Models with time-dependent parameters using transform methods: Application to Heston's model

Alberto Elices

Model Validation Group
Risk Division
Grupo Santander

Kick-off-Workshop
Special Semester on Stochastics with empphasis on Finance
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
September 8-12th 2008, Linz, Austria

Grupo Santander

Outline of the presentation

- Introduction.
- Characteristic functions of models with time-dependent parameters.
- Application to Heston's model.
- Case study: Calibration to Eurostoxx 50.
- Application to Forward start options.
- Forward skew of Heston's model.
- Conclusions.

Introduction

- Exotic valuation: usually carried out with Monte Carlo.
- Calibration: fast analytic models are needed for valuation of vanilla products.
- Analytic models depend on just a few parameters which cannot fit the whole set of market parameters.
- More degrees of freedom are needed in order to calibrate the market across all maturities.
- The most natural way of introducing more parameters is to let them depend on time.

- **Characteristic function methods:**
  - Useful when the characteristic function is analytic.
  - The Inversion of the characteristic function is carried out through the inverse Fourier transform.

- **Characteristic function:**

$$\varphi_{uv}(\mathbf{X}/\mathbf{x}_u) = \mathbf{E}\left(e^{i\mathbf{X}\cdot\mathbf{x}_v}\right) = \int_{\mathbf{R}^N} e^{i\mathbf{X}\cdot\mathbf{x}_v} f_{uv}(\mathbf{X}/\mathbf{x}_u)\,d\mathbf{x}_v$$

- **Family of characteristic functions for which the methodology can be applied:**

$$\varphi_{uv}(\mathbf{X}/\mathbf{x}_u) = \exp\left(C_{uv}(\mathbf{X}) + \mathbf{D}_{uv}(\mathbf{X}) \cdot \mathbf{x}_u\right)$$

$$\mathbf{x}(t) = (x_1(t), \cdots, x_N(t)) \qquad \mathbf{X} = (X_1, \cdots, X_N)$$

$$\mathbf{D}_{uv}(\mathbf{X}) = (D_{uv,1}(\mathbf{X}), \cdots, D_{uv,N}(\mathbf{X}))$$

- **The method proposed introduces time-dependent parameters for a wide variety of models which admit analytic characteristic function:**
  - Merton jump model. $\varphi_{uv}(G/g_u) = \exp\left(C_{uv}(G) + iGg_u\right)$

g_u *: sum of all Poisson distributed jumps up to time* t_u .

- Cox Ingersoll Ross model.

$$\varphi_{uv}(R/r_u) = \exp\left(C_{uv}(R) + iD_{uv}(R)r_u\right)$$

r_u : *short rate interest rate at time* t_u.

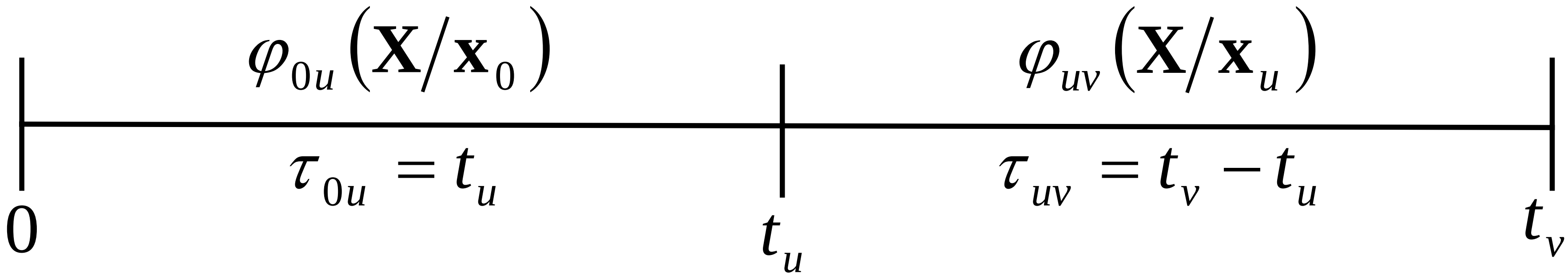
- Heston stochastic volatility model.

$$\varphi_{uv}(X,V/x_u,v_u) = \exp\left(C_{uv}(X,V) + D_{uv}(X,V)v_u + iXx_u\right)$$

x_u : *logarithm of underlying.* v_u : *variance process.*

- Hybrids with jumps, stochastic interest rates and volatility.

$$\varphi_{uv}(X,V,R,G/x_u,v_u,r_u,g_u) = e^{C_{uv}+D_{uv,2}r_u+D_{uv,1}v_u+iXx_u+iGg_u}$$



- **All relevant information of a Markov process with independent increments at an instant t_v is given by the joint probability distribution: $\varphi_{0v}(\mathbf{X}/\mathbf{x}_0)$**

- **Objective: Find $\varphi_{0v}(\mathbf{X}/\mathbf{x}_0)$ in terms of** $\varphi_{0u}(\mathbf{X}/\mathbf{x}_0)$ and $\varphi_{uv}(\mathbf{X}/\mathbf{x}_u)$

- **Characteristic function under search:**

$$\varphi_{0v}\left(\mathbf{X}/\mathbf{x}_0\right)=\int_{\mathbf{R}^N} d\mathbf{x}_v e^{i\mathbf{X}\cdot\mathbf{x}_v} f_{0v}\left(\mathbf{x}_v/\mathbf{x}_0\right)$$

- **Joint density $t_0 \rightarrow t_v$ in terms of densities $t_0 \rightarrow t_u$ and $t_u \rightarrow t_v$ (independent increments):**

$$f_{0v}\left(\mathbf{x}_v/\mathbf{x}_0\right)=\int_{\mathbf{R}^N} d\mathbf{x}_u f_{0u}\left(\mathbf{x}_u/\mathbf{x}_0\right) f_{uv}\left(\mathbf{x}_v/\mathbf{x}_u\right)$$

