh}|^A = \frac{|u(x + h) - u(x - h)|}{2h}. \]

However this scheme is not monotone nor stable.

**Fact:** In general, higher order finite difference schemes for HJ equations are neither monotone nor stable.
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$$\left| u^h_A \right| = \frac{\left| u(x + h) - u(x - h) \right|}{2h}.$$ 

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**Fact:** In general, higher order finite difference schemes for HJ equations are neither monotone nor stable.

Up to a scaling in $h$, the difference of the schemes is the centered finite difference approximation for $\left| u_{xx} \right|$:

$$\left| u^h_A \right| - \left| u^h_M \right| = \frac{h}{2} \frac{\left| u(x + h) - 2u(x) + u(x - h) \right|}{h^2} = \frac{h}{2} u^h_{xx}(x).$$

We can then use this as a (local) criteria to decide whether or not to use the accurate scheme.
The filtered scheme, $|\nabla u^h|^F$, then uses the following simple formula:

$$
|u^h_x|^F = \begin{cases} 
|u^h_x|^A, & \text{if } |u^h_x|^A - |u^h_x|^M \leq \sqrt{h} \\
|u^h_x|^M, & \text{otherwise}.
\end{cases}
$$

The filtered scheme, which is consistent provided both underlying schemes are consistent, is usually not monotone. However it is almost monotone, since

$$
|u^h_x|^F = |u^h_x|^M + \mathcal{O}(\sqrt{h}).
$$
Filtered Schemes

Definition

$S$ is a filter function if it is a bounded function that is equal to the identity in a neighborhood of the origin and zero outside.

Definition

The filtered scheme is given by

$$F^h[u] = F^h_M[u] + \epsilon(h)S \left( \frac{F^h_A[u] - F^h_M[u]}{\epsilon(h)} \right).$$
Theorem (Froese, Oberman 2013)

Let $F^h_M$ be a consistent, stable, monotone scheme. Assume $S$ is a continuous filter function and $\epsilon(h) \to 0$ as $h \to 0$. For each $h > 0$, let $u_h$ be a solution of $F^h[u] = 0$, where the filtered scheme is given by

$$F^h[u] = F^h_M[u] + \epsilon(h)S \left( \frac{F^h_A[u] - F^h_M[u]}{\epsilon(h)} \right).$$

Then $u_h$ converges to the unique viscosity solution of the underlying PDE.
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Why use filtered schemes?
- Simplicity.
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Why use filtered schemes?
- Simplicity.
- Provably convergent.
- Achieve higher accuracy where the solution is smooth.
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$$F^h[u] = F^h_M[u] + \epsilon(h)S \left( \frac{F^h_A[u] - F^h_M[u]}{\epsilon(h)} \right).$$

Then $u_h$ converges to the unique viscosity solution of the underlying PDE.

Why use filtered schemes?

- Simplicity.
- Provably convergent.
- Achieve higher accuracy where the solution is smooth.
- In practice, it avoids the use of a wide stencil in second order equations (e.g. Monge-Ampère equation).
Contents

1 Hamilton-Jacobi equations

2 Filtered Schemes

3 Numerical schemes
   ● Schemes for the Eikonal equation
   ● Schemes for Hamilton-Jacobi equations

4 Explicit formulas
   ● Eikonal equation
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Schemes for the Eikonal equation

- Monotone scheme

\[ |u_h^x|^M = \max \{ -D^+ u(x), D^- u(x), 0 \} \]
Schemes for the Eikonal equation

- Monotone scheme
  \[ |u^h_x|^M = \text{max} \{ -D^+ u(x), D^- u(x), 0 \} \]

- Accurate scheme (centred scheme)
  \[ |u^h_x|_{C,2} = \frac{|u(x + h) - u(x - h)|}{2h} \]
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$$|u^h_x|_{\text{c,2}} = \frac{|u(x + h) - u(x - h)|}{2h}$$

- High-order upwind scheme

$$|u^h_x|_{U,n} = \max \left\{ -\frac{d}{dx} P^{+,n}[u](x), \frac{d}{dx} P^{-,n}[u](x), 0 \right\}$$
Schemes for the Eikonal equation

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  \[ |u_x^h|^M = \max \{ -D^+ u(x), D^- u(x), 0 \} \]

