Two-Scale Wave Equation Modeling for Seismic Inversion

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Outline

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II. Operator Upscaling for the Acoustic Wave Equation

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- B. Numerical Examples (Accuracy)
- C. Numerical Experiments (Convergence)
- D. Matrix Analysis To Illustrate Physics of Solution
- E. Solution of the Inverse Problem

III. Conclusions

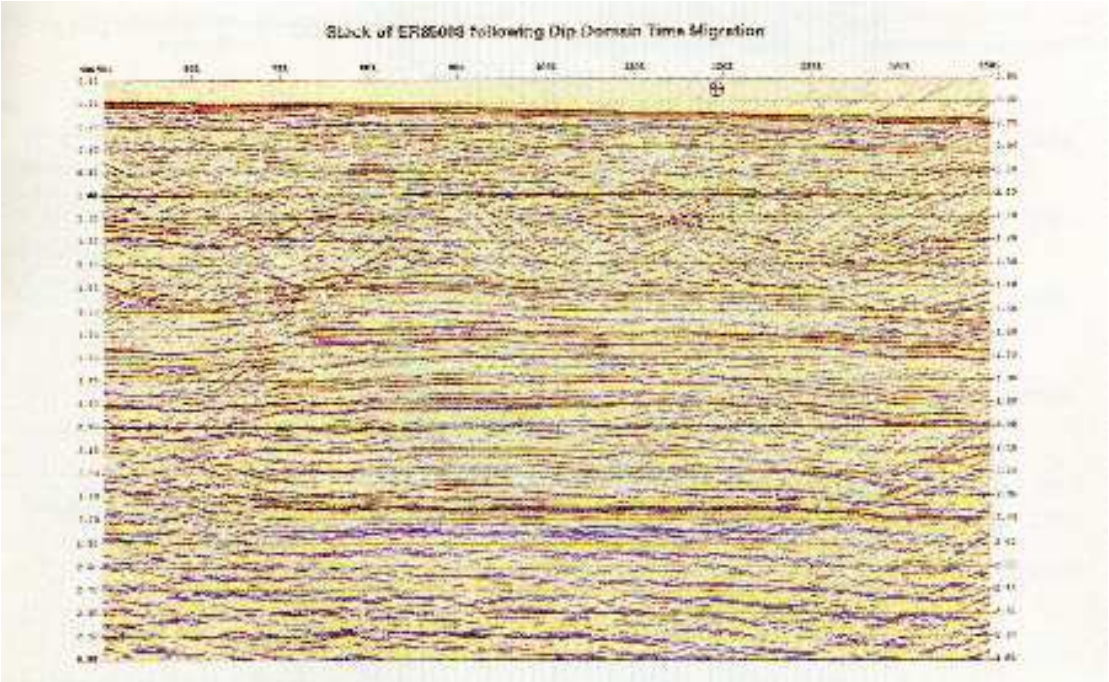
Motivation: A Seismic Inversion Example

- To illuminate a section of the subsurface, geophysicists introduce energy into the ground.
- Seismic source is always a part of the resulting data.
- Good estimate of the energy source essential to recovery of mechanical earth parameters.
- Source's shape (signature) and direction-dependence (radiation pattern) are of no use in themselves.
- Seismic inverse problems generally do not have unique solutions. The model estimate may have unique average behavior.

Inputs to Numerical Experiments:

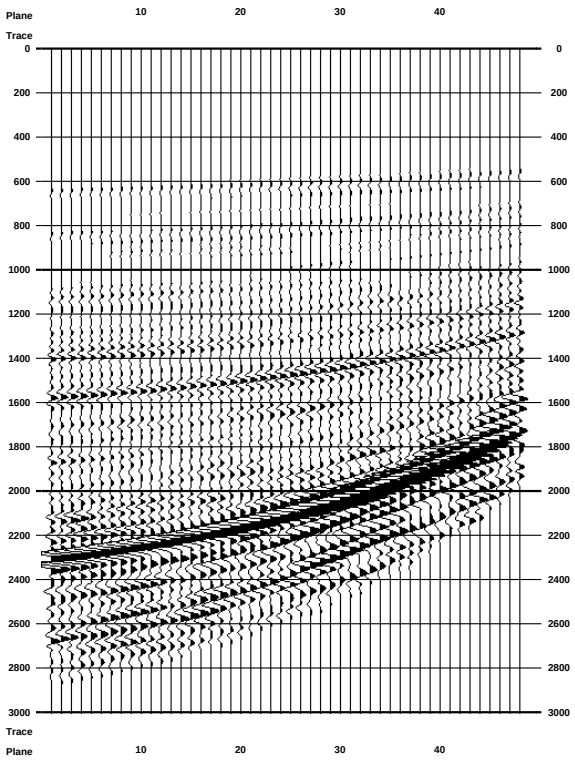
- Gulf of Mexico Data from Exxon Production Research Co.
- Eleven common-midpoint data gathers.
- Radon transform to yield 48 plane-wave traces (per gather) with slow ness values from $p_{min} = .1158$ ms/m to $p_{max} = .36468$ ms/m.
- Estimate of anisotropic air gun source in a 31-component Legendre expansion in slowness, p.
- Viscoelastic model (with estimated attenuation coefficient) used as forward simulator for inversion.

Stacked Section of Gulf of Mexico Data



Seismic data in $\tau - p$ domain

Figure: The $\tau - p$ transformed seismic data from common midpoint gather 6. Data was filtered by convolving it with a 15 Hz Ricker filter.



Inversion Algorithm:

1. Estimate *P*-wave background velocity and elastic parameter reflectivities by *Differential Semblance Optimization*

$$J_{DSO}[v,r] = \frac{1}{2}\{\|S[v,r] - S_{data}\|^2 + \lambda^2\|Wr\|^2$$

$$+\sigma^2\|\partial r/\partial p\|^2\}$$

2. Estimate seismic source and elastic parameter reflectivities (again) by alternation and *Output Least Squares Inversion*

$$J_{OLS}[r,f] = \frac{1}{2}\{\|S[r,f] - S_{data}\|^2 + \lambda^2\|W[r,f]\|^2\}$$

Alternation Algorithm:

Repeat until convergence:

1. *Given the current source, f_c,and current reflectivity, r_c, invert for a new estimate of the reflectivity r_+ using* Output Least Squares *(i.e., $J_{OLS}[r,f] = \frac{1}{2}\|d^{pred}[r,f] - d^{obs}\|^2$)*
2. *Replace r_c by r_+.*
3. *Given the current source and reflectivity guesses, f_c, r_c, invert for a new estimate of the source f_+ using OLS.*
4. *Replace f_c by f_+.*

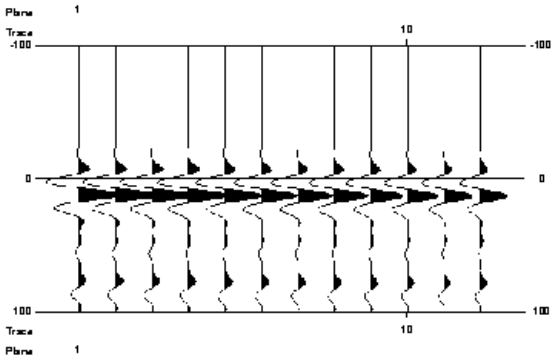
Two Experiments:

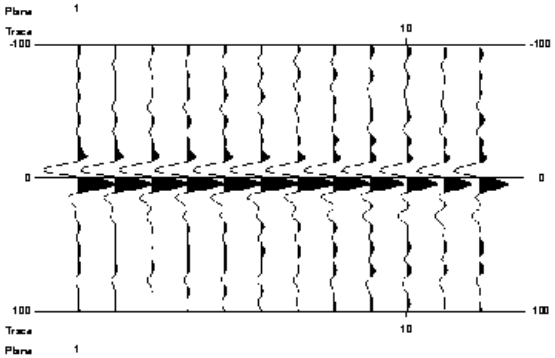
1. Model m = three elastic parameter reflectivities. (Source not updated in inversion)
2. Model m = three elastic parameter reflectivities *and seismic source*.

In both cases the data is the same. The fixed background velocity is the same. The starting guesses for the reflectivities are the same (zero). The algorithmic stopping tolerances are the same.

Sources

Figure: Left: air gun model source; Right: anisotropic source from inversion





Results:

Inversion-estimated source appears to allow for a better corresponding reflectivity estimate:

- Data Fit:
  1. Experiment 1 (air gun source)

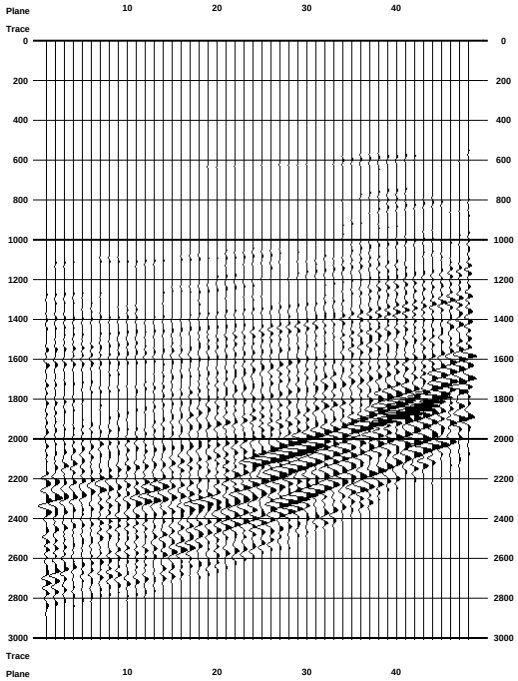
     55% rms error.
  2. Experiment 2 (inversion-estimated source)

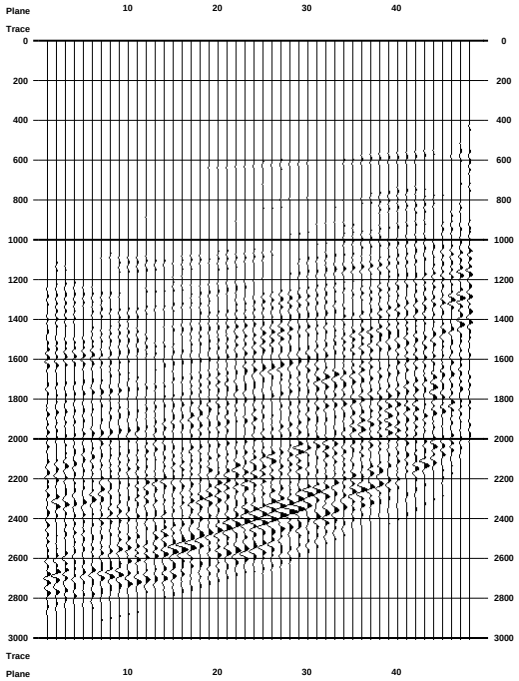
     27% rms error.
  3. In fact in Experiment 2 (inversion-estimated source)

     10% rms error in region around gas sand target.
- Model Fit: Experiment 2 reflectivity estimates match well log better than Experiment 1 estimates.

Data Misfit Comparison for Two experiments

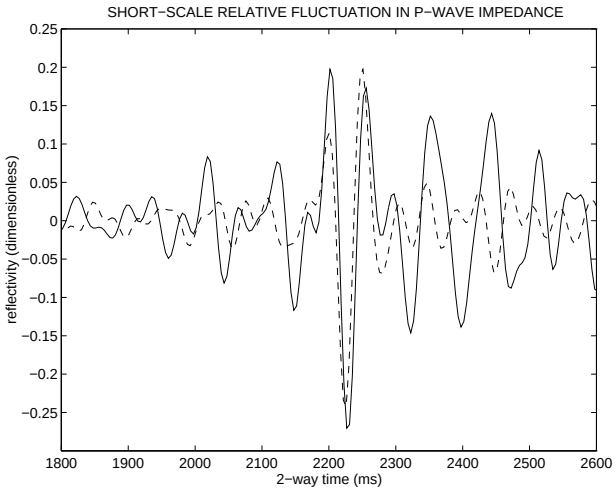
Figure: Left: 55% data misfit. Right: 29% data misfit

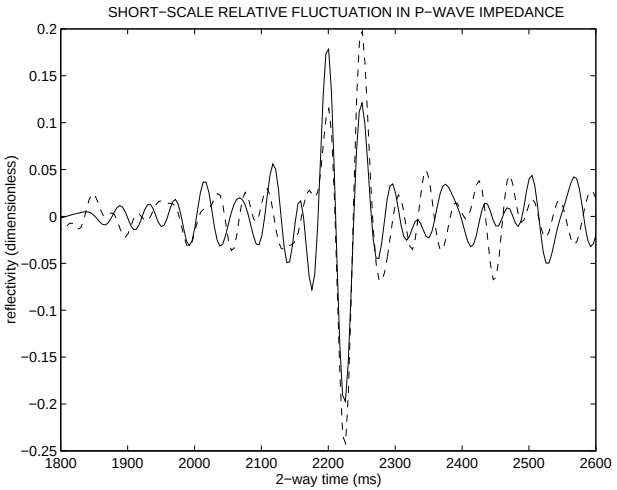




Well log comparisons for P-wave impedance

Figure: Left: air gun model source Right: inversion estimated source





The solid line shows the inversion result. The dashed line shows the detrended well log.

For More Detail See...

1. Minkoff, S. E. and Symes, W. W., "Full Waveform Inversion of Marine Reflection Data in the Plane-Wave Domain", *Geophysics*, **62** pp. 540–553, 1997.

2. Minkoff, S. E., "A Computationally Feasible Approximate Resolution Matrix for Seismic Inverse Problems", *Geophysical Journal International*, **126**, pp. 345–359, 1996.

3. Minkoff, S. E. and Symes, W. W., "Estimating the Energy Source and Reflectivity by Seismic Inversion", *Inverse Problems* **11**, pp. 383–395, 1995.