- **Subtituting $f_{0v}\left(\mathbf{x}_v/\mathbf{x}_0\right)$ in $\varphi_{0v}\left(\mathbf{X}/\mathbf{x}_0\right)$:**

$$\varphi_{0v}\left(\mathbf{X}/\mathbf{x}_0\right)=\int_{\mathbf{R}^N} d\mathbf{x}_u f_{0u}\left(\mathbf{x}_u/\mathbf{x}_0\right) \underbrace{\int_{\mathbf{R}^N} d\mathbf{x}_v e^{i\mathbf{X}\cdot\mathbf{x}_v} f_{uv}\left(\mathbf{x}_v/\mathbf{x}_u\right)}_{\varphi_{uv}\left(\mathbf{X}/\mathbf{x}_u\right)=\exp\left(C_{uv}(\mathbf{X})+\mathbf{D}_{uv}(\mathbf{X})\cdot\mathbf{x}_u\right)}$$

- **After substituting** $\varphi_{uv}\left(\mathbf{X}/\mathbf{x}_u\right)$ **:**

$$\varphi_{0v}(\mathbf{X}/\mathbf{x}_0) = \int_{\mathbf{R}^N} d\mathbf{x}_u\, f_{0u}(\mathbf{x}_u/\mathbf{x}_0)\exp\left(C_{uv}(\mathbf{X}) + \mathbf{D}_{uv}(\mathbf{X})\cdot\mathbf{x}_u\right)$$

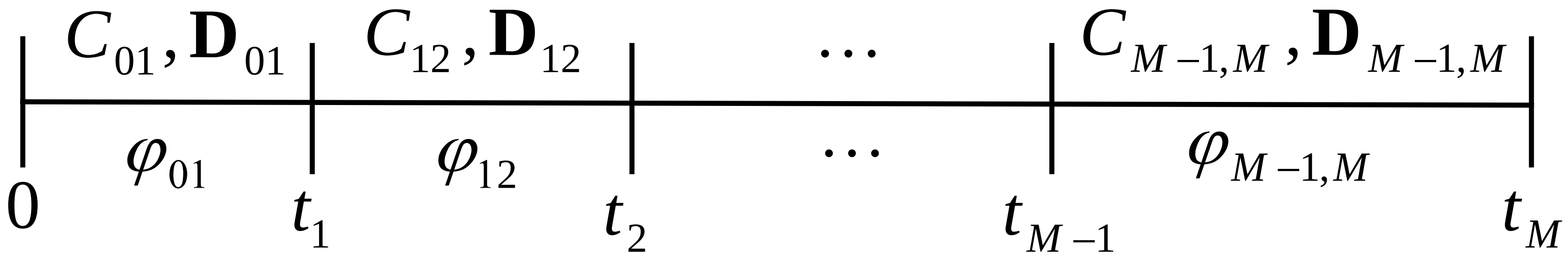
$$= \exp\left(C_{uv}(\mathbf{X})\right)\underbrace{\int_{\mathbf{R}^N} d\mathbf{x}_u\, f_{0u}(\mathbf{x}_u/\mathbf{x}_0)\exp\left(i(i^{-1}\mathbf{D}_{uv}(\mathbf{X}))\cdot\mathbf{x}_u\right)}_{\varphi_{0u}\left(i^{-1}\mathbf{D}_{uv}(\mathbf{X})/\mathbf{x}_0\right)}$$

$$= \exp\left(\underbrace{C_{uv}(\mathbf{X}) + C_{0u}\left(i^{-1}\mathbf{D}_{uv}(\mathbf{X})\right)}_{C_{0v}(\mathbf{X})} + \underbrace{\mathbf{D}_{0u}\left(i^{-1}\mathbf{D}_{uv}(\mathbf{X})\right)}_{D_{0v}(\mathbf{X})}\cdot\mathbf{x}_0\right)$$

- **Identifying terms:**

$$\varphi_{0v}(\mathbf{X}/\mathbf{x}_0) = \exp\left(C_{0v}(\mathbf{X}) + \mathbf{D}_{0v}(\mathbf{X})\cdot\mathbf{x}_0\right)$$

$$\begin{cases} C_{0v}(\mathbf{X}) = C_{uv}(\mathbf{X}) + C_{0u}\left(i^{-1}\mathbf{D}_{uv}(\mathbf{X})\right) \\ \mathbf{D}_{0v}(\mathbf{X}) = \mathbf{D}_{0u}\left(i^{-1}\mathbf{D}_{uv}(\mathbf{X})\right) \end{cases}$$



- **Heston process:**

$$\begin{cases} dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t \\ dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}\, dY_t \end{cases} \qquad d\langle W_t, Y_t\rangle = \rho dt$$

- **The two state variables for Heston's process are the logarithm of the stock price** $x_t = \log(S_t)$ **and the variance process** v_t **:**

$$\mathbf{x}_u = (x(t_u), v(t_u))$$

- **These two state variables translate into** X **and** V **for the characteristic function:**

$$\mathbf{X} = (X, V)$$

- **Joint characteristic function for Heston process:**

$$\varphi_{uv}\left(X,V/x(t_u),v(t_u)\right)=e^{C_{uv}(X,V)+D_{uv,2}(X,V)v(t_u)+D_{uv,1}(X,V)x(t_u)}$$

$$D_{uv,2}\left(X,V\right)=\frac{\kappa-\rho\sigma Xi+d}{\sigma^2}\left(\frac{g-\tilde{g}e^{-d\tau}}{1-\tilde{g}e^{-d\tau}}\right)\qquad D_{uv,1}\left(X,V\right)=iX$$

$$C_{uv}\left(X,V\right)=i\mu X\tau+\frac{\kappa\theta}{\sigma^2}\left(-2\ln\left(\frac{1-\tilde{g}e^{-d\tau}}{1-\tilde{g}}\right)+\left(\kappa-\rho\sigma Xi-d\right)\tau\right)$$

$$\tilde{g}=\frac{\kappa-\rho\sigma Xi-d-iV\sigma^2}{\kappa-\rho\sigma Xi+d-iV\sigma^2}\qquad g=\frac{\kappa-\rho\sigma Xi-d}{\kappa-\rho\sigma Xi+d}$$

