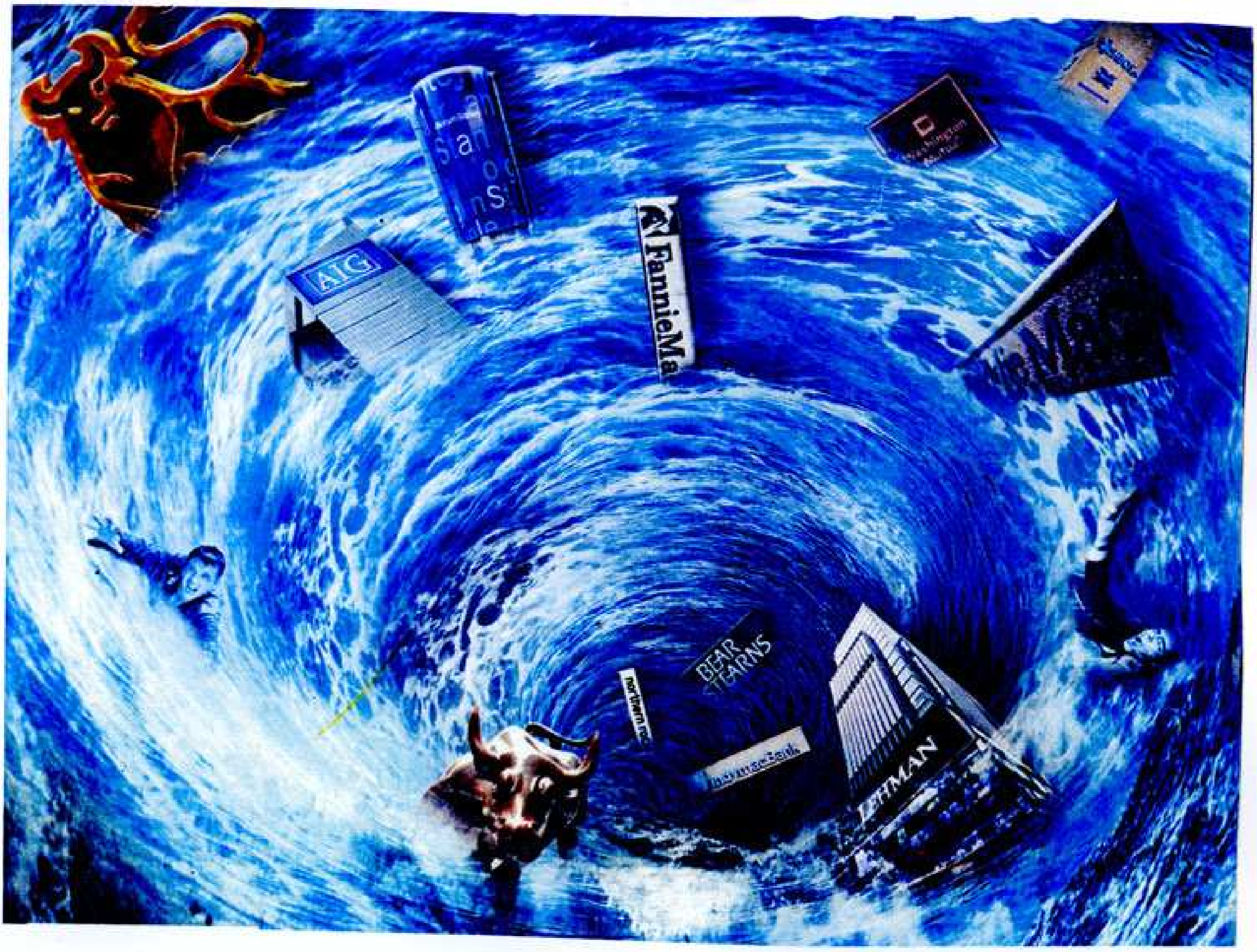
Thanks

to

Hansjörg Albrecher
Wolfgang Runggaldier
Walter Schachermayer

for organizing such a
splendid week at the
Radon Institute,

a pleasant refugium
from


AIG
FannieMa
BEAR STEARNS

nationalization
("as fast as you can say Hugo Chavez")
failure or rescue of

Fannie Mae
Freddie Mac
Lehmann Brothers
AIG

within 10 days

moving in:
Mitsubishi / Nomura / Mizuho

yesterday:

US Government seizes Washington Mutual

= largest bank seizure in US history

pieces sold to JP Morgan Chase

"Licht am Ende des Tunnels? Nicht absehbar"

(Peer Steinbrück)
25/09/08

"Nichts bleibt wie es war!"

first regulation measures:

- no short sales
  (until end of year)
- existing short positions
  (> 0.25% of capitalization)
  must be declared:

e.g. John Paulson:
short on

Barclays	440 Mio
RBS	368 "
Lloyds TSB	328 "
HBOS	113 "

e.g. Man Group
(→ Man Institute, Oxford)
asks FSA to be
on protected list (-27,2%)

There is
no such thing
as a
free crunch

The Economist, 20/09/2008

Derivatives:

A nuclear winter?

"financial weapons of mass destruction"

(Warren Buffett)

The Economist, 20/09/2008

"Finance needs to shrink"

The industry will not be able to make money after the boom

(share of total American corporate profits rose from 10% in early 80s to 40% last year)

unless it is far smaller

(The Economist, 20/09/2008)

"Wheel of Blame"
Time, March 31, 2008:

- President Bush

for blocking regulation

- Wall Street Wizards

"securitization":
slicing / packaging / selling of
subprime mortgage securities

+ rating agencies: "AAA"
"marked to model"

- Alan Greenspan

for making lending too available
flushing liquidity
("houses as ATM")

"Mathematical Finance"

a great success story over the last 20 years

correlated with dramatic growth of financial industry

Mathematics $\rightleftarrows$ * **Finance**

Banks / Supervising agencies

* Mathematical Finance as a sustainable research field within Mathematics

?

(Academia Europaea, ...)

since last year:

Black shadows

- emergence of
  "subprime crisis"
- wrong incentives
  (rating agencies, stock options, ...)
- fraud
  (Société Générale, CDO's,
  now, since Sept. 24:
  FBI investigates 26,
  including Lehman Brothers,
  AIG, Fannie Mae, Freddie Mac)


CRIME SCENE DO NOT CROSS
CRI
WALL ST
NASSAU ST
ONE WAY
WAY

- **greed** (excessive $\left\{\begin{array}{l}\text{benchmarks} \\ \text{bonus systems,}\end{array}\right.$ golden parachutes, ...)

$$\frac{\text{Average CEO pay}}{\text{average income}} =$$

1965 now

≈ 25 ≈ 275

at the top in the financial industry:

$$\approx \frac{500 \cdot 10^6}{5 \cdot 10^4} = 10^4$$

(Richard Fuld, Lehman Brothers)

Mathematics in Finance:
a force for
clarity and transparence

"enlightenment"

("lumières" / "Aufklärung")

?

White knights
vs.
black shadows

?

- overconfidence
in mathematical models

?

Have we done enough
to convey the needed*

humility

?

* Alan Greenspan

On a deeper level:

does Mathematics
create black shadows?

powerful focus:
"spotlights on reality"
(A. Connes, J. Brüning
FAZ, 26.2.2000)

$\Downarrow$

shadows!

"complete" financial market model

à la Black - Scholes:

powerful focus on

pricing / hedging of financial derivatives

message (for a long time, up to mid 80's):

- $\nexists$ risk

  (can be hedged away)

- even robust w.r.t. models $Q \approx P$

(only P^* = unique equivalent martingale measure matters)

incomplete financial markets:

$\mathcal{P}^*$ = a whole class of equivalent martingale measures

$\Downarrow$

choice of model $Q \approx P$ matters:

utility-based methods, optimal portfolio choice, partial (efficient) hedging, ...

$$Q = ?$$

model risk / ambiguity!

new emphasis on

<u>model uncertainty</u>,

e.g. in the

quantification of financial risk by

<u>risk measures</u>:

(conceptually)

"Risk"

of a financial position

$$X : \Omega \to \mathbb{R}^1$$

Ω = set of scenarios

$X(\omega)$ = discounted net worth at end of trading period "marked to $\begin{cases}\text{market"} \\ \text{model}\end{cases}$ *

?