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  \[ |u_x^h|^{U,n} = \max \left\{ -\frac{d}{dx} P^{+,n}[u](x), \frac{d}{dx} P^{-,n}[u](x), 0 \right\} \]

- ENO schemes
  \[ |u_x^h|^{E,n} = \max \left\{ -\frac{d}{dx} E^{n,\frac{1}{2}}[u](x), \frac{d}{dx} E^{n,-\frac{1}{2}}[u](x), 0 \right\} \]
Schemes for Hamilton-Jacobi equations

- Monotone scheme (Lax-Friedrichs)

\[ H_{LF}^h[u](x) = H_{LF}^h(x, D^+ u(x), D^- u(x)) = H \left( x, \frac{D^+ u(x) + D^- u(x)}{2} \right) - \frac{1}{2} \sigma_x (D^+ u(x) - D^- u(x)) \]
Schemes for Hamilton-Jacobi equations

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\[ H^{h}_{C,2}[u](x) = H \left( x, \frac{u(x + h) - u(x - h)}{2h} \right) \]
Schemes for Hamilton-Jacobi equations

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- High-order upwind scheme

\[ H^h_{U,n}[u](x) = H^h_{LF}\left( x, \frac{d}{dx} P^+, n[u](x), \frac{d}{dx} P^-, n[u](x) \right) \]
Schemes for Hamilton-Jacobi equations

- Monotone scheme (Lax-Friedrichs)
  \[ H^h_{LF}[u](x) = H^h_{LF}(x, D^+ u(x), D^- u(x)) = H \left( x, \frac{D^+ u(x) + D^- u(x)}{2} \right) - \frac{1}{2} \sigma_x (D^+ u(x) - D^- u(x)) \]

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- High-order upwind scheme
  \[ H^h_{U,n}[u](x) = H^h_{LF} \left( x, \frac{d}{dx} P^{+,n}[u](x), \frac{d}{dx} P^{-,n}[u](x) \right) \]

- ENO schemes
  \[ H^h_{E,n}[u](x) = H^h_{LF} \left( x, \frac{d}{dx} E^{n,\frac{1}{2}}[u](x), \frac{d}{dx} E^{n,-\frac{1}{2}}[u](x) \right) \]
Two-dimensional schemes

- Eikonal equation

\[ |\nabla u^h| = N \left( |u_x^h|, |u_y^h| \right) \]

where \( N(x, y) = \sqrt{x^2 + y^2} \)
Two-dimensional schemes

- Eikonal equation
  
  \[ \left| \nabla u^h \right| = N \left( \left| u_x^h \right|, \left| u_y^h \right| \right) \quad \text{where} \quad N(x, y) = \sqrt{x^2 + y^2} \]

- Hamilton-Jacobi equations
  
  \[ H_{LF}^h[u](x) = H_{LF}^h(x, p^+, p^-, q^+, q^-) \]
  
  \[ = H \left( x, \frac{p^+ + p^-}{2}, \frac{q^+ + q^-}{2} \right) - \sigma_x \frac{p^+ - p^-}{2} - \sigma_y \frac{q^+ - q^-}{2} \]
  
  where \( p^\pm = D_x^\pm u(x) \) and \( q^\pm = D_y^\pm u(x) \).
Contents

1 Hamilton-Jacobi equations
2 Filtered Schemes
3 Numerical schemes
   • Schemes for the Eikonal equation
   • Schemes for Hamilton-Jacobi equations
4 Explicit formulas
   • Eikonal equation
   • HJ equations
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Solving $|u^h_x|^M = f$ for the reference variable, $u(x)$, leads to

$$ |u^h_x|^M = f \iff \max \left\{ -\frac{u(x + h) - u(x)}{h}, \frac{u(x) - u(x - h)}{h}, 0 \right\} = f(x) $$

$$ \iff \max \{u(x) - u(x + h), u(x) - u(x - h), 0\} = hf(x) $$

$$ \iff u(x) = \min\{u(x + h), u(x - h)\} + hf(x) $$
Solving \( |u^h_x|^M = f \) for the reference variable, \( u(x) \), leads to

