# The Maxwell Compactness Property in Bounded Weak Lipschitz Domains with Mixed Boundary Conditions

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**Abstract**

Let $\Omega \subset \mathbb{R}^3$ be a bounded weak Lipschitz domain with boundary $\Gamma := \partial\Omega$ divided into two weak Lipschitz submanifolds $\Gamma_\tau$ and $\Gamma_\nu$ and let $\epsilon$ denote an $\mathsf{L}^\infty$-matrix field inducing an inner product in $\mathsf{L}^2(\Omega)$. The main result of this contribution is the so called 'Maxwell compactness property', that is, the Hilbert space

$$\left\{E \in \mathsf{L}^2(\Omega) \,:\, \operatorname{rot} E \in \mathsf{L}^2(\Omega),\, \operatorname{div} \epsilon E \in \mathsf{L}^2(\Omega),\, \nu \times E|_{\Gamma_\tau} = 0,\, \nu \cdot \epsilon E|_{\Gamma_\nu} = 0\right\}$$

is compactly embedded into $\mathsf{L}^2(\Omega)$. We will also prove some canonical applications, such as the Maxwell estimate, the Helmholtz decomposition and a static solution theory as well. Furthermore, a Fredholm alternative for the underlying time-harmonic Maxwell problem and all corresponding and related results for exterior domains formulated in weighted Sobolev spaces are straight forward.