

*“Homogenization and the spectrum of the Neumann Poincaré operator”*

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#### Abstract

Resonances of metallic nano-particles has been an active topic of investigation in the last decade, as this phenomenon allows localization of strong electro-magnetic fields in very small regions of space, an exciting feature for many applications. Asymptotically as the size of the particles tends to 0, the resonant frequencies are related to the spectral properties of the Neumann-Poincaré operator (NP). In this talk, we discuss the spectrum of that integral operator, when one considers a periodic distribution of inclusions made of metamaterials in a dielectric background medium.

We show that under the assumptions that the inclusions are fully embedded in the periodicity cells, the spectra  $\sigma_\varepsilon$  of the NP operators for a collection of period  $\varepsilon$  converge to a limiting set composed of 2 parts : the union of the Bloch spectra of NP operators defined over periodicity cells with quasi-periodic boundary conditions and a boundary spectrum associated with eigenfunctions which spend a not too small part of their energies near the boundary.

If the conductivity inside the inhomogeneities lies outside the spectrum of the periodic NP operator, we show that uniformly bounded sequences of solutions weakly converge to a homogenized limit, whose effective matrix is defined by a cell-problem of the usual form, albeit the non-elliptic setting. Conversely, if the homogenized source problem is not well-posed, then the conductivity inside the inclusions must lie in  $\lim_{\varepsilon \rightarrow 0} \sigma_\varepsilon$ . This cannot happen when the inclusions are strictly contained in the periodicity cells and if their conductivity is sufficiently positive or negative.

This is joint work with Charles Dapogny and Faouzi Triki.