---

* Protter, Jarrow: Liquidity Risk and Risk measure computation (2005)
Acerbi, Scandolo (2007)

Axiomatic approach:

what do we really want?

- focus on <u>downside risk</u>
- <u>monetary</u> perspective
  (capital requirement,
  point of view of supervising
  agency, cf. Basel II)
- <u>consistency</u>
  (across positions, in time)

$\downarrow$ Artzner, Delbaen, Eber, Heath
F-Schied, Frittelli

<u>convex / coherent risk measures</u>

$\mathcal{X}$ = a linear space of financial positions

$X : \Omega \to \mathbb{R}$, $1 \in \mathcal{X}$

uniformly bounded

$\mathcal{A} \subseteq \mathcal{X}$: "acceptable" positions

$X \in \mathcal{A}, Y \geq X \Rightarrow Y \in \mathcal{A}$

$\Downarrow$

$$\rho(X) := \inf \{ m \mid X + m \in \mathcal{A} \}$$

= capital requirement:

minimal amount which should be added (and invested in a risk-free manner) to make the position acceptable

i) $X \leq Y \Rightarrow \rho(X) \geq \rho(Y)$ "monotone"

ii) $\rho(X+m) = \rho(X) - m$ "cash-invariant"

ρ = monetary risk measure

diversification is
not discouraged:

$$X, Y \in \mathcal{A} \Rightarrow \lambda X + (1-\lambda) Y \in \mathcal{A}$$

i.e. $\mathcal{A}$ is convex

$\Downarrow$

$$\rho : \mathcal{X} \to \mathbb{R}$$

"convex risk measure"

If, in addition,

2 | $X \in \mathcal{A},\ \lambda \geq 0 \Rightarrow \lambda X \in \mathcal{A}$

$\mathcal{A}$ convex cone

then

ρ "coherent risk measure" (ADEH

convex + pos. homogeneous

general remark:

ρ = convex functional on $\mathcal{X}$

If ρ is $\sigma(\mathcal{X}, \mathcal{X}')$-lower-semicontinuous

then, by convex duality,

$$\boxed{\rho = \rho^{**}}$$

where

$$\rho^*(L) = \sup_{X \in \mathcal{X}} (L(X) - \rho(X)) \quad \text{for } L \in \mathcal{X}'$$

= convex conjugate on $\mathcal{X}'$
(Fenchel-Legendre transform)

i.e.

$$\boxed{\rho(X) = \sup_{L \in \mathcal{X}'} (L(X) - \rho^*(L))}$$

"monetary" $\Rightarrow$ for $\rho^*(L) < \infty$:

$$-L \geq 0 \,, \quad -L(1) = 1$$

+ mild continuity conditions
(e.g. $\mathcal{X} = L^\infty(\Omega, \mathcal{F}, P)$, Fatou property

$\Rightarrow$ "robust representation"

$$\rho(X) = \sup_{Q \in \mathcal{Q}} \left(E_Q[-X] - \alpha(Q) \right)$$

$\shortparallel$

a whole class of probability measures on $(\Omega, \mathcal{F})$, taken more or less seriously

$$\alpha(Q) = \sup_{X \in \mathcal{A}} E_Q[-X]$$

"robust" view:

not one model Q,
but a whole

class $\mathcal{Q}$

of probability measures Q
on $(\Omega, \mathcal{F})$

cf.