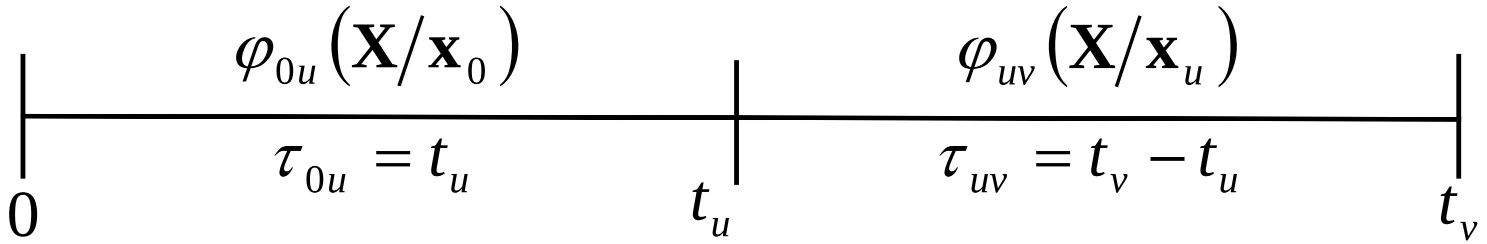
$$d=\sqrt{\left(\kappa-\rho\sigma Xi\right)^2+\sigma^2X\left(i+X\right)}$$

Application to Heston´s model

- **Characteristic function with time-dependent parameters at maturity t_v :**

$$\varphi_{0v}\left(X,V/x(t_0),v(t_0)\right)=e^{C_{0v}(X,V)+D_{0v,2}(X,V)v(t_0)+D_{uv,1}(X,V)x(t_0)}$$

$$\begin{cases} C_{0v}\left(X,V\right)=C_{uv}\left(X,V\right)+C_{0u}\left(X,i^{-1}D_{uv,2}(X,V)\right) \\ D_{0v,2}\left(X,V\right)=D_{0u,2}\left(X,i^{-1}D_{uv,2}(X,V)\right) \\ D_{0v,1}(X,V)=iX \end{cases}$$



- **Valuation of vanilla options:**

$$C = DF_T \mathbf{E}\left((S_T - K)^+\right) = DF_T \left(\underbrace{\mathbf{E}\left(e^{x_T} \mathbf{1}_{\{x_T > \ln K\}}\right)}_{\text{Asset or nothing}} - K \underbrace{\mathbf{E}\left(\mathbf{1}_{\{x_T > \ln K\}}\right)}_{\text{Cash or nothing}} \right)$$

- **Characteristic function for cash or nothing option:**

$$\varphi_{0T}^{\mathbf{CN}}(X/\mathbf{x}_0) = \varphi_{0T}(X,0/\mathbf{x}_0) = \int_{\mathbf{R}} e^{iX \cdot x_T} f_{0T}(x_T, v_T/\mathbf{x}_0) dx_T dv_T$$

- **Inversion formula: cummulative density in terms of characteristic function.**

$$P(x > a) = \frac{1}{2} + \frac{1}{2\pi} \int_0^\infty \frac{1}{iX} \left(\frac{\varphi(X)}{e^{iXa}} - \frac{\varphi(-X)}{e^{-iXa}} \right) dX$$

- **Characteristic function for asset or nothing option:**

$$\varphi_{0T}^{\mathbf{AN}}(X/\mathbf{x}_0) = \frac{\varphi_{0T}(X-i,0/\mathbf{x}_0)}{\varphi_{0T}(-i,0/\mathbf{x}_0)} = \frac{\mathbf{E}\left(e^{i(X-i)x_T}\right)}{\mathbf{E}(e^{x_T})} = \frac{\mathbf{E}\left(e^{iXx_T}e^{x_T}\right)}{\mathbf{E}(S_T)}$$

$$\varphi_{0T}^{\mathbf{AN}}(X/x_0,v_0) = \int_R e^{iXx_T}dx_T \int_R \overbrace{f_{0T}\left(x_T,v_T/x_0,v_0\right)\frac{e^{x_T}}{\mathbf{E}(S_T)}}^{f_{0T}^{\mathbf{AN}}(x_T,v_T/x_0,v_0)}dv_T$$

- **Final expression of vanillas:**

$$C = DF_T\left(\mathbf{E}(S_T)P^{\mathbf{AN}}(x_T > \ln K) - KP^{\mathbf{CN}}(x_T > \ln K)\right)$$

Application to Heston´s model

- **Valuation of FX quanto options (S_T in USD per EUR):**

$$C = DF_T^{\$}\mathbf{E}\left((S_T - K)^+ S_T\right) = DF_T^{\$}\left(\underbrace{\mathbf{E}\left(e^{2x_T}\mathbf{1}_{\{x_T > \ln K\}}\right)}_{\text{Asset^2 or nothing}} - K \underbrace{\mathbf{E}\left(e^{x_T}\mathbf{1}_{\{x_T > \ln K\}}\right)}_{\text{Asset or nothing}} \right)$$

- **Characteristic function for asset^2 or nothing option:**

$$\varphi_{0T}^{\mathbf{A^2N}}(X/\mathbf{x}_0) = \frac{\varphi_{0T}(X - 2i/\mathbf{x}_0)}{\varphi_{0T}(-2i/\mathbf{x}_0)} = \frac{\mathbf{E}\left(e^{i(X-2i)x_T}\right)}{\mathbf{E}(e^{2x_T})} = \frac{\mathbf{E}\left(e^{iXx_T}e^{2x_T}\right)}{\mathbf{E}(S_T^2)}$$

- **Final expression for FX quanto vanillas:**

$$C = DF_T^{\$}\left(\mathbf{E}(S_T^2)P^{\mathbf{A^2N}}\left(x_T > \ln K\right) - K\mathbf{E}(S_T)P^{\mathbf{AN}}\left(x_T > \ln K\right)\right)$$

- **A bootstrapping algorithm is proposed:**
  - Periods in between vanilla maturities are chosen to let parameters change.
  - 1. n = 1
  - 2. Search model parameters $(\theta, \kappa, \sigma, \rho)$ from T_{i-1} to T_i to fit vanillas at T_i minimizing the following objective function:

$$FO = \sum_{i=1}^{M} \frac{w_i}{\sum w_j} \left(price_i^{model} - price_i^{market} \right)^2$$

N.B. w_i chosen to give more weight to options closer to ATM.

