Weierstrass semigroups at several points, total inflection points on curves and coding theory

Cícero Carvalho
Faculdade de Matemática
Universidade Federal de Uberlândia

Algebraic curves over finite fields
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Weierstrass semigroup at several points

Let X be a smooth complete irreducible curve of genus $g \geq 1$ defined over a field $\mathbb{F}$, assumed to be the full field of constants of $\mathbb{F}(X)$.

Let $P_1, \ldots, P_m$ be distinct rational points of X.

Definition The Weierstrass semigroup at $P_1, \ldots, P_m$ is defined as

$$H = H(P_1, \ldots, P_m) := \{(\alpha_1, \ldots, \alpha_m) \in \mathbb{N}_0^m \mid \exists\, f \in \mathbb{F}(X) \text{ with } \operatorname{div}_\infty(f) = \alpha_1 P_1 + \cdots + \alpha_m P_m\}$$

Its systematic study was initiated by S. J. Kim and M. Homma in mid 90's. They studied specially the case $m = 2$; investigated properties of H and its relationship with the theory of algebraic geometry (Goppa) codes.

In a joint work with F. Torres we extended their results for any value of m, and also applied the results to obtain better lower bounds for the minimum distance of certain algebraic geometry codes.
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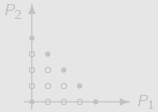

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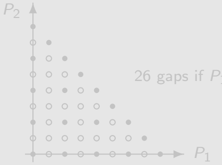

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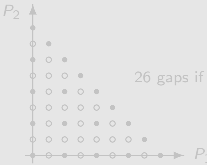

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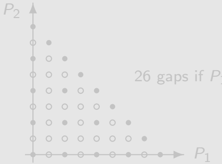

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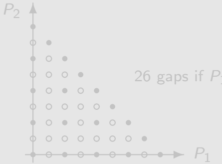

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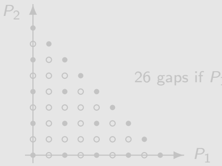

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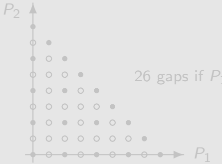

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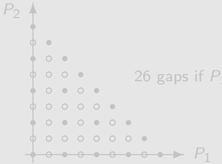

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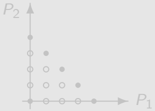

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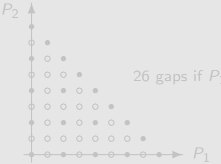

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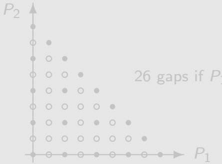

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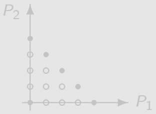

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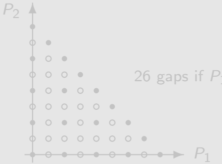

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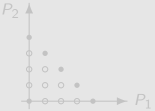

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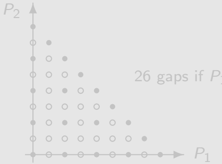

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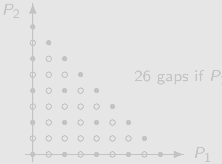

26 gaps if P_1 and P_2 are both W. points of X

Weierstrass semigroup at several points

We will write $\mathbf{n} := (n_1, \ldots, n_m) \in \mathbb{N}_0^m$, $\mathbf{e}_i \in \mathbb{N}_0^m$ for the m-tuple that has 1 in the i-th position and 0 in the others, $L(\mathbf{n}) := L(n_1 P_1 + \cdots + n_m P_m)$ and $\ell(\mathbf{n}) := \dim L(\mathbf{n})$.

Lemma. The following are equivalent:
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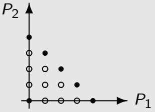

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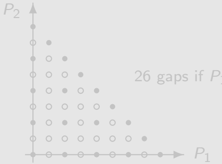

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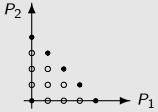

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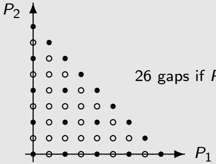

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Weierstrass semigroup at several points

From now on we assume that $\#(\mathbb{F}) \geq m$.

Properties of H:

• For all $i = 1, \ldots, m$ we get that $a \in H(P_i)$ if and only if $a.\mathbf{e}_i \in H$.

• Let $(n_1, \ldots, n_m)$, $(p_1, \ldots, p_m) \in H$ and set $q_i := \max\{n_i, p_i\}$, $i = 1, \ldots, m$. Then $(q_1, \ldots, q_m) \in H$.

Define $(n_1, \ldots, n_m) \preceq (p_1, \ldots, p_m)$ if $n_i \leq p_i$ $\forall i = 1, \ldots, m$. Then $\preceq$ is a partial order in $\mathbb{N}_0^m$.