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|u^h_x|^M = f \iff \max \left\{ -\frac{u(x + h) - u(x)}{h}, \frac{u(x) - u(x - h)}{h}, 0 \right\} = f(x)
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\[
\iff \max \{ u(x) - u(x + h), u(x) - u(x - h), 0 \} = hf(x)
\]

\[
\iff u(x) = \min \{ u(x + h), u(x - h) \} + hf(x)
\]

Similarly, for the second order upwind scheme we have

\[
|u^h_x|^{U,2} = f \iff \max \{3u(x) - 4u(x \pm h) + u(x \pm 2h), 0\} = 2hf(x)
\]

\[
\iff u(x) = \frac{1}{3} \min \{4u(x \pm h) - u(x \pm 2h)\} + \frac{2}{3} hf(x)
\]
Solving

\[ |\nabla u^h| = f \iff |u_x^h|^2 + |u_y^h|^2 = f(x, y)^2 \]

for the reference variable \( u(x, y) \) requires solving nonlinear equation

\[ \max\{z - a, 0\}^2 + \max\{z - b, 0\}^2 = c^2 \]

for the unknown \( z \) where \( a, b \) and \( c > 0 \) are constants. The unique solution is given by

\[
 z = \begin{cases} 
 \min\{a, b\} + c, & |a - b| \geq c, \\
 \frac{a+b+\sqrt{2c^2-(a-b)^2}}{2}, & |a - b| < c.
\end{cases}
\]
Recall that
\[ D^+ u(x) = \frac{u(x + h) - u(x)}{h}, \quad D^- u(x) = \frac{u(x) - u(x - h)}{h}. \]

Solving \( H^h_{LF}[u] = f \) for the reference variable, \( u(x) \), leads to
\[
H \left( x, \frac{D^+ u(x) + D^- u(x)}{2} \right) - \frac{1}{2} \sigma_x \left( D^+ u(x) - D^- u(x) \right) = f(x) \\
\iff H \left( x, \frac{u(x + h) - u(x - h)}{2h} \right) - \frac{1}{2} \sigma_x \frac{u(x + h) - 2u(x) + u(x - h)}{h} = f(x) \\
\iff u(x) = \frac{1}{\sigma_x} \left[ f(x) - H \left( x, \frac{u(x + h) - u(x - h)}{2h} \right) + \sigma_x \frac{u(x + h) + u(x - h)}{2h} \right].
\]
Consider now the second order upwind scheme, $H_{U,2}[u] = f$, rewritten as

$$H \left( x, \frac{d}{dx} P^{+,2}[u](x) + \frac{d}{dx} P^{-,2}[u](x) \right) - \frac{1}{2} \sigma_x \left( \frac{d}{dx} P^{+,2}[u](x) - \frac{d}{dx} P^{-,2}[u](x) \right) = f(x).$$

where

$$\frac{d}{dx} P^{+,2}[u](x) = \frac{-3u(x) + 4u(x + h) - u(x + 2h)}{2h},$$

$$\frac{d}{dx} P^{-,2}[u](x) = \frac{3u(x) - 4u(x - h) + u(x - 2h)}{2h}.$$

Thus, solving for the reference variable, $u(x)$, leads to

$$u(x) = \frac{2}{3\sigma_x} \left[ f(x) - H \left( x, \frac{-u(x + 2h) + 4u(x + h) - 4u(x - h) + u(x - 2h)}{4h} \right) \right] + \sigma_x \frac{-u(x + 2h) + 4u(x + h) + 4u(x - h) - u(x - 2h)}{4h}.$$

Contents

1 Hamilton-Jacobi equations

2 Filtered Schemes

3 Numerical schemes
   - Schemes for the Eikonal equation
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4 Explicit formulas
   - Eikonal equation
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The 1\textsuperscript{st} example is given by

\[ |u_x| = 1 + \cos(x), \quad u(x) = 3 - |x + \sin(x)| \]