  - 3. The parameters up to T_i are fixed
  - 4. n = n + 1
  - 5. Return to step 2

Case study: Calibration to Eurostoxx 50.

- **Time dependent Heston model is calibrated to the following Eurostoxx 50 volatility surface:**

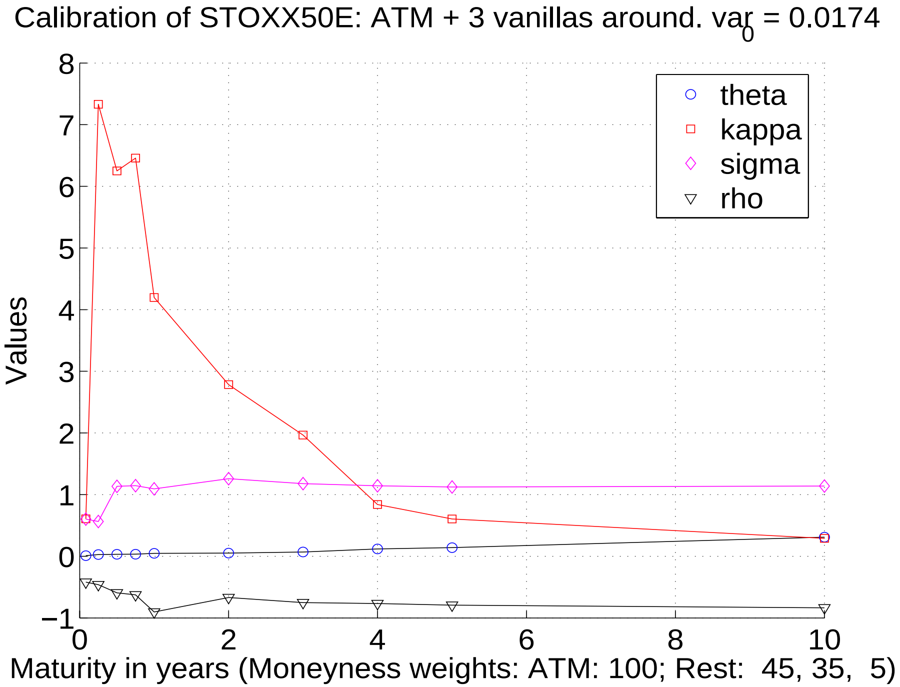
K \ Mat	1m	3m	6m	9m	1y	2y	3y	4y	5y	10y
0.85	23.0	18.7	18.5	18.6	19.1	19.7	20.6	21.5	22.2	25.8
0.90	18.9	16.7	17.0	17.2	17.8	18.8	19.8	20.8	21.5	25.3
0.95	15.2	14.7	15.5	16.0	16.6	17.8	19.0	20.0	20.8	24.7
1.00	12.2	13.2	14.1	14.8	15.5	16.9	18.2	19.3	20.2	24.2
1.05	11.6	12.3	13.1	13.9	14.4	16.1	17.5	18.7	19.5	23.7
1.10	13.3	12.3	12.6	13.2	13.7	15.4	16.9	18.1	19.0	23.2
1.15	15.6	12.9	12.4	12.7	13.2	14.8	16.3	17.5	18.5	22.7

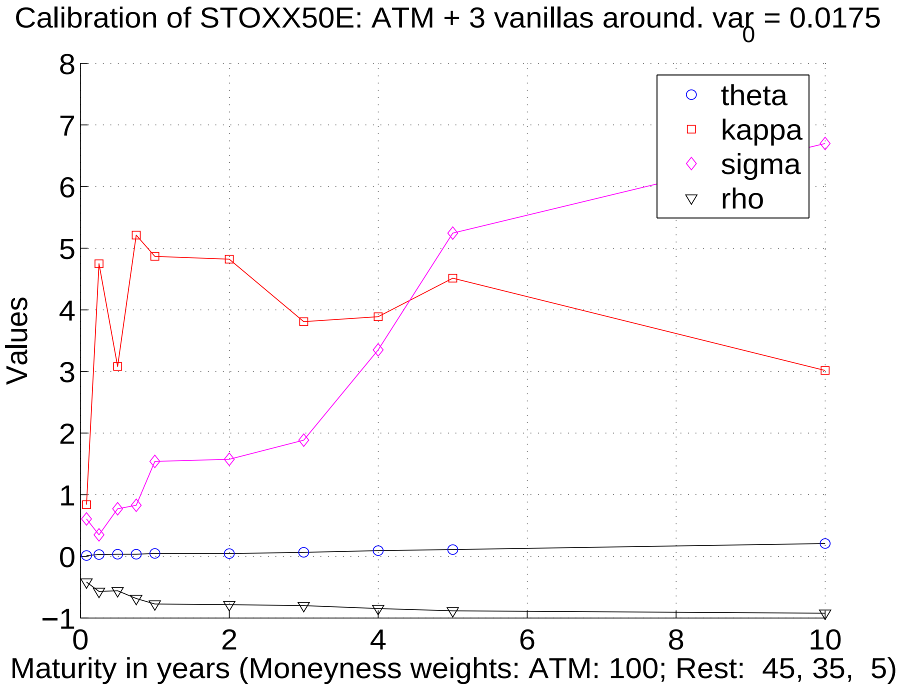
- **To avoid problems with discrete dividend payments, what is calibrated is the forward delivered at the last maturity rather than the underlying itself.**
- **Two calibrations are carried out:**
  - Left: constrained calibration (esp. with respect to σ and κ).
  - Right: unconstrained calibration

	v_0	θ	κ	σ	ρ	v_0	θ	κ	σ	ρ
max	1	1	20	1.5	1	100	100	100	100	1
min	0	0	0	0	-1	0	0	0	0	-1

Case study: Calibration to Eurostoxx 50.