Problem: Running inversion experiments is expensive...

Upscaling

- One would like to simulate wave propagation on a coarser scale than the one on which the parameters are defined.
- We adapted the subgrid upscaling technique developed for elliptic problems (flow in porous media) to the wave equation.
- Goal is to be able to solve the problem on the coarser grid while still capturing some of the small scale features internal to coarse grid blocks.
- Operator upscaling is one option. Original reference: Arbogast, Minkoff & Keenan, "An Operator-Based Approach to Upscaling the Pressure Equation", *Computational Methods in Water Resources XII* (1998).
- Advantages: No Scale Separation or Periodic Medium Requirements.

Model problem

The Acoustic Wave Equation

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} - \nabla \cdot \left(\frac{1}{\rho}\nabla p\right) = f$$

p is the pressure
$c(x, y)$ is the sound velocity,
$\rho(x, y)$ is the density,
f is the source of acoustic energy

The First Order System

$$\begin{cases} \vec{v} = -\dfrac{1}{\rho}\nabla p & \text{in } \Omega, \\ \dfrac{1}{\rho c^2}\dfrac{\partial^2 p}{\partial t^2} = -\nabla \cdot \vec{v} + f & \text{in } \Omega, \end{cases}$$

Boundary conditions

$$\vec{v} \cdot \nu = 0, \text{ on } \partial\Omega$$

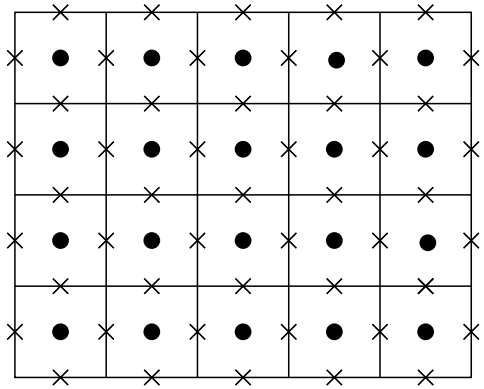
The finite element method

Find $\vec{v} \in V$ and $p \in W$ such that

$$\begin{cases} \langle \rho \vec{v}, \vec{u} \rangle = \langle p, \nabla \cdot \vec{u} \rangle, \\ \left\langle \dfrac{1}{\rho c^2} \dfrac{\partial^2 p}{\partial t^2}, w \right\rangle = -\langle \nabla \cdot \vec{v}, w \rangle + \langle f, w \rangle \end{cases}$$

for all $\vec{u} \in V$ and $w \in W$

- W={piecewise discontinuous constant functions}
- V={piecewise continuous linear functions of the form $(a_1 x + b_1, a_2 y + b_2)$}

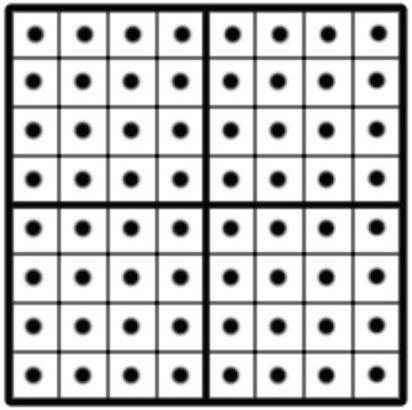


Full fine grid

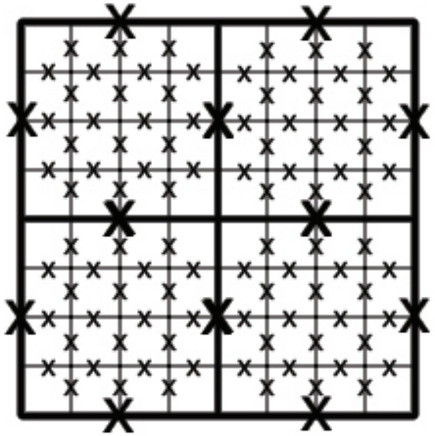
- Pressure
- × Acceleration

Upscaling technique

- **Goal**: Capture the fine scale behavior on the coarser grid



Pressure Unknowns



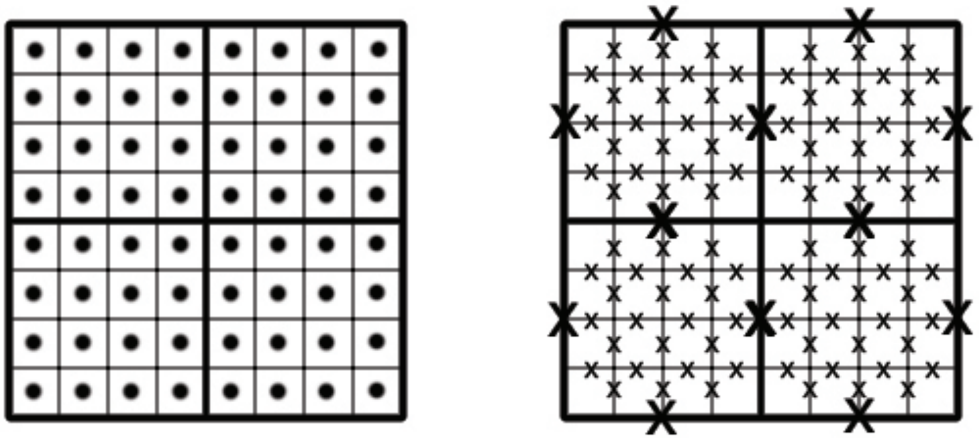
Acceleration Unknowns

- **Idea**: Decompose the solution

$$\vec{v} = \vec{v}^c + \delta\vec{v}$$

  - $\vec{v}^c \in V^c$ is the coarse-scale solution
  - $\delta\vec{v} \in \delta V$ are the fine-grid unknowns internal to each coarse-grid cell – subgrid unknowns

Upscaling technique



- **Simplifying assumption:** $\delta\vec{v} \cdot \nu = 0$ on the boundary of each coarse cell.
- This assumption allows us to decouple the subgrid problems from coarse-grid block to coarse-grid block.
- We use full fine-grid pressure.

Two steps of upscaling

- **Step 1**: On each coarse element solve the subgrid problem:
  - Find $\delta\vec{v} \in \delta V$ and $p \in W$ such that

$$\begin{cases} \langle \rho(\vec{v}^c + \delta\vec{v}), \delta\vec{u} \rangle = \langle p, \nabla \cdot \delta\vec{u} \rangle, \\ \left\langle \dfrac{1}{\rho c^2} \dfrac{\partial^2 p}{\partial t^2}, w \right\rangle = -\langle \nabla \cdot (\vec{v}^c + \delta\vec{v}), w \rangle + \langle f, w \rangle \end{cases}$$

    for all $\delta\vec{u} \in \delta V$ and $w \in W$
  - **Note**: the value of $\vec{v}^c$ is unknown at this stage.
  - The subgrid problem fully determines p.
- **Step 2**: Use the solution of subgrid problem to solve the coarse problem:
  - Find $\vec{v}^c \in V^c$ such that

$$\langle \rho(\vec{v}^c + \delta\vec{v}), \vec{u}^c \rangle = \langle p, \nabla \cdot \vec{u}^c \rangle, \quad \text{for all} \quad \vec{u}^c \in V^c$$

Parallel implementation

Serial cost

$$(N_p + 2N_u)N_\delta\, N_c + O(N_c)$$

Parallel cost

$$\frac{(N_p + 2N_u)N_\delta\, N_c}{p} + O(N_c)$$

N_p and N_u are the number of flops required to solve for pressure and acceleration respectively,
N_δ and N_c are the sizes of the subgrid and coarse problems,
p is the number of processors.

