ADAPTIVE AMG FOR DIFFUSION EQUATIONS WITH STOCHASTIC COEFFICIENTS

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ABSTRACT. We are interested in solving a 2^{nd}-order diffusion equation where there is some uncertainty in the conductivity field, k:

$$-\text{div}\left(k\left(x,\omega\right)\nabla p\left(x,\omega\right)\right) \quad = \quad f\left(x\right) \text{ in } D\times\Omega, \tag{1}$$

subject to some boundary conditions that may or may not contain uncertainty as well. One popular technique for quantifying uncertainty in (1) is the Markov Chain Monte Carlo (MCMC) method, which requires solving the deterministic version of (1) for many thousands of realizations of k. If the correlation lengths in the stochastic process defining k are small compared to the size of the domain, the solution of even a single realization of (1) is nontrivial and may require adaptivity. Our goal is: assuming that we have constructed an efficient multilevel solver for (1) based on some realization of $k\left(x,\omega\right)$, we would like to efficiently construct a multilevel solver for (1) based on some new realization, $k(x,\hat{\omega})$. The nature of MCMC makes this possible because consecutive realizations of k are often similar.

We propose an adaptive algebraic multigrid method which constructs a sequence of coarse spaces by identifying elements on the coarse mesh with agglomerates of elements on the fine mesh. Interpolation between spaces is defined by representing a small part of the spectrum of the local stiffness matrix associated with each fine-mesh agglomerate on the coarse mesh. This requires solving a generalized eigenproblem on each agglomerate. If adaptivity is needed we can expose algebraically smooth error by applying several iterations of the current method to the homogeneous problem $A(\omega)e = 0$. The resulting error mode can then be added to the coarse space to improve performance.

The method is entirely defined by two components: the hierarchy of meshes and the collection of interpolation operators. In the MCMC method, only the entries of the interpolation operators need change between realizations of k. Then, assuming an efficient solver for (1) based on $k\left(x,\omega\right)$, we solve (1) based on $k(x,\hat{\omega})$ by first testing the current method on $A(\hat{\omega})e = 0$. If the performance is acceptable we retain the previous method. If convergence stalls we take the resultant smooth error component and incorporate it into the coarse space. This requires the solution of an additional generalized eigenproblem on each agglomerate, but of a much smaller size than in the initial adaptive setup. We demonstrate the effectiveness of the method by using MCMC to condition conductivity samples to dynamic pressure data with random walk and Langevin transition kernels.

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