- **Maximum error for both calibrations: 8 bp for most OTM options.**
- **Both calibrations are equivalent from a qualitative point of view:**
  - Market is pricing in increasing uncertainty of volatility: σ is constant while κ reduces (left) vs κ is constant while σ increases (right).
  - Market is pricing in increasing volatility (from 11% to around 45% at 10y) and increasing skew.





Application to Forward start options.

- **Forward start option:**

$$p = P(0,t_v)\mathbf{E}\left((e^{x_v} - Ke^{x_u})^+\right) = P(0,t_v)\mathbf{E}\left(e^{x_u}(e^{x_v - x_u} - K)^+\right)$$

- **Applying the tower law:**

$$p = P(0,t_v)\mathbf{E}\left(e^{x_u}\mathbf{E}\left[(e^{x_v - x_u} - K)^+ / \mathbf{x}_u\right]\right) = P(0,t_v)E$$

- **The expectation *E* can be calculated integrating over the state variables x_u and x_v at times t_u and t_v .**

$$E = F_u \int_{R^2} d\mathbf{x}_u\, f_{0u}(\mathbf{x}_u / \mathbf{x}_0) \frac{e^{x_u}}{F_u} \int_{R^2} \left(e^{x_v - x_u} - K\right)^+ f_{uv}(\mathbf{x}_v / \mathbf{x}_u) d\mathbf{x}_v$$

Application to Forward start options.

- **The increment $\tilde{x}_v = x_v - x_u$ depends on ν_u but not on x_u:**

$$f_{uv}(x_v, \nu_v / x_u, \nu_u) = f_{uv}(x_v - x_u, \nu_v / 0, \nu_u) = f_{uv}(\tilde{\mathbf{x}}_v / 0, \nu_u)$$

- **Doing the change of variable $\tilde{x}_v = x_v - x_u$:**

$$E = F_u \int_{\mathbf{R}^2} d\mathbf{x}_u \, f_{0u}(\mathbf{x}_u / \mathbf{x}_0) \frac{e^{x_u}}{F_u} \int_{\mathbf{R}^2} \left(e^{\tilde{x}_v} - K\right)^+ f_{uv}(\tilde{\mathbf{x}}_v / 0, v_u) d\tilde{\mathbf{x}}_v$$

- **Exchanging the order of integration, the expectation *E* can be calculated as a regular vanilla with respect to a new measure $\tilde{f}$.**

$$E = F_u \int_{\mathbf{R}^2} d\tilde{\mathbf{x}}_v \left(e^{\tilde{x}_v} - K\right)^+ \tilde{f}(\tilde{\mathbf{x}}_v / \mathbf{x}_0) = F_u \tilde{\mathbf{E}}\left((e^{\tilde{x}_v} - K)^+\right)$$

$$\tilde{f}(\tilde{\mathbf{x}}_v / \mathbf{x}_0) = \int_{\mathbf{R}^2} \frac{e^{x_u}}{F_u} f_{0u}(\mathbf{x}_u / \mathbf{x}_0) f_{uv}(\tilde{\mathbf{x}}_v / 0, v_u) d\mathbf{x}_u$$

Application to Forward start options.

- **Definition of the characteristic function of $\tilde{f}$:**

$$\tilde{\varphi}(\mathbf{X}/\mathbf{x}_0) = \int_{\mathbf{R}^2} e^{i\tilde{\mathbf{x}}_v \cdot \mathbf{X}} \tilde{f}(\tilde{\mathbf{x}}_v/\mathbf{x}_0) d\tilde{\mathbf{x}}_v$$

- **Substituting $\tilde{f}$ and exchanging the order of integration:**

$$\tilde{\varphi}(\mathbf{X}/\mathbf{x}_0) = \int_{\mathbf{R}^2} d\mathbf{x}_u \frac{e^{x_u}}{F_u} f_{0u}(\mathbf{x}_u/\mathbf{x}_0) \underbrace{\int_{\mathbf{R}^2} e^{i\tilde{\mathbf{x}}_v \cdot \mathbf{X}} f_{uv}(\tilde{\mathbf{x}}_v/0, v_u) d\tilde{\mathbf{x}}_v}_{\varphi_{uv}(\mathbf{X}/0,v_u)=\exp\left(C_{uv}(\mathbf{X})+D_{uv,2}(\mathbf{X})v_u\right)}$$

- **Replacing the definition of $\varphi_{uv}(\mathbf{X}/0, v_u)$ and reordering:**

$$\tilde{\varphi}(\mathbf{X}/\mathbf{x}_0) = \frac{e^{C_{uv}(\mathbf{X})}}{F_u} \underbrace{\int_{\mathbf{R}^2} d\mathbf{x}_u f_{0u}(\mathbf{x}_u/\mathbf{x}_0) \exp\left(i(-i)x_u + i(-iD_{uv,2}(\mathbf{X})v_u)\right)}_{\varphi_{0u}\left(-i,-iD_{uv,2}(\mathbf{X})/\mathbf{x}_0\right)=\exp\left(C_{0u}\left(-i,-iD_{uv,2}(\mathbf{X})\right)+x_0+D_{0u,2}\left(-i,-iD_{uv,2}(\mathbf{X})\right)v_0\right)}$$

Application to Forward start options.