- **Subgrid problems:** Embarrassingly parallel.
  - No communication between processors.
  - No ghost-cell memory allocation required.
- **Coarse problem:** Solve in serial.
- **Post-processing:** Cheap and fast.

Timing Studies

- Numerical grid contains 3600×3600 fine grid blocks and 36×36 coarse grid blocks.
- timings are for 20 time steps.

number of processors	total time	subgrid problems	coarse problem	post-processing
1	29.70	29.69	0.00060	0.0026
2	15.46	15.38	0.00045	0.0711
4	7.63	7.56	0.00049	0.0707
6	5.23	5.14	0.00048	0.0749
8	4.37	4.26	0.00046	0.0896
12	3.07	2.94	0.00045	0.1150

References: Vdovina, T., Minkoff, S., and Korostyshevskaya, O., "Operator Upscaling for the Acoustic Wave Equation", *Multiscale Modeling and Simulation*, **4**, pp. 1305–1338, 2005.

Vdovina, T. and Minkoff, S., "An A Priori Error Analysis of Operator Upscaling for the Acoustic Wave Equation", *International Journal of Numerical Analysis and Modeling*, **5**, pp. 543–569, 2008.

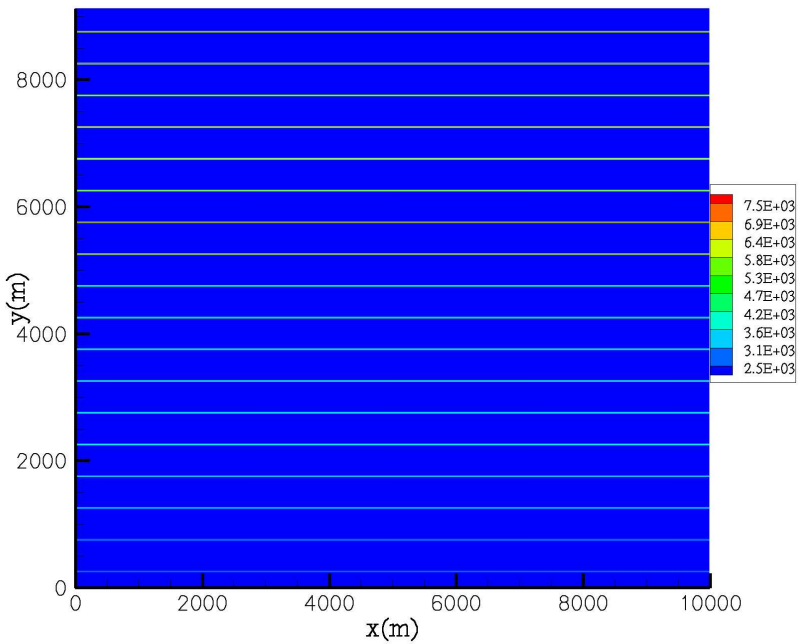
Timing Studies:

number of processors	Finite-difference code	Upscaling code number of fine blocks per coarse block			
	100×100	100×100	60×60	50×50	40×40
1	29.43	29.69	29.92	30.31	31.01
2	–	15.46	15.35	15.61	16.13
4	–	7.63	7.97	8.59	8.84
6	–	5.23	5.62	5.88	6.37
8	–	4.37	5.12	7.52	7.20
12	–	3.07	3.82	3.97	4.43

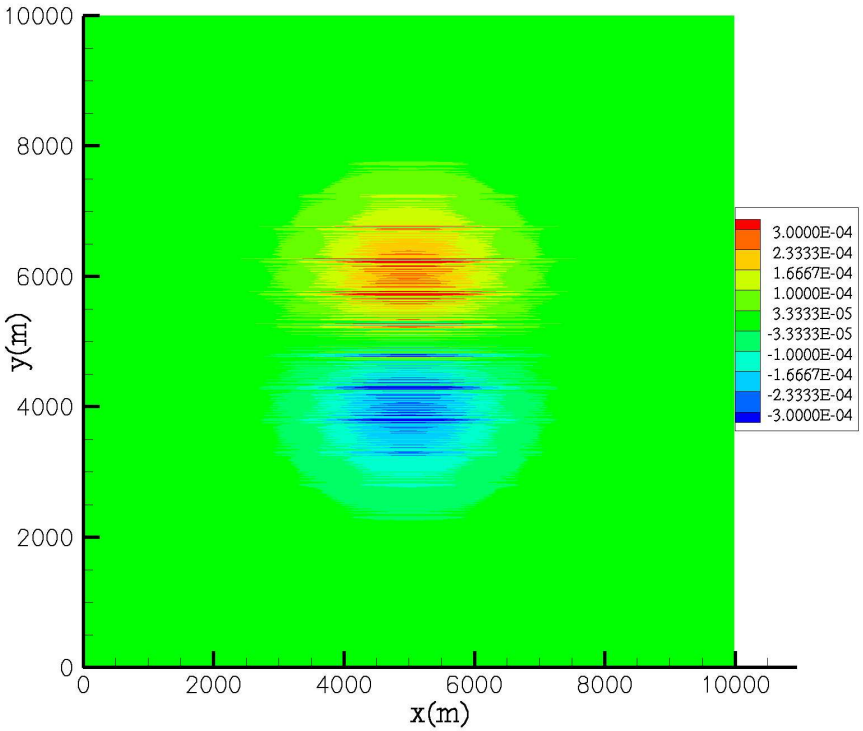
Note: Performance improves as coarse block size increases.

