Welcome to the

Special Semester on New Trends in Calculus of Variations

Linz, October 13 – December 12, 2014

http://www.ricam.oeaw.ac.at/specsem/specsem2014

Organizer

Maïtine Bergounioux, Universite d'Orleans, France

Local Organizer

Karl Kunisch, University of Graz & RICAM, Austria

Otmar Scherzer, University of Vienna & RICAM, Austria

Workshops

Workshop 1: Shape and topological optimization
October 13-17, 2014

Workshop 2: Variational methods in imaging
October 27-31, 2014

Workshop 3: Geometric control and related fields
November 17-21, 2014

Workshop 4: Optimal Transport in the Applied Sciences
December 08-12, 2014

Schools

School 1: Imaging
October 22-24, 2014

School 2: Optimal Transport in the Applied Sciences
December 02-05, 2014

Workshop 3: Geometric control and related fields
November 17-21, 2014

Main topics

This workshop intends to gather people from geometric control and related fields including Riemannian geometry and generalizations, Hamilton-Jacobi theory, celestial mechanics, quantum control, algebraic and numerical methods for ode's and control. Young researchers are especially welcome to participate.

Organizers

Jean-Baptiste Caillau, University of Dijon, France
Thomas Haberkorn, Université d'Orléans, France

Local Organziers

Dante Kalise, RICAM, Austria
Axel Kröner, RICAM, Austria
Zhiping Rao, RICAM, Austria

List of speakers

Bernard Bonnard (Dijon)
Thierry Combot (Dijon)
Florin Diacu (Victoria, Canada)
Jacques Fejoz (Paris)
Marek Grochowski (Warsaw)
Didier Henrion (Toulouse / Prague)
Angel Jorba (Barcelona)
Christoph Koutschan (Linz)
Wojciech Krynski (Warsaw)
Andrea Mennucci (Pisa)
Mazyar Mirrahimi (Rocquencourt / Yale)
Luca Rizzi (Trieste)
Gianna Stefani (Firenze)
Hasnaa Zidani (Paris)

Bernard Bonnard (Dijon). *Dynamics of spins with Ising coupling and spheres with Liouville metrics.*

In this talk we introduce an almost Riemannian Liouville type metric on the sphere $\mathbf{S}^2$ associated with the time minimal transfer in a linear chain of three spins with Ising coupling. Integrability properties of the dynamics are discussed in connection with invariant sub-Riemannian metrics on SO(3), Poisson sphere and Liouville normal form. Finally we describe the conjugate and cut loci using the analogy with the conjugate and cut loci on the ellipsoids.

Thierry Combot (Dijon). *Non-integrability of some n-body problem through higher variational equations.*

We will prove the non-integrability of the 3 and 4 collinear three body problem and planar n-body problem (with positive masses) through the application of the Morales-Ramis-Simo theorem. We will prove in particular that in these problems, we can bound the eigenvalues of the Hessian matrices of the potential at central configurations. In the case of the planar n-body problem, this is sufficient to prove non integrability. The collinear case is much more difficult: There are 3 one dimensional families of masses in the 3 body problem and 20 one dimensional families of masses in the 4 body problem for which we cannot conclude directly. Higher variational equations in these cases are computed and solved up to order 5, allowing to compute strong necessary conditions to integrability: In particular, these conditions cannot be satisfied by real potentials. This allows to prove non-integrability without complete computation of the central configurations, which is computationally challenging.

Florin Diacu (Victoria). *Newton's equations in spaces of constant curvature.*

We consider a natural extension of Newton's equations of the N-body problem to spaces of constant curvature. We first present some qualitative results regarding the motion of the bodies, focusing on relative equilibria and rotopulsators, which generalize the notion of homographic orbits from Euclidean to curved space. Then we write the equations in intrinsic coordinates and discuss the advantages and disadvantages of this approach. Finally we come up with a new and simple form of the equations that brings together the Euclidean case (of Gaussian curvature $k = 0$), the hyperbolic case (of Gaussian curvature $k < 0$), and the elliptic case (of Gaussian curvature $k > 0$). Thus Newton's classical equations can be regarded in a broader context, namely that in which the motion of the bodies takes place in spaces of constant curvature. The equations of motion depend on the curvature k, and the Euclidean case is recovered when $k = 0$. This conclusion could not be analytically drawn from previously known forms of the equations of motion in curved space since taking $k \to 0$, for both $k > 0$ and $k < 0$, led to undetermined expressions, although it was geometrically and mechanically clear that the Newtonian equations are recovered in the limit. This new form of the equations of motion allows the study of the classical case, $k = 0$, in a larger framework and will help us better understand Newton's original approach.