- **Final expression obtained for $\tilde{\varphi}(\mathbf{X}/\mathbf{x}_0)$:**

$$\tilde{\varphi}(\mathbf{X}/\mathbf{x}_0) = \exp\left(\tilde{C}(\mathbf{X}) + x_0 + \tilde{D}(\mathbf{X})v_0\right)$$

$$\begin{cases} \tilde{C}(\mathbf{X}) = -\ln(F_u) + C_{uv}(\mathbf{X}) + C_{0u}\left(-i, -iD_{uv,2}(\mathbf{X})\right) \\ \tilde{D}(\mathbf{X}) = D_{0u,2}\left(-i, -iD_{uv,2}(\mathbf{X})\right) \end{cases}$$

- **The marginal distribution of the underlying $\tilde{x}_t$ is obtained by setting $V = 0$ in $\mathbf{X} = (X, V)$.**

- **The final forward start option price is:**

$$p = P(0, t_v) F_u \tilde{\mathbf{E}}\left((e^{\tilde{x}_v} - K)^+\right)$$

Grupo Santander

Forward skew of Heston's model.

- Consider the price of the forward start option when the underlying $\tilde{x}_t$ is driven by BS process with constant vol:

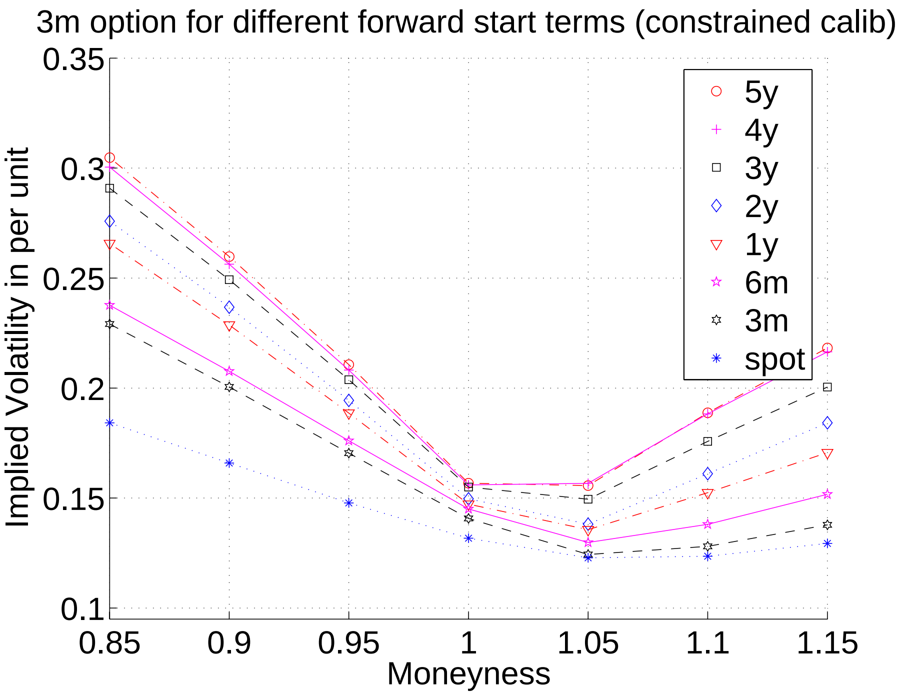
$$p = DF_{t_v} E\left(\left(e^{x_v} - Ke^{x_u}\right)^+\right) = DF_{t_v} E\left(e^{x_u}\right) E\left(\left(e^{\tilde{x}_v} - K\right)^+\right)$$

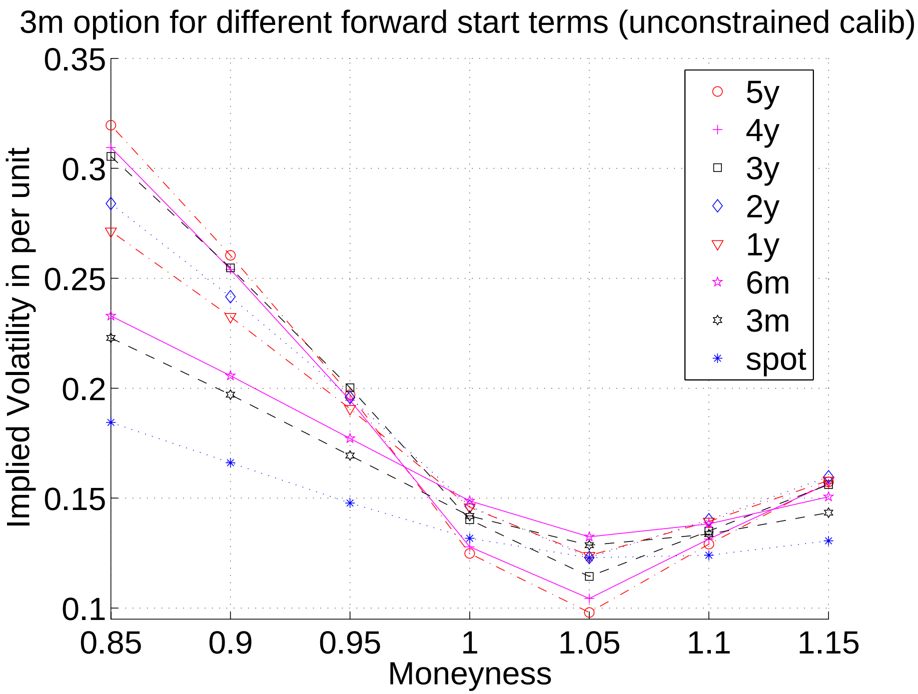
$$\tilde{x}_t = \left(\mu - \frac{1}{2}\sigma_{BS}{}^2\right) dt + \sigma_{BS} dW_t$$

- It is understood by forward skew the implied volatility surface that results when the forward start option price above, equals the price of the same forward start option when $\tilde{x}_t$ is a Heston process.

Forward skew of Heston's model.