Numerical Experiment I (Acoustic Wave Equation)

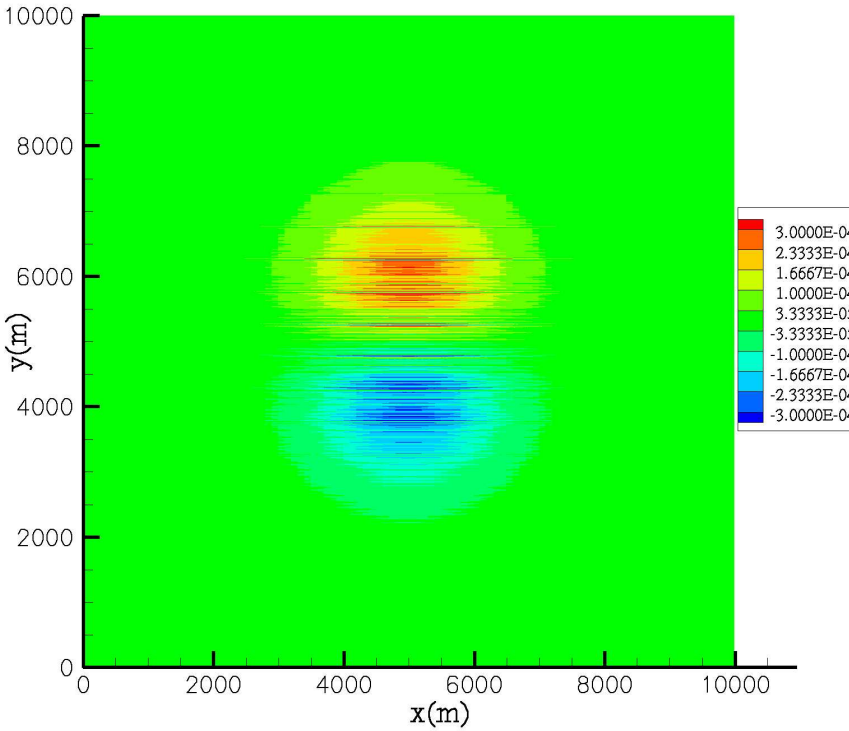
- Domain is of size 10×10 km.
- Fine grid: 1000×1000. Coarse grid: 100×100.
- Gaussian source, 200 time steps.
- Sound velocity strips range from 3500 m/s to 7500 m/s.



Vertical Acceleration



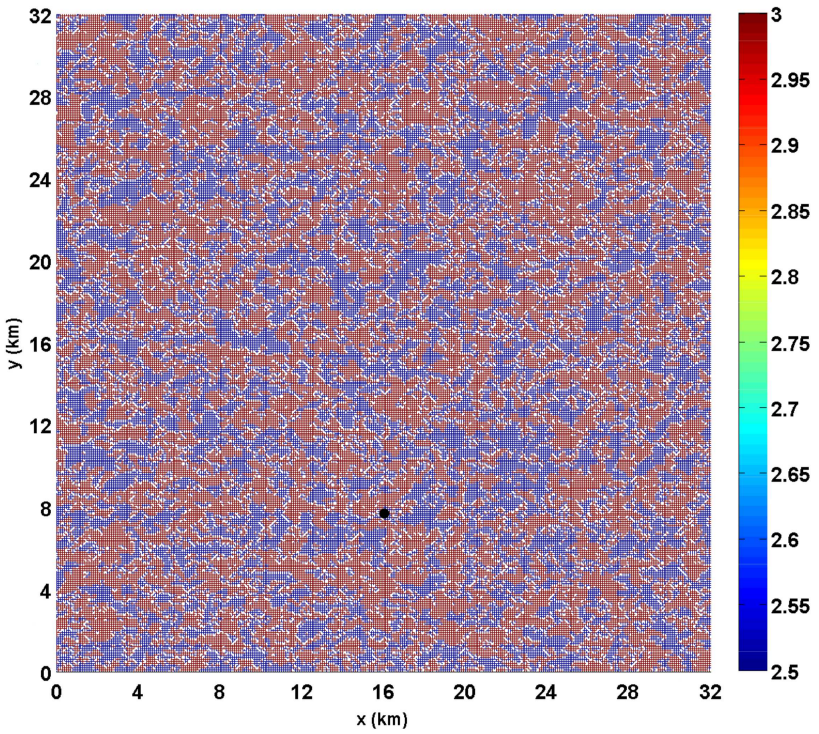
Full finite-difference soln



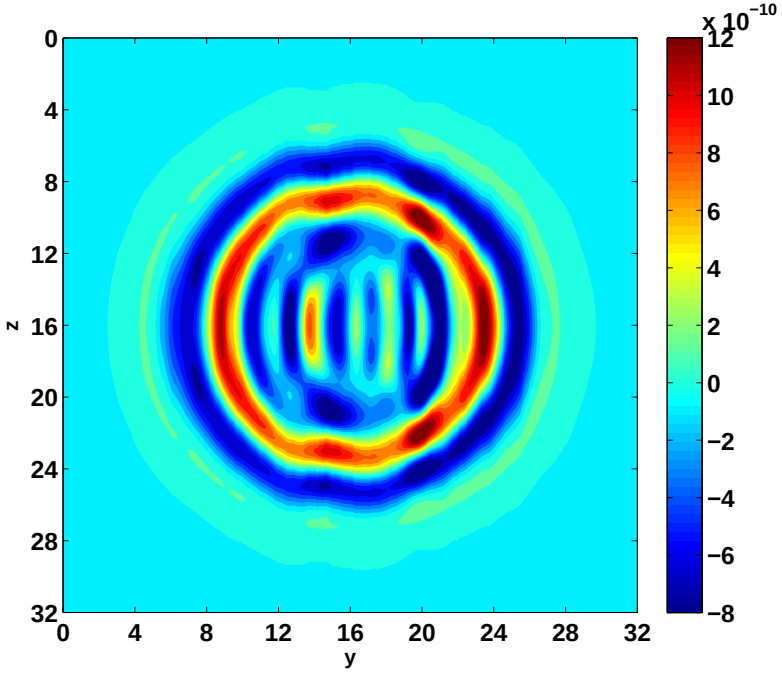
Reconstructed Upscaled Soln

Numerical Experiment II (Elastic Wave Equation)

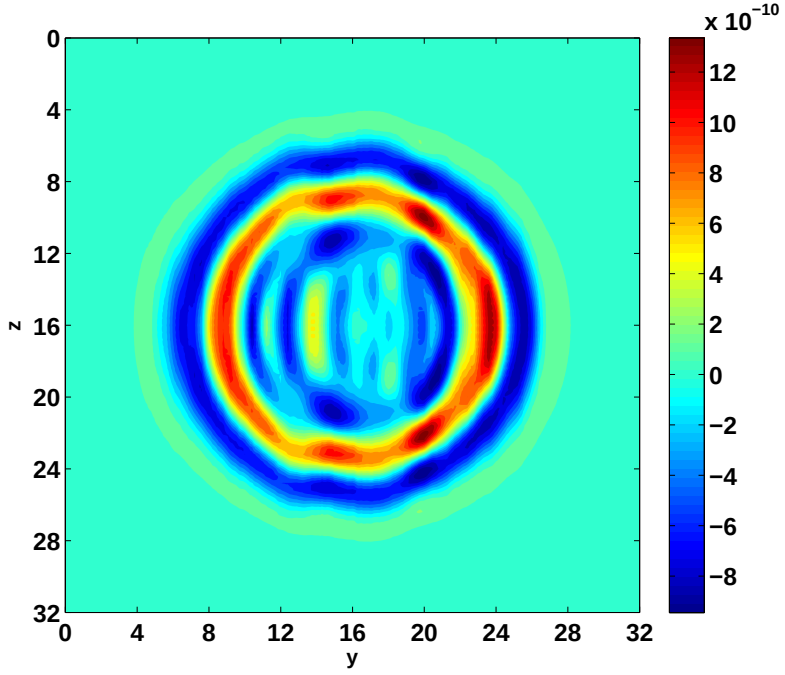
- Fine grid: $320 \times 320 \times 320$. Coarse grid: $80 \times 80 \times 80$.
- Source is Gaussian in space, Ricker in time.
- Wave propagates about 7 wavelengths.
- Mixture of two materials with $V_p = 2.5$ km/s or 3 km/s; $V_s = 1.5$ and 1.75 km/s respectively, $\rho = 2.2 \cdot 10^{12}$ and $2.3 \cdot 10^{12}$ kg/km^3.



