"*Approximating the divergence of electromagnetic fields by edge elements*"

Patrick Ciarlet Laboratoire POEMS, ENSTA
828, boulevard des Maréchaux
91762 Palaiseau Cedex, FRANCE

Abstract

We study the approximation of electromagnetic fields in a bounded domain Ω, made of a classical medium: it is well-known that edge elements are globally conforming in $\mathbf{H}(\mathbf{curl}, \Omega)$, so error estimates are usually derived in $\|\cdot\|_{\mathbf{H}(\mathbf{curl},\Omega)}$ norm. What is less studied is how well these finite elements allow to approximate the divergence of the fields. In the recent paper "Edge element methods for Maxwell's equations with strong convergence for Gauss' laws" by H. Wu, J. Zou and P. Ciarlet, Jr. (*SIAM J. Numer. Anal.*, **52**, p. 779-807, 2014), we address this issue for the electric field $\boldsymbol{E}$. In particular, instead of relying on the classical mixed approach, we use a perturbed approach. The latter approach relies on the addition of a term like $\gamma(h)(\varepsilon \boldsymbol{E}_h, \boldsymbol{v}_h)_{\mathbf{L}^2(\Omega)}$ in the discrete variational formulations, here indexed by h. For this approach, there is no need to introduce a Lagrange multiplier. On the other hand, the parameter $\gamma(h) > 0$ must be chosen with care.

In this talk we first recall how the divergence of the fields can be approximated in general by edge elements. Then we show how to choose the parameter $\gamma(h)$ and compare the convergence results that one may obtain for the mixed and perturbed approaches. In the above cited reference, the results are obtained for piecewise $\mathbf{H}^{\delta}$-regular fields, for $1/2 < \delta \leq 1$: we explain how those results can be generalized to low-regularity fields. We propose numerical results to highlight the abstract convergence results, and compare the mixed and perturbed approaches. Finally, we conclude by a discussion on the possible optimization of the parameter $\gamma(h)$ and on the applicability of this method to interface problems with sign-changing coefficients.