▪ **Lower maturity options are more sensitive to the variance distribution as the forward start term increases.**

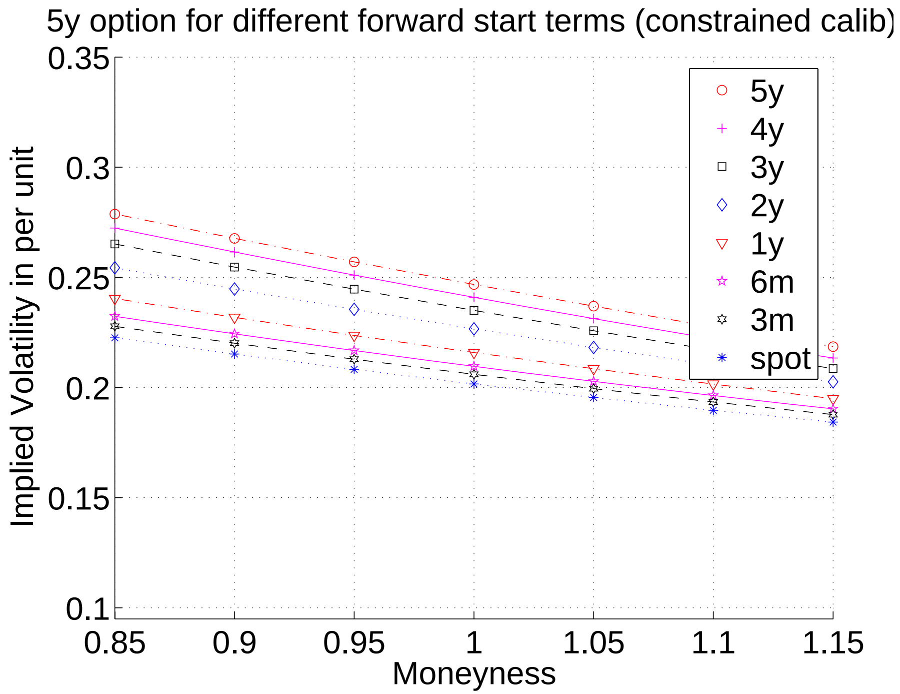


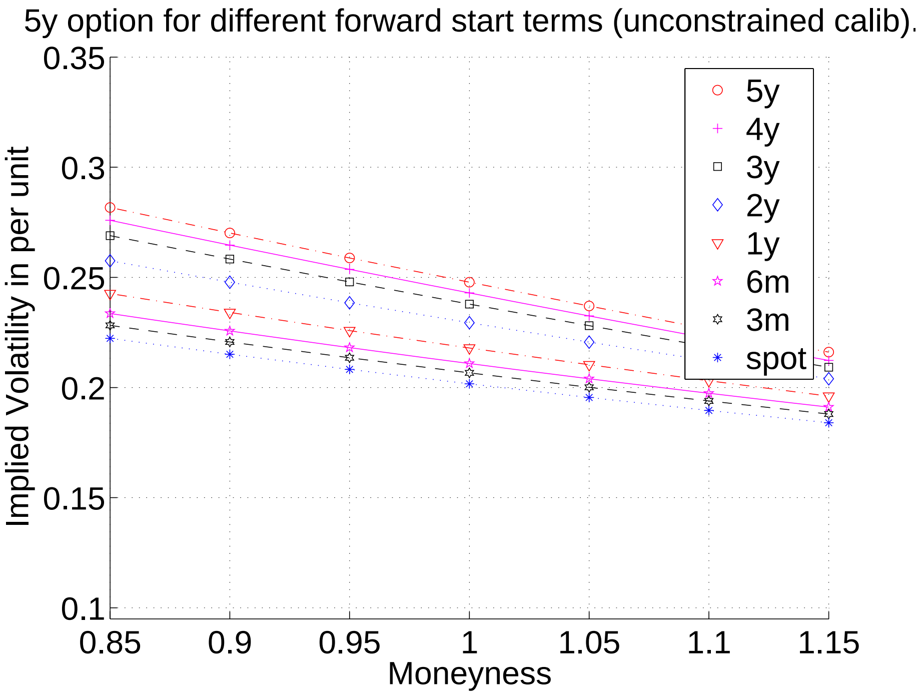


▪ **Constrained calibration (left) seems a lot more reasonable than unconstrained calibration (right).**

Forward skew of Heston's model.

- **Longer maturity options are less sensitive to the variance distribution as the forward start term increases.**



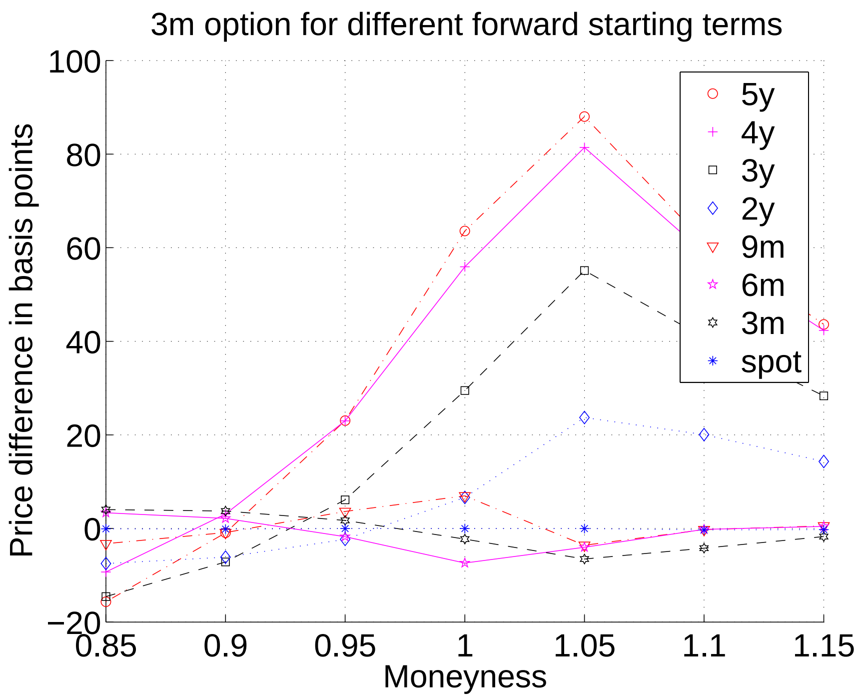


- **Constrained and unconstrained calibrations seem to agree a lot more for longer maturity options.**

Forward skew of Heston's model.

- **Between both calibrations: big difference for short maturity forward start options.**
  - Both calibrations fit the marginal distribution of the underlying but,
  - the variance distribution is not specifically calibrated in either case.