\textbf{Figure:} Profile of the solution.
Figure: Exact solution and solutions obtained with the monotone scheme and the $2^{nd}$ order upwind filtered scheme with 50 grid points.
Figure: Active scheme for the solutions of the 1\textsuperscript{st} example: filtered scheme with centred scheme (left) and filtered scheme with 2\textsuperscript{nd} order upwind scheme (right).
The figure shows a loglog plot of the errors for the 1st Example. The different lines represent various filtered schemes for Hamilton-Jacobi equations, including Monotone, Filtered (2nd), Filtered (2ndUpwind), Filtered (3rdUpwind), Filtered (4thUpwind), Filtered (2ndENO), Filtered (3rdENO), and Filtered (4thENO).
The 2\textsuperscript{nd} example is given by

\[ |u_x| = 1 + e^{|x|}, \quad u(x) = 10 - |x| - e^{|x|} \]
Figure: Loglog plot of the errors for the 2\textsuperscript{nd} Example.
The 3rd example is given by

$$|u_x| = 3x^2 + a,$$

$$u(x) = \begin{cases} 
  x^3 + ax, & x \in [0, x_0], \\
  1 + a - ax - x^3, & x \in [x_0, 1], 
\end{cases}$$

with $a = \frac{1 - 2x_0^3}{2x_0 - 1}$, $x_0 = \frac{3\sqrt{2} + 2}{4\sqrt{2}}$.

**Figure:** Profile of the solution.
Figure: Loglog plot of the errors for the 3rd Example.
The 4\textsuperscript{th} example is given by

\[ |u_x|^2 = e^x, \quad u(x) = \begin{cases} 
2e^{\frac{x}{2}} + 16, & x \in [-2, 0], \\
-2e^{\frac{x}{2}} + 20, & x \in [0, 2].
\end{cases} \]

\textbf{Figure:} Profile of the solution.
Figure: Loglog plot of the errors for the 4th Example.
For the last example we take

\[ \cos^2(u_x) + |u_x| = \cos^2 \left( e^{-|x|} \right) + e^{-|x|}, \quad u(x) = e^{-|x|} \]

**Figure:** Profile of the solution.
Figure: Loglog plot of the errors for the 5th Example.
Two-dimensional examples

We consider three distinct examples all of them solution to the Eikonal equation

$$\begin{cases} 
|\nabla u| = 1 \quad \text{for } x \text{ outside } \Gamma, \\
u = 0 \quad \text{for } x \text{ on } \Gamma 
\end{cases}$$

with the computational domain being $[-2, 2]^2$ taking for $\Gamma$:

1. $\Gamma = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$,
2. $\Gamma = \{(\frac{1}{2}, \frac{1}{2}), (-\frac{1}{2}, -\frac{1}{2})\}$,
3. $\Gamma = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, x \geq 0\}$. 

Tiago Salvador
Filtered schemes for Hamilton-Jacobi equations
RICAM, 21-25 November 2016
Figure: Profile of the solutions of the three examples considered in 2D.
Distance to a circle

Figure: Log-log plot of the errors for the first example in the $L^\infty$ norm.
Distance to two points

Figure: Log-log plot of the errors for the second example in the $L^\infty$ norm (left) and $L^1$ norm (right).
Distance to a semi-circle

Figure: Log-log plot of the errors for the third example in the $L^\infty$ norm in $[-2, 2]$ (left) and $L^\infty$ norm in $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \geq 1, x > 0.1\}$ (right).
Results summary

- One-dimensional case

The order of convergence in the filtered upwind schemes is the order of accuracy of the accurate scheme used (special result to the Eikonal equation).

We found an example where the error for the ENO was greater than the formal accuracy.

Two-dimensional case

We obtain second order convergence for smooth solutions. In general, solutions are piecewise smooth and we only recover second order convergence in the smooth regions.
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- **Two-dimensional case**
  - We obtain second order convergence for smooth solutions.
  - In general, solutions are piecewise smooth and we only recover second order convergence in the smooth regions.
Contents

1 Hamilton-Jacobi equations
2 Filtered Schemes
3 Numerical schemes
   • Schemes for the Eikonal equation
   • Schemes for Hamilton-Jacobi equations
4 Explicit formulas
   • Eikonal equation
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- Are provably convergent;
- Are simple and easy to implement;
- Allow for a wide choice of accurate discretizations;
- Achieve higher accuracy where the solution is smooth;
- In practice, it avoids the use of a wide stencil in second order equations (e.g. Monge-Ampere equation).

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RICAM, 21-25 November 2016
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