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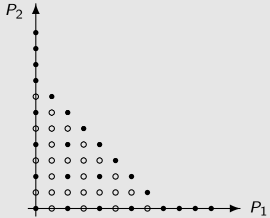

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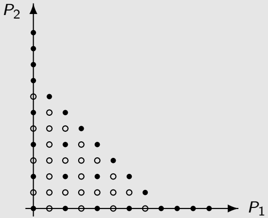

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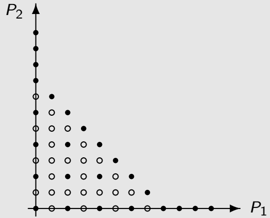

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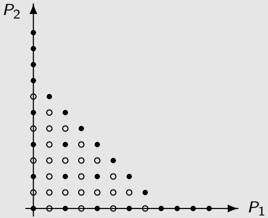

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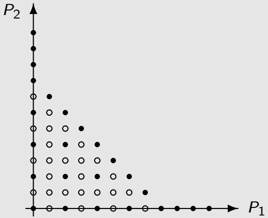

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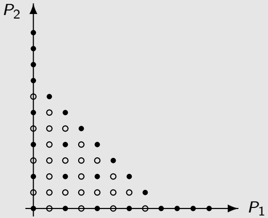

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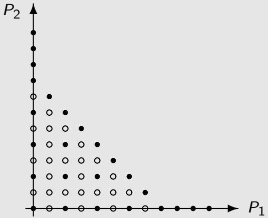

Weierstrass semigroup and AG codes

Given $\mathbf{n} = (n_1, \ldots, n_m) \in \mathbb{N}_0^m$, define
$\nabla_i(\mathbf{n}) := \{(p_1, \ldots, p_m) \in H \mid p_i = n_i \text{ and } p_j \leq n_j \; \forall j \neq i\}$

Lemma. Let $\mathbf{n} \in \mathbb{N}_0^m$. The following are equivalent:
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We say that $\mathbf{n} \in \mathbb{N}_0^m$ is a pure gap if $\nabla_i(\mathbf{n}) = \emptyset$ for all $i \in \{1, \ldots, m\}$.

Denote the set of pure gaps by $\mathbf{G}_0$.

Lemma: Let $\mathbf{n} = (n_1, \ldots, n_m) \in \mathbb{N}_0^m$.
(i) If $\mathbf{n} \in \mathbf{G}_0$ then $n_i \notin H(P_i)$ for all $i = 1, \ldots, m$.
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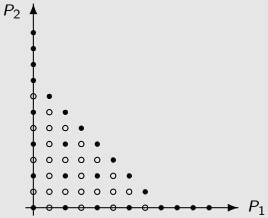

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$\nabla_i(\mathbf{n}) := \{(p_1, \ldots, p_m) \in H \mid p_i = n_i \text{ and } p_j \leq n_j \ \forall j \neq i\}$

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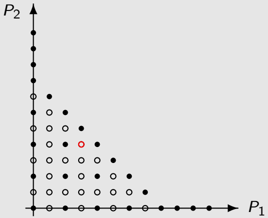

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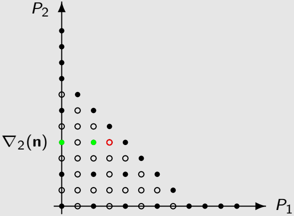

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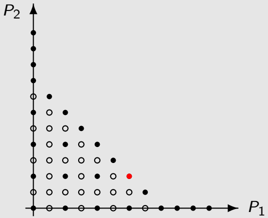

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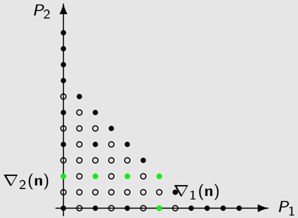

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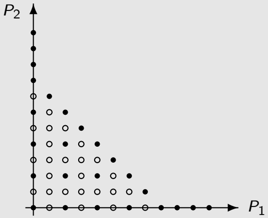

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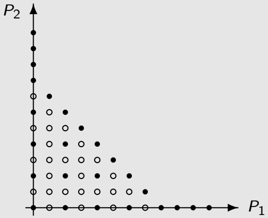

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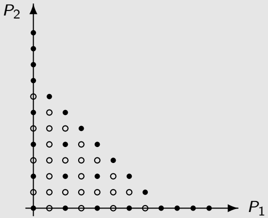

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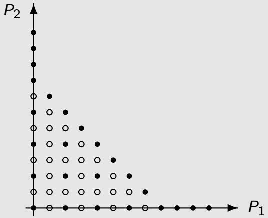