yz slices of first component of velocity solution



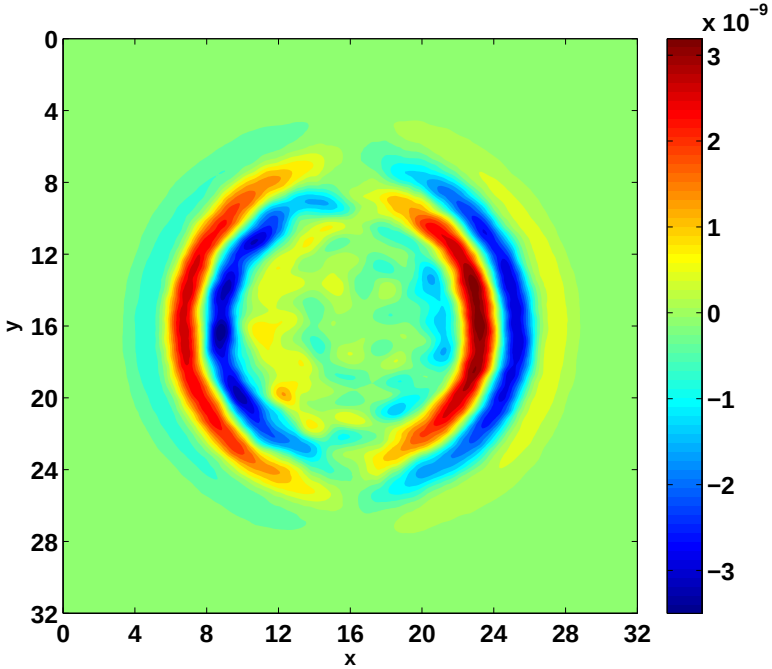
Full finite-element soln



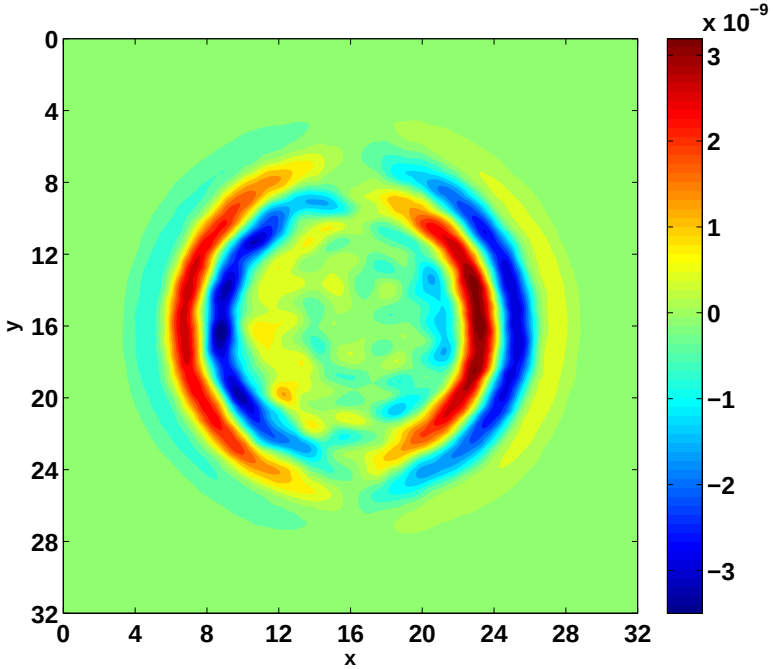
Augmented Upscaled Soln

Reference: Vdovina, T., Minkoff, S., and Griffith, S., "A Two–Scale Solution Algorithm for the Elastic Wave Equation," *SIAM Journal on Scientific Computing*, **31**, pp. 3356-3386, 2009.

xy slices of first component of velocity solution



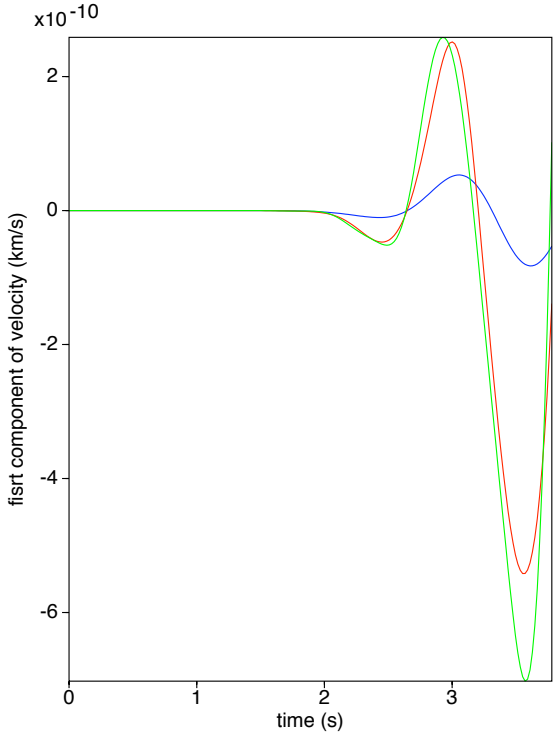
Full finite-element soln



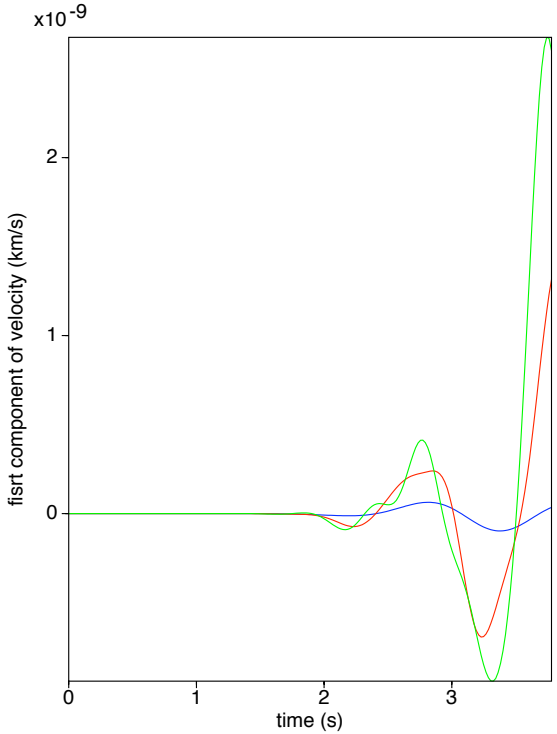
Augmented Upscaled Soln

Comparison of time traces for the first component of velocity

Red curve = full finite element solution. Green curve = coarse solution. Blue curve = full finite element solution for the homogeneous medium with a single (average) value for each of the three input parameter.