- convex risk measures
- microeconomic theory
  of preferences (Gilboa, Schmeidler)
- robust statistics (Huber, Hampel)

preferences

$\succsim$ on $\mathcal{X}$

Axioms for "Rationality":
("paradoxa")

- v. Neumann - Morgenstern
- Savage, Aumann
- Gilboa, Schmeidler (1989)
- Maccheroni, Marinacci, Rustichini (2006, Econometrica)

$\Downarrow$

numerical representation

$U(X)$

by some "utility functional":

"weak" rationality (MMR) $\Rightarrow$
(flexible)

A. Schied

$$U(X) = -\rho\big(u(X)\big)$$

for some utility function u
and some convex risk measure ρ

$$\underset{\text{above}}{=} \inf_{Q \in \mathcal{Q}} \Big(E_Q[u(X)] + \alpha(Q) \Big)$$

= worst case over some
class of probabilistic models,
suitably penalized

Gilboa-Schmeidler $\Rightarrow$ ρ coherent,
i.e.

$$U(X) = \inf_{Q \in \mathcal{Q}} E_Q[u(X)]$$

robust utility maximization:

maximize

$$U(H) := \inf_{Q \in \mathcal{Q}} \left(E_Q[u(H)] + \alpha(Q) \right)$$

over

$$\mathcal{H}(x_0) := \left\{ H \leq x_0 + \int_0^T \xi_t \, dX_t \;\middle|\; \xi \text{ admissible} \right\}$$

$$\underset{\text{"superhedging duality"}}{=} \left\{ H \;\middle|\; \sup_{P^* \in \mathcal{P}^*} E^*[H] \leq x_0 \right\}$$

= all contingent claims that can be financed with initial capital x_0

complete case: $\mathcal{P}^* = \{P^*\}$

a) standard situation: $\mathcal{Q} = \{Q\}$
solution is well known:

$$H_0 = (u')^{-1}\left(\lambda_0 \frac{dP^*}{dQ}\right)$$

b) robust case: $|\mathcal{Q}| > 1$
A. Schied: $\sim$ "robust statistics"
solution via

$Q_0 :=$ "least favorable measure" in $\mathcal{Q}$ w.r.t. P^*

$:\Longleftrightarrow \quad \varphi^* := \frac{dP^*}{dQ_0}$ satisfies

$$Q_0[\varphi^* \leq c] = \inf_{Q \in \mathcal{Q}} Q[\varphi^* \leq c] \quad \forall c$$

$$\Longleftrightarrow \quad E_{Q_0}[f(\varphi^*)] = \inf_{Q \in \mathcal{Q}} E_Q[f(\varphi^*)] \quad \forall f \downarrow$$

cf. Huber-Strassen: robust Neyman-Pearson lemma

If Q_0 exists then

$$X_T^* = (u')^{-1}(\lambda \varphi^*)$$

is optimal:

$$\begin{aligned}
U(X_T) &= \inf_{Q \in \mathcal{Q}} E_Q[u(X_T)] \\
&\leq E_{Q_0}[u(X_T)] \\
&\leq E_{Q_0}[\underbrace{u(X_T^*)}_{= f(\varphi^*),\ f \searrow}] \\
&= \inf_{Q \in \mathcal{Q}} E_Q[u(X_T^*)] \\
&= U(X_T^*)
\end{aligned}$$

Note: Q_0 does not depend on u!

"incomplete" (realistic) case:

$$|\mathcal{P}^*| = \infty$$

a) standard preferences: $\mathcal{Q} = \{Q\}$
("expected utility")

idea:

"divergence functional" in terms of conjugate of utility function

$\mathcal{P}_0^* :=$ "projection" of Q on $\mathcal{P}^*$

= "least favorable martingale measure

take

$$H_0 = (u')^{-1}\left(\lambda_0 \frac{dP_0^*}{dQ}\right)$$

= classical solution for P_0^*, Q

→ Karatzas
Frittelli - Biagini
Goll - Rüschendorf
Kramkov - Schachermayer

f = convex conjugate of u

<u>f- divergence:</u>

$$f(P\,|\,Q) := \int f\left(\frac{dP}{dR}, \frac{dQ}{dR}\right) dR$$

if $P, Q \ll R$ (e.g. $= \frac{1}{2}(P+Q)$)

$$= \int f\left(\frac{dP^c}{dQ}\right) dQ + \lim_{z \to \infty} \frac{f(z)}{z} \, P^s[\Omega]$$

