"*Encounters with Maxwell Equations*"

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Abstract

The talk presents my personal history with Maxwell's equations, presented in the form of two stories.

The first story centers around the notion of strong ellipticity. It started in 1982 from trying to understand a paper by MacCamy and Stephan that transformed a system of boundary integral equations for the time-harmonic Maxwell equations with PEC boundary conditions into a strongly elliptic system of pseudodifferential operators by modifying the set of Dirichlet data. The consequence is that Galerkin approximation by boundary elements of any nature is stable and convergent. This first led to the construction of a strongly elliptic system of boundary integral equations for Maxwell transmission problems arising from scattering by a dielectric body. Further steps were on the positive side the construction of a bilinear form for regularized Maxwell equations that is coercive on H^1 (1991), on the negative side the observation that standard finite element methods on polyhedra do not converge for the regularized Maxwell equations and that they compute wrong eigenvalues (1995–2001), and finally the invention and analysis of the weighted regularization method (2002, with Monique Dauge).

The second story is centered around the approximation of Maxwell eigenvalue problems by the p and hp versions of the finite element method using edge elements (2002–2011, with Daniele Boffi, Monique Dauge, Leszek Demkowicz and Ralf Hiptmair). For the property of discrete compactness, a crucial lemma was needed that stated the p independent boundedness of the inversion of the curl operator in polynomial spaces. For the proof, the class of regularized Poincaré integral operators are employed that among other interesting properties had been found to be pseudo-differential operators of order -1, to satisfy a homotopy relation with respect to exterior derivation, and to respect polynomials (2010, with Alan McIntosh).