- **Market volatility surface:**
  - Gives info about the marginal distribution of the underlying.
  - No info is given about the distribution of the variance (this info could be given by forward start or cliquet option quotes).



Grupo Santander

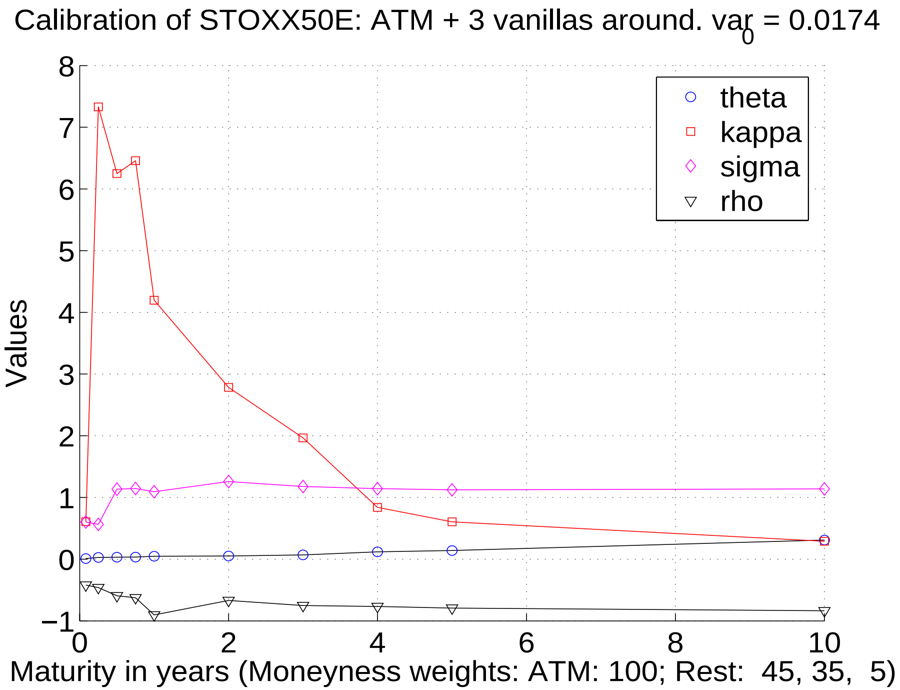
- **What´s different from both calibrations?**
- **Consider the instantaneous volatility** $\tilde{\sigma}_t = \sqrt{v_t}$ **(obtained applying Itô):**

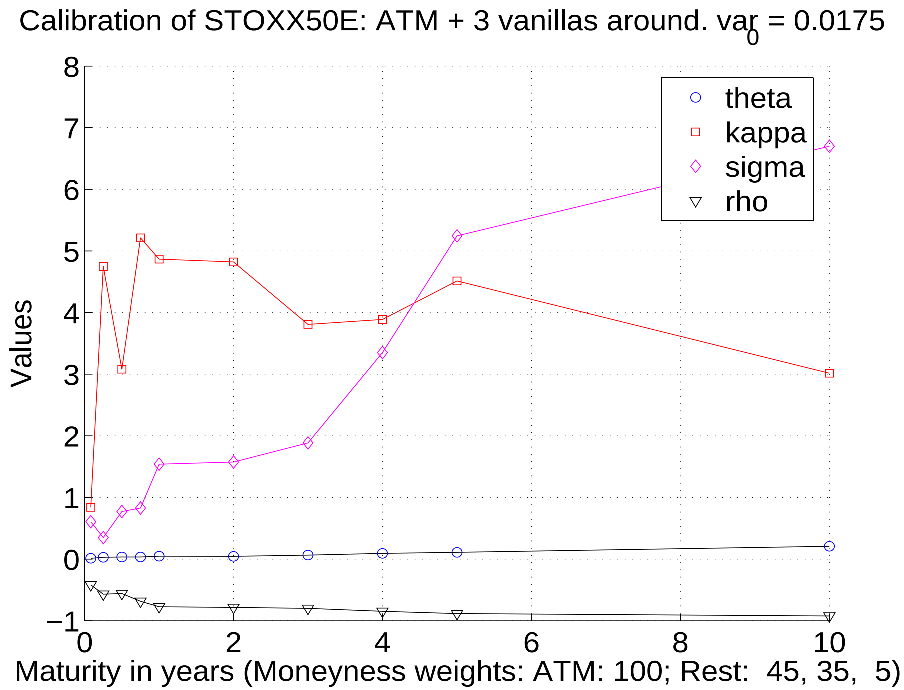
$$d\tilde{\sigma}_t = \left(\frac{4\kappa\theta - \sigma^2}{8\tilde{\sigma}_t} - \frac{\kappa}{2}\tilde{\sigma}_t \right) dt + \frac{1}{2}\sigma\, dY_t$$

- **Calibrations with** σ **greater than one can lead to very negative drift:**
  - Constrained: σ cannot be higher than 1.5.
  - Unconstrained: σ far exceeds 1 at higher maturities:
    - Variance is biased towards values near zero at long maturities.
    - Forward implied volatility is lower and forward skew does not make sense.

Forward skew of Heston's model.

- **Calibration of the uncertainty of volatility (κ and σ):**
  - Left: constraining σ to moderate values and calibrating κ seems to provide a better forward skew.
  - Right: fixing κ and calibrating σ may bias the forward skew towards artificially lower implied volatilities.





Grupo Santander

Conclusions

- **A new method to introduce piecewise constant time-dependent parameters using transform methods is presented:**
  - The characteristic function of the underlying for a time horizon is calculated in terms of the characteristic functions of the sub-periods where the parameters change.
  - Analytic tractability is preserved for a wide family of models such as hybrids with stochastic vol, interest rates an jumps.
- **The method has been applied to Heston's model.**
- **Two calibrations were carried out on the Eurostoxx 50.**
- **The method has also been applied to valuation of forward start options.**
- **The forward skew of both calibrations is explored.**