$P = P^c + P^s$ = Hahn-Lebesgue decomposition w.r.t. Q
(independent of R)

<u>robust f- divergence:</u>

$$f(P\,|\,\mathcal{Q}) := \inf_{Q \in \mathcal{Q}} f(P\,|\,Q)$$

$P^* \in \mathcal{P}$ <u>robust f- projection of $\mathcal{Q}$ on $\mathcal{P}$</u>

$$:\Longleftrightarrow \quad f(P^*\,|\,\mathcal{Q}) = \inf_{P \in \mathcal{P}} f(P\,|\,\mathcal{Q})$$

b) robust preferences: $|\mathcal{Q}| > 1$

project $\mathcal{Q}$ on $\mathcal{P}^*$!

new probabilistic
projection problem
(extending Csiszar, ...).

H.F. + Anne Gundel
(Illinois J. Math, 2006)
volume in honor of J.L. Doob

existence

of robust f-projections ?
(extending Csiszar, ...)

Theorem (H.F., Anne Gundel)
Assume

i) $\mathcal{P}^*$ variation-closed

ii) $\left\{ \frac{dQ}{dQ_0} \,\middle|\, Q \in \mathcal{Q} \right\}$ $\sigma(L^1(Q_0), L^\infty(Q_0))$-compact

If

$$\lim_{x \uparrow \infty} \frac{f(x)}{x} = \infty$$

then $\exists!$ robust f-projection of $\mathcal{P}^*$ on $\mathcal{Q}$

In general:

$\exists!$ on projective limit of initial model
(supermartingale $\hat{=}$ measure on F. predictable sets in $\Omega \times (0, T]$)

involves

duality approach

Beyond $\mathcal{P}^*$:

Kramkov, Schachermayer (1999)
for $|\mathcal{Q}| = 1$,

Quenez (2005)
Schied, Wu (2005)
for $|\mathcal{Q}| > 1$

Schied (2006): Beyond Gilboa-Schmeidler

H.F., Anne Gundel (2006):
in terms of

"extended martingale measures"
on $\Omega \times (0, \infty]$

"extended martingale measure"

$$\underbrace{\bar{P}^* \text{ on } (\bar{\Omega}, \bar{\mathcal{F}})}_{}$$

= predictable σ-field

$\bar{P}^*$ on $(\bar{\Omega}, \bar{\mathcal{F}})$, $P^*_t \underset{\mathcal{F}_t}{\ll} P \quad \forall t$.

$\bar{V}$ is $\bar{P}^*$-supermartingale for any value process V of an admissible trading strategy:

$$V_t = x_0 + \int_0^t \xi_s \, dX_s \, , \quad \text{bounded from below}$$

$$\underbrace{\hphantom{V_t = x_0 + \int_0^t \xi_s \, dX_s \, , \quad \text{bounded from below}}}_{}$$

$$\boxed{\bar{\mathcal{P}}^*} \supseteq \{ P^* \times \delta_\infty \mid P^* \in \mathcal{P}^* \}$$

$$\bar{\mathcal{Q}} := \{ Q \times \delta_\infty \mid Q \in \mathcal{Q} \}$$

robust utility maximization

translates into

robust projection problem for

$$f(\bar{P}|\bar{Q}) := f(P_T|Q)$$

Theorem (F.-Gundel):

1) $\exists$ robust f-projection $\bar{P}_0^*$ of $\bar{Q}$ on $\bar{\mathcal{Q}}^*$

2) $$H_0 := (u')^{-1}\left(\lambda_0 \frac{dP_{0,T}}{dQ_0}\right)$$
is optimal

reverse f-projection

cf.