Receiver at (16 km, 8 km, 12 km)



Receiver at (16 km, 8 km, 18 km)

Order of Accuracy Experiment 1

- Number of coarse grid blocks is fixed at 100 in x and y. Number of fine blocks per coarse block changes in each experiment.

Number of fine blocks	Number of coarse blocks	Time step (s)	Number of time steps	$\frac{\|\|P-p\|\|_0}{\|\|p\|\|_1}$	$\frac{\|\|\mathbf{U}-\mathbf{u}\|\|_0}{\|\|\mathbf{u}\|\|_1}$
400×400	100×100	$1.77 \cdot 10^{-4}$	200	$5.48 \cdot 10^{-3}$	$1.28 \cdot 10^{-2}$
800×800	100×100	$8.83 \cdot 10^{-5}$	400	$2.75 \cdot 10^{-3}$	$1.11 \cdot 10^{-2}$
1600×1600	100×100	$4.42 \cdot 10^{-5}$	800	$1.40 \cdot 10^{-3}$	$1.06 \cdot 10^{-2}$
3200×3200	100×100	$2.21 \cdot 10^{-5}$	1600	$7.27 \cdot 10^{-4}$	$1.05 \cdot 10^{-2}$
6400×6400	100×100	$1.10 \cdot 10^{-5}$	3200	$3.97 \cdot 10^{-4}$	$1.04 \cdot 10^{-2}$

- Linear convergence in pressure (h).

Reference: Vdovina, T. and Minkoff, S., "An A Priori Error Analysis of Operator Upscaling for the Acoustic Wave Equation", *International Journal of Numerical Analysis and Modeling*, **5**, pp. 543–569, 2008.

Order of Accuracy Experiment 2

- Number of fine grid blocks is fixed at 6400 in x and y. Number of coarse blocks varies in each experiment.

Number of fine blocks	Number of coarse blocks	$\frac{\|\|P-p\|\|_0}{\|\|p\|\|_1}$	$\frac{\|\|\mathbf{U}-\mathbf{u}\|\|_0}{\|\|\mathbf{u}\|\|_1}$
6400×6400	100×100	$3.97 \cdot 10^{-4}$	$1.04 \cdot 10^{-2}$
6400×6400	200×200	$3.50 \cdot 10^{-4}$	$5.24 \cdot 10^{-3}$
6400×6400	400×400	$3.41 \cdot 10^{-4}$	$2.65 \cdot 10^{-3}$
6400×6400	800×800	$3.39 \cdot 10^{-4}$	$1.38 \cdot 10^{-3}$
6400×6400	1600×1600	$3.39 \cdot 10^{-4}$	$7.97 \cdot 10^{-4}$

- Linear convergence in acceleration (H).

An Alternate Analysis:

Reference: Korostyshevskaya, O. and Minkoff, S., "A Matrix Analysis of Operator-Based Upscaling for the Wave Equation", *SIAM J. Numer. Anal.*, **44**, pp. 586–612, 2006.

Recall two steps of upscaling algorithm:

- **Subgrid problems**: On each coarse element solve the subgrid problem:
  - Find $\delta\vec{v} \in \delta V$ and $p \in W$ such that

$$\begin{cases} \langle \rho(\vec{v}^c + \delta\vec{v}), \delta\vec{u} \rangle = \langle p, \nabla \cdot \delta\vec{u} \rangle, \\ \left\langle \frac{1}{\rho c^2} \frac{\partial^2 p}{\partial t^2}, w \right\rangle = -\langle \nabla \cdot (\vec{v}^c + \delta\vec{v}), w \rangle + \langle f, w \rangle \end{cases}$$

  for all $\delta\vec{u} \in \delta V$ and $w \in W$
- **Coarse problem**: Use the solution of subgrid problem to solve the coarse problem:
  - Find $\vec{v}^c \in V^c$ such that

$$\langle \rho(\vec{v}^c + \delta\vec{v}), \vec{u}^c \rangle = \langle p, \nabla \cdot \vec{u}^c \rangle, \quad \text{for all} \quad \vec{u}^c \in V^c$$

Use finite element expansions to obtain the matrix-vector form

- **Subgrid problems**:

$$A^{cf}\mathbf{v_x^c} + A^{ff}\delta\mathbf{v_x} = B^f\mathbf{p}$$

$$W\frac{\partial^2 \mathbf{p}}{\partial t^2} = -(B^c)^T\mathbf{v_x^c} - (B^f)^T\delta\mathbf{v_x} - (v_y \text{ terms}) + F$$

- **Coarse problem**:

$$A^{cc}\mathbf{v_x^c} + (A^{cf})^T\delta\mathbf{v_x} = B^c\mathbf{p}$$

- Matrix entries

$$a^{ff}_{l,n} = \langle \rho(\delta u_x)_n, (\delta u_x)_l \rangle \qquad a^{cf}_{l,i} = \langle \rho(u^c_x)_i, (\delta u_x)_l \rangle \qquad a^{cc}_{l,i} = \langle \rho(u^c_x)_i, (u^c_x)_l \rangle$$

$$w_{l,m} = \left\langle \frac{1}{\rho c^2} w_m, w_l \right\rangle \qquad b^f_{l,m} = \left\langle w_m, \frac{\partial(\delta u_x)_l}{\partial x} \right\rangle \qquad b^c_{l,m} = \left\langle w_m, \frac{\partial(u^c_x)_l}{\partial x} \right\rangle$$

$$f_l = \langle f, w_l \rangle$$

Matrix equation

- Idea: Eliminate the subgrid acceleration from the coarse equation
  - Subgrid problem

$$\delta \mathbf{v_x} = -(A^{ff})^{-1} A^{cf} \mathbf{v_x^c} + (A^{ff})^{-1} B^f \mathbf{p}$$

  - Coarse problem

$$(A^{cc} - (A^{cf})^T (A^{ff})^{-1} A^{cf}) \mathbf{v_x^c} = (B^c - (A^{cf})^T (A^{ff})^{-1} B^f) \mathbf{p}$$

- Obtain a matrix equation for the coarse acceleration and pressure only

$$U \mathbf{v_x^c} = D \mathbf{p}$$

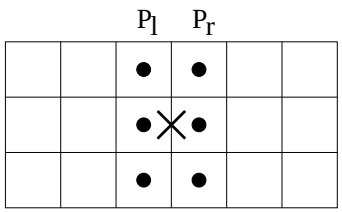
- Find explicit formulas for the entries of U and D to derive a difference equation

Differential equation

Differential equation (from Taylor series expansion)

$$\rho^{ups} v_x^c = -\frac{\partial p}{\partial x}$$

ρ^{ups} is the upscaled density given by the average of the density values on the boundary of the coarse cell.



