Preconditioned space-time boundary element methods for the heat equation

S. Dohr  TU Graz, Austria

Abstract

Regarding time-dependent initial boundary value problems, there are different numerical approaches to compute an approximate solution. In addition to finite element methods and time-stepping schemes one can use boundary element methods to solve time-dependent problems. As for stationary problems, one can use the fundamental solution of the partial differential equation and the given boundary and initial conditions to derive boundary integral equations and apply some discretization method to compute an approximate solution of those equations.

In this talk, we describe the boundary element method for the discretization of the time-dependent heat equation. In contrast to standard time-stepping schemes we consider an arbitrary decomposition of the space-time cylinder into boundary elements. Besides adaptive refinement strategies, this approach allows us to parallelize the computation of the global solution of the whole space-time system. In addition to the analysis of the boundary integral operators and the derivation of boundary element methods for the Dirichlet initial boundary value problem, we state convergence properties and error estimates of the approximations. Those estimates are based on the approximation properties of boundary element spaces in anisotropic Sobolov-spaces, in particular in $H^{\frac{1}{2}+\delta} (\Sigma)$ and $H^{-\frac{1}{2}-\delta} (\Sigma)$.

The systems of linear equations, which arise from the discretization of the boundary integral equations, are being solved with GMRES. For an efficient computation of the solution we need preconditioners. Based on the mapping properties of the single layer- and hypersingular boundary integral operator we construct and analyse a preconditioner for the discretization of the first boundary integral equation. Finally we present numerical examples for the spatial one-dimensional heat equation to confirm the theoretical results.