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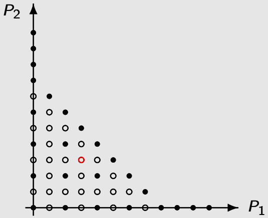

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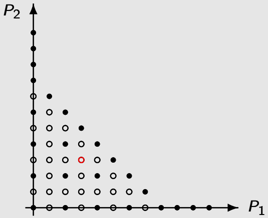

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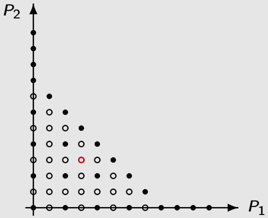

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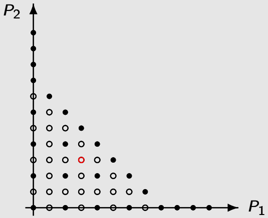

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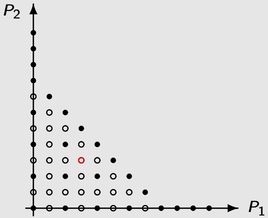

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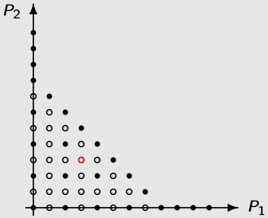

Weierstrass semigroup and AG codes

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Building on previous work by Homma, Kim and Matthews, F. Torres and myself proved the following results.

Theorem. Assume that $(n_1, \ldots, n_m)$ and $(p_1, \ldots, p_m)$ are pure gaps of H, and let $G = \sum_{i=1}^m (n_i + p_i - 1)P_i$. Then $C_\Omega(D, G)$ is an $[n, k, d]$-code, with $d \geq \deg(G) - (2g - 2) + m$.

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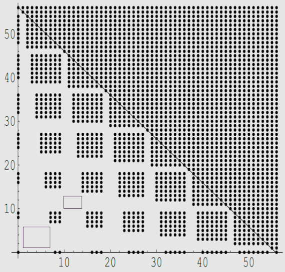

Weierstrass semigroup of two rational points in $Y^8 + Y = X^9$ over $\mathbb{F}_{64}$.

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Weierstrass semigroup and AG codes

A similar improvement can be made to bounds for the generalized Hamming distance of AG codes.

Let r be a positive integer, and $C \subset \mathbb{F}^m$ a linear code. Let U be a subcode of C, the support of U is defined as
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The r-th generalized Hamming distance of C is defined as

$$d_r(C) = \min\{\#(\mathrm{supp}(U)) \mid U \text{ is a subcode of } C,\ \dim(U) = r\}.$$

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Theorem. The r-th generalized Hamming distance of an AG code of length n defined over X satisfies
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Total inflection points and pure gaps

Assume that X is a smooth plane, projective curve, of degree $r > 4$.

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Take $s_1 = s$, $s_2 = \cdots = s_m = 0$, from the above theorem we get that $(sr + \alpha_1, \alpha_2, \ldots, \alpha_m)$ is a pure gap at $H(P_1, \ldots, P_m)$ whenever $1 \leq \alpha_i \leq q - i - s$ $(i = 1, \ldots, m)$ (i.e. only have pure gaps "between" $(sr + 1, 1, \ldots, 1)$ and $(sr + q - 1 - s, q - 2 - s, \ldots, q - m - s)$).
Let $G = (2sr + q - 1 - s)P_1 + \sum_{i=2}^{m}(q - i - s)P_i$ and let D be the sum of the other $q^3 + 1 - m$ rational points of X. From the work together with F. Torres we know that $C_\Omega(D, G)$ is an $[q^3 + 1 - m, k, d]$ code with $d \geq 2s(q+1) + m(2q - 2s - 1) - m^2 - (q+1)(q-2)$.

Total inflection points and pure gaps

So $d \geq 2s(q+1) + m(2q-2s-1) - m^2 - (q+1)(q-2)$
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We compared $C_\Omega(D, G)$ with codes on the Hermitian curve supported on one point and having the same dimension k, finding many situations where $C_\Omega(D, G)$ has better parameters.

For example, asssume that q is odd and $q \geq 5$, take $m = s = (q-1)/2$.
Then $C_\Omega(D, G)$ is an $[q^3 + 1 - (q-1)/2, k, d]$-code with
$k = q^3 - (5q+13)(q-1)/8$ e $d \geq q^2/4 + q + 3/4$.

Taking $F = (q^3 - (q^2/8 + 3q/2 - 5/8))P$, where P is a rational point of X and E is the sum of the other rational points, we get that $C_\Omega(F, E)$ is an $[q^3, k, d']$ code, where $d' = q^2/8 + 3q/2 - 5/8$ (from works by Stichtenoth, Yang and Kummar) so that $d - d' \geq (q(q-4) + 11)/8$.

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THANK YOU!