Jacques Féjoz (Paris). *On bounded motions in the N-body problem.*

In the spatial N-body problem, for any given masses, there is a set of positive Lebesgue measure of initial conditions leading to quasiperiodic motions. This is a variant of Arnold's theorem, where one mass is supposed much larger than the others.

Marek Grochowski (Warsaw). *Invariants and infinitesimal symmetries for contact sub-Lorentzian structures in dimension* 3.

I will present some elementary aspects of a theory of symmetry in sub-Lorentzian geometry.

Luis Guijarro (Madrid). *Balanced split loci in Riemannian manifolds.*

We will review some results on the cut loci of points or hypersurfaces in Riemannian manifolds. More precisely, we characterize the cut locus as a balanced split locus; conversely, we examine what other possible balanced split loci appear for a given metric. This is joint work with Pablo Angulo-Ardoy (UAM).

Didier Henrion (Toulouse / Prague). *Occupation measures and semidefinite programming for nonlinear optimal control.*

In the first part of the talk, we survey the theory of occupation measures in calculus of variations and optimal control, starting with the pioneering works of L.C. Young on relaxed arcs, L. Kantorovich on optimal mass transportation, and culminating with linear programming formulations of nonlinear optimal control. In the second part of the talk, we show how J.-B. Lasserre's hierarchies of semidefinite programming relaxations can be used to solve numerically nonlinear optimal control problems featuring explicit state constraints and oscillation (chattering controls) or concentration (impulsive controls) phenomena.

Angel Jorba (Barcelona). *Jet Transport and Control.*

Jet transport is a technique to compute high order differentials of a given numerical algorithm with respect to initial data and/or parameters. In this presentation we will explain first how to combine Jet Transport with a numerical integrator for ODEs (for instance, Taylor method) to compute, at a given time, the Taylor expansion of the solution with respect to the initial data. Then, we will show how to use this information to derive control strategies for some concrete problems of astrodynamics.

Wojciech Kryński (Warsaw). *Invariants of control affine systems and related geometric structures.*

We solve an equivalence problem for a class of control affine systems and analyse the occurring invariants. We also show that the geometry is related to Einstein-Weyl structures and classical geometry of webs. We exploit this relation to construct new solutions to Einstein-Weyl equation.

Andrea Mennucci (Pisa). *Hamilton-Jacobi equations and asymmetric distances.*

Let $H(x, Du(x)) = 0$ be an Hamilton-Jacobi partial differential equation on an n-dimensional manifold M; the existence and uniqueness of the viscosity solutions u of this equation is related to the metric completeness of the space M equipped with an asymmetric metric b induced by H, while the regularity of u may be inferred from studying the Hamiltonian flow induced by H on M.

Mazyar Mirrahimi (Rocquencourt / Yale). *Quantum systems: measurement and feedback.*

One of the most needed requirements for operating quantum information processing machines is to prepare various non-classical states with high-fidelities, manipulate them reliably and protect them against decoherence over arbitrary long times. All these requirements lead to the redesign and adaptation of various system theory concepts, such as state and parameter estimation, feedback control, active and passive stabilization, for such physical systems. The goal of this talk is to present some of the particular features of the quantum systems that one needs to study with extra care while discussing these system theory concepts. Some of the most important features are related to the concept of measurement. While such a measurement is essential for any active stabilizing feedback procedure it introduces modeling complications. I will introduce the concept of measurement back-action and I will show how it can be taken into account in a dynamical model. The back-action being intrinsically a random procedure, such a dynamical model is necessarily a stochastic one. Next, I will focus on an active feedback stabilization problem for such a quantum system and I will study a rather general Lyapunov feedback method that ensures the stabilization of the system around its equilibrium state.