Recall one of two original continuous pde's in our system: $\vec{v} = -\frac{1}{\rho}\nabla p$

Pressure Equation Analysis

Assuming appropriate smoothness on pressure p and density ρ, the upscaling algorithm solves the following differential equation for pressure inside a coarse block:

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial}{\partial x}(\rho^{-1}\frac{\partial p}{\partial x}) + \frac{\partial}{\partial y}(\rho^{-1}\frac{\partial p}{\partial y}) + f,$$

[order of approximation $O(h_x^2 + h_y^2)$].

Along coarse block edges upscaling satisfies the following equation for pressure:

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial}{\partial x}\left((\rho^{ups})^{-1}\frac{\partial p}{\partial x}\right) + \left((\rho^{ups})^{-1}\frac{\partial^2 p}{\partial x \partial y}\right)K + \frac{\partial}{\partial y}\left((\rho^{ups})^{-1}\frac{\partial p}{\partial y}\right) + f,$$

[order of approximation $O(h_x + h_y)$] and K is a constant which depends on the location of the pressure node within a single coarse cell.

Solving the Inverse Problem

Want to find c given some observed p.

- Easier to find $m = \frac{1}{c^2}$, squared slowness.
- The least squares functional is given by

$$\mathcal{J}(m) = \frac{1}{2}\sum_{s,r}\left[\int_0^T (S_{s,r}p - d_{s,r})^2 + (T_{s,r}(\nabla p) - b_{s,r})^2\, dt\right].$$

- s and r are indices over source and receiver position.
- $b_{s,r}$ and $d_{s,r}$ are observed values for p and ∇p.
- $S_{s,r}$ and $T_{s,r}$ are sampling operators, restricting the values of p and ∇p to locations with observed values.

(Reference: Plessix, *A Review of the Adjoint State Method For Computing the Gradient of a Functional with Geophysical Applications*)

The Formal Optimization Problem

$$\text{Find } \min_m \mathcal{J}(\phi, m) \text{ s.t. } F(\phi, m) = 0.$$

- ϕ is the state variable (pressure).
- m is the control variable (squared slowness).
- $F(\phi, m) = 0$ is the forward problem (the wave equation).

$$\text{Method: } \delta_m \mathcal{J} = \delta_\phi \mathcal{J} \circ \delta_m \phi + \delta_m^e \mathcal{J}.$$

- $\delta_m^e \mathcal{J}$ is the explicit variation of $\mathcal{J}$ w.r.t. m.

$$\text{Problem: How do we find } \delta_m \phi?$$

(Reference: Gunzburger's *Perspectives in Flow Control and Optimization*.)

The Adjoint Problem

- The adjoint problem is:

$$
\begin{aligned}
\frac{1}{c^2}\frac{\partial^2 \lambda}{\partial t^2} - \Delta\lambda &= \sum_r S^*_{s,r}\left(S_{s,r}p - d_{s,r}\right) - \sum_r \nabla\cdot T^*_{s,r}\left(T_{s,r}\left(\nabla p\right) - b_{s,r}\right) \\
\lambda\,|_{t=T} &= 0 \\
\frac{\partial\lambda}{\partial t}\,|_{t=T} &= 0 \\
\left(\nabla\lambda + T^*_{s,r}\left(T_{s,r}\left(\nabla p\right) - b_{s,r}\right)\right)\cdot\nu &= 0 \text{ on } \Gamma.
\end{aligned}
$$

- The auxiliary conditions are:

$$
\begin{aligned}
\mu_0 &= m\frac{\partial\lambda}{\partial t}\,|_{t=0} \\
\mu_1 &= -m\lambda\,|_{t=0} \\
\psi &= -\lambda \text{ on } \Gamma.
\end{aligned}
$$

Obtaining an Expression for the Gradient

Question: How do we get $D_m\mathcal{J}^{(n)}(x,y)$?

- To get $D_m\mathcal{J}^{(n)}(x,y)$ from $\delta_m\mathcal{J}$, we let $\tilde{m}$ be $\delta(x,y)$.
- This gives:

$$(D_m\mathcal{J})(x,y) = -\sum_s \left[\int_0^T \frac{\partial^2 p(x,y)}{\partial t^2}\lambda(x,y)\,dt\right].$$

- To calculate the gradient, we must have access to both the forward and adjoint solutions at all time steps.
- Use a checkpointing scheme to optimally determine what to store and what to recalculate.

(Reference: Gunzburger's *Perspectives in Flow Control and Optimization*.)

An Optimization Algorithm

- Beginning with $m^{(0)}(x,y)$, we repeat the following:
  1. Solve the forward problem to get the state variables $p^{(n)}(x,y,t)$ and then solve the adjoint problem to get the adjoint variable $\lambda^{(n)}(x,y,t)$.
  2. Calculate the gradient $D_m\mathcal{J}^{(n)}(x,y) = -\sum_s \left[\int_0^T \frac{\partial^2 p(x,y)}{\partial t^2}\lambda(x,y)\,dt\right]$.
  3. Update squared slowness for the next step:

$$m^{(n+1)}(x,y) = m^{(n)}(x,y) + \beta_n \sum_s \left[\int_0^T \frac{\partial^2 p(x,y)}{\partial t^2}\lambda(x,y)\,dt\right],$$

using some step size β_n.

The Upscaled Adjoint Problem

Introduce $\chi = -\nabla\lambda$ and decompose χ into $\delta\chi + \chi^c$.

- Then, for each subgrid problem,

$$
\begin{aligned}
(\chi^c + \delta\chi, \delta u) &= (\lambda, \nabla \cdot \delta\mathbf{u}) \\
\left(\frac{1}{c^2}\frac{\partial^2\lambda}{\partial t^2}, w\right) &= -\left(\nabla \cdot (\chi^c + \delta\chi), w\right) \\
&\quad + \left(\sum_r S^*_{s,r}\left(S_{s,r}p - d_{s,r}\right) - \sum_r \nabla \cdot T^*_{s,r}\left(T_{s,r}\left(\nabla p\right) - b_{s,r}\right), w\right)
\end{aligned}
$$

for $w \in W$ and $\delta\mathbf{u} \in \delta V$.

- And, for each coarse problem,

$$
(\chi^c + \delta\chi, \mathbf{u}^c) = (\lambda, \nabla \cdot \mathbf{u}^c)
$$

for $\mathbf{u}^c \in V^c$.

Conclusions

- To speed up an expensive iterative process like seismic inversion, operator upscaling can be applied to the wave equation.
- The upscaled solution captures some of the sub-wavelength heterogeneities of the full solution.
- For the acoustic implementation, the method exhibits good parallel performance due to the independence of the subgrid problems (most expensive part of the algorithm).
- A matrix analysis of the algorithm indicates that, in fact, the upscaling technique is solving the continuous differential equation with density given by averaging along coarse block edges.
- The pressure equation corresponds to the standard acoustic wave equation at nodes internal to coarse blocks.
- Along coarse cell boundaries, the upscaled solution solves a modified wave equation which includes a mixed second-derivative term.
- Using the adjoint method we can reuse much of the upscaling code to solve the inverse problem.

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