The Maxwell Compactness Property in Bounded Weak Lipschitz Domains with Mixed Boundary Conditions

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October 18, 2016

Abstract

Let $\Omega \subset \mathbb{R}^3$ be a bounded weak Lipschitz domain with boundary $\Gamma := \partial \Omega$ divided into two weak Lipschitz submanifolds $\Gamma_\tau$ and $\Gamma_\nu$ and let $\epsilon$ denote an $L^\infty$-matrix field inducing an inner product in $L^2(\Omega)$. The main result of this contribution is the so called ‘Maxwell compactness property’, that is, the Hilbert space

$$\{ E \in L^2(\Omega) : \text{rot} \, E \in L^2(\Omega), \text{div} \, \epsilon E \in L^2(\Omega), \nu \times E|_{\Gamma_\tau} = 0, \nu \cdot \epsilon E|_{\Gamma_\nu} = 0 \}$$

is compactly embedded into $L^2(\Omega)$. We will also prove some canonical applications, such as the Maxwell estimate, the Helmholtz decomposition and a static solution theory as well. Furthermore, a Fredholm alternative for the underlying time-harmonic Maxwell problem and all corresponding and related results for exterior domains formulated in weighted Sobolev spaces are straightforward.