Luca Rizzi (Trieste). *Curvature: A variational approach.*

We will present a definition of curvature for affine optimal control problems with Tonelli Lagrangian. This extends the well known concept of Riemannian sectional curvature to much more general spaces, including sub-Riemannian, Finsler and sub-Finsler structures. This is a work in collaboration with A. Agrachev and D. Barilari.

Christoph Koutschan (Linz). *Creative Telescoping.*

Creative telescoping is a methodology to deal with parametrized symbolic sums and integrals. Under certain conditions, it can deliver a recurrence or a differential equation that the sum/integral of interest satisfies. In this lecture, we give an introduction to the method, discuss different algorithmic approaches and present some examples where creative telescoping has been applied successfully.

Gianna Stefani (Firenze). *Optimality conditions for trajectories with a singular arc: An overview and perspectives.*

In this talk I'll consider an optimal control problem associated to an affine system on a n-dimensional manifold M

$$\begin{cases} \dot{\xi} = (f_0 + \sum_{i=1}^{m} u_i f_i) \circ \xi(t) \\ \xi(0) \in N_0, \quad \xi(T) \in N_f \\ \mathbf{u} = (u_1, \ldots, u_m) \in U \subset \mathbf{R}^m \end{cases}$$

where N_0 and N_f are sub-manifolds of M and all the data are smooth. I'll show how the Hamiltonian approach is suitable to prove second order sufficient conditions for strong optimality and to study structural stability of the optimizers, in the case when the extremal trajectory contains a singular arc. Some open problems will be described. The results have been obtained together with Laura Poggiolini from Florence and Francesca Carlotta Chittaro from Toulon.

Hasna Zidani (Paris). *Optimal control problems on generalized networks.*

This talk concerns some optimal control problems on d-dimensional networks. We will present a new characterization of the value function as solution to a system of Hamilton-Jacobi-Bellman equations with appropriate junction conditions. This result doesn't require any condition on the junctions and still holds when the value function is discontinuous.

Public lecture

Florin Diacu (Victoria). *Historical chronology: Truth or fable?*

We learn in school that Rome was founded in 753 BC, that the Battle of Hastings took place in 1066, and that the Turks reached the gates of Vienna in 1683. While the last date in this sequence is most surely correct, the further we step back in time, the more likely it is that we deal with approximations. But how is this possible? Haven't historians always kept track of events? The answer is surprisingly "no." Historical chronology, as we know it today, was founded by the time of the Gregorian calendar reform, towards the end of the 16th century. Until then it was far from clear when the events depicted in documents had taken place. The reason is simple: documents describe events in terms of various calendars and relative to unknown dates, such as "the battle took place 32 years after the death of Alexander." Alexander who, and when did he die? Unfortunately, the span of our collective memory proves to be amazingly short.

In this talk we will discuss the intriguing aspects of how global historical chronology was created and the huge difficulties its founders encountered, how it survived in spite of ferocious opposition from historians and scientists, including Isaac Newton, and why it was and still is attacked by various figures, including cranks like Immanuel Velikovsky, and academics, such as the mathematician Anatoly Fomenko. We will emphasize the role mathematics and science play in historical chronology today, and why celestial mechanics was and remains fundamental in dating historical events.

Posters

Maxime Chupin (Paris). *Interplanetary transfer with low consumption using three-body problem properties.*

Ariadna Farres (Barcelona). *Surfing on invariant manifolds.*

Damien Goubinat (Toulouse). *TBA.*

Helen Henninger (Nice). *Propulsion requirement for time-minimal transfers in quasi-circular planar low-earth orbits under* J_2*-perturbation.*

Nathalie Khalil (Brest). *Normality for necessary conditions in calculus of variations problems.*

Sébastien Le Bihan (Toulouse). *Low-energy, low-thrust tranfers to the Moon.*

Jérémy Rouot (Nice). *Lunar perturbations of the metric associated to the averaged orbital transfer.*

Chen Zheng (Orsay). *Chattering arcs in fuel-optimal orbital transfers.*

Jiamin Zhu (Paris). *Time optimal control of the rocket attitude reorientation.*