"Robust Preferences and Robust Portfolio Choice"

H. F., A. Schied, S. Weber

to appear in: Handbook of Computational Finance (ed. A. Bensoussan)

What happens in the

long run ?

of course "we all die"
but:

asymptotics of
robust projection problem

?

"least favorable"
in the long run

?

Back to basics:

$T \uparrow \infty \implies$

asymptotic arbitrage

since (typically)

$$P^* \perp P$$

globally, i.e., on

$$\mathcal{F}_\infty = \sigma\left(\bigcup \mathcal{F}_t\right)$$

---

how exactly?

$\sim$ large deviations

A.F. and W. Schachermayer:

"Asymptotic arbitrage and large deviations"

Mathematics and Financial Economics 1 (2007)

---

a mathematical warm-up exercise

($\longleftarrow$)

(S_t) $\mathbb{R}^d$-valued semimartingale

$\mathcal{P}^* \neq \emptyset$

for fixed horizon T:

"$\exists$ $(\varepsilon_1, \varepsilon_2)$ - arbitrage"

$:\Longleftrightarrow$ $\exists$ X_T, attainable at 0 cost, such that

$$X_T \geq -\varepsilon_2$$

$$P[X_T \geq 1 - \varepsilon_2] \geq 1 - \varepsilon_1$$

$\Longleftrightarrow$ $\exists A \in \mathcal{F}_T : \forall P^* \in \mathcal{P}^*:$

$$P[A] \leq \varepsilon_1, \quad P^*[A] \geq 1 - \varepsilon_2$$

$\Longleftrightarrow$ $\forall P^* \in \mathcal{P}^* : \exists A \in \mathcal{F}_T$

$\overset{?}{\Longleftarrow}$ $P[A] \leq \hat{\varepsilon}_1$, $P^*[A] \geq 1 - \hat{\varepsilon}_2$

Walter!

application to (possibly incomplete)

$\mathbb{R}^d$-valued <u>diffusion</u>

$$dS_t = \sigma(S_t)\,(dW_t + \varphi(S_t)\,dt)$$

"<u>market price of risk</u>"

Assume:

above threshold $c > 0$ $:\Leftrightarrow$

$$\lim_{T\uparrow\infty} P\left[\frac{1}{T}\int_0^T \|\varphi_t\|^2 dt < c\right] = 0$$

"law of large numbers"

$$\Downarrow$$

(rough) exponential estimates:

$\forall \varepsilon > 0$, $\gamma_1 + \gamma_2 < \frac{c}{2}$, T large

$\exists$ X_T attainable at 0 cost

i) $X_T \geq -e^{-\gamma_1 T}$

ii) $P[X_T \geq e^{\gamma_2 T}] \geq 1-\varepsilon$

<u>Proof:</u> via construction, for any $P^* \in \mathcal{P}^*$, $\gamma < \frac{c}{2}$ of

$$A_T = \left\{ \frac{dP^*}{dP}\Big|_{\mathcal{F}_\tau} > e^{-\gamma T} \right\}$$

such that τ = stopping time $\leq T$

$$P^*[A_T] \geq 1 - e^{-\gamma T}$$

$$P[A_T] \leq \varepsilon$$

exponential decay via <u>large deviation</u> estimates for market price of risk?

more precisely, for complete ergodic case:

$$\varrho_T(\omega) := \frac{1}{T}\int_0^T \delta_{S_t(\omega)}\,dt$$ empirical distribution

<u>ergodic</u> $\Rightarrow$

$$\varrho_T(\omega) \xrightarrow[P\text{-a.s.}]{\text{weakly}} \mu = \text{unique invariant measure}$$

<u>LDP</u> ($\sim$ Sanov's theorem):

$$\frac{1}{T}\log P[\varrho_T \in A] \asymp -\inf_{\nu\in A} I(\nu)$$

with some convex rate function I on $\mathcal{M}_1(\mathbb{R}^1)$:

$\Downarrow$ <u>Contraction principle</u>

large deviations estimate for Sharpe ratio:

$$\frac{1}{T}\log P\left[\frac{1}{T}\int_0^T \varphi_t^2\,dt \in B\right] \asymp -\inf\left\{I(\nu) \,\middle|\, \int \varphi^2 d\nu \in B\right\}$$

However:

$$P[Z_T \geq e^{-\gamma T}] =$$

$$P\Big[\underbrace{\tfrac{1}{T}\int_0^T \varphi_t \, dW_t}_{?} + \underbrace{\tfrac{1}{2}\tfrac{1}{T}\int_0^T \varphi_t^2 \, dt}_{\text{LDP}} \leq \gamma\Big]$$

Itô's formula:

$$\int_0^T \varphi_t \, dW_t = \int_0^T \frac{\varphi_t}{\sigma_t}(dS - \sigma_t \varphi_t \, dt)$$

$f := \frac{\varphi}{\sigma} \in C^1$, $F' = f$

$$= \int_0^T f(S_t)\, dS_t - \int_0^T \varphi_t^2 \, dt$$

$$= F(S_T) - F(S_0) - \frac{1}{2}\int_0^T f'(S_t)\sigma_t^2 \, dt - \int_0^T \varphi_t^2 \, dt$$

$\Longrightarrow$

$$P[Z_T \geq e^{-\gamma T}] =$$

$$P\Big[\underbrace{\tfrac{1}{T}(F(S_T) - F(S_0))}_{\text{non-trivial perturbation}} - \underbrace{\tfrac{1}{2}\tfrac{1}{T}\int_0^T h(S_t)\, dt}_{\text{LDP}} \leq \gamma\Big]$$

$h := f'\sigma + \frac{1}{2}\varphi^2$

joint LDP!

explicit case study:

geometric Ornstein-Uhlenbeck process

$$S_t = e^{Y_t}, \quad dY_t = \sigma\, dW_t - c Y_t\, dt$$

based on Florens-Landais, Pham, Rouault, Fleming-Sheu

Theorem: $\forall \gamma \in \left(0, \frac{\sigma^2}{8} + \frac{c}{4}\right)$

the sets

$$A_T := \{ Z_T \geq -\gamma T \}$$

satisfy

$$P^*[A_T] \geq 1 - e^{-\gamma T}$$

and

$$\lim_{T \uparrow \infty} \frac{1}{T} \log P[A_T] = \underbrace{-\frac{\left(\frac{\sigma^2}{8} - \gamma + \frac{c}{4}\right)^2}{\frac{\sigma^2}{8} - \gamma + \frac{c}{2}}}_{=: I(\gamma)}$$

Theorem: For $\gamma \in \left(0, \frac{\sigma^2}{8} + \frac{c}{4}\right)$ and $\gamma_1 < \gamma$, there exist contingent claims X_T at zero cost such that

i) $$X_T \geq -\alpha_T := -e^{\gamma_1 T} \frac{P^*[A_T^c]}{P^*[A_T]}$$

where

$$A_T = \{ Z_T \geq e^{-\gamma T} \}$$

$$\overline{\lim_{T \uparrow \infty}} \cancel{\frac{1}{T}} \log \alpha_T \leq -(\gamma - \gamma_1)$$

ii) $$\lim_{T \uparrow \infty} \frac{1}{T} \log P[X_T < e^{\gamma_1 T}] = -I(\gamma)$$

iii) The decay rate in ii) is optimal under the constraint i)

Proof:

$$X_T := \underbrace{e^{\gamma_1 T} I_{A_T^c}}_{\text{typical for } P} - \underbrace{\alpha_T I_{A_T}}_{\text{typical for } P^*}$$

$$\geq -\alpha_T$$

where

$$\frac{1}{T}\log \alpha_T = \gamma_1 + \frac{1}{T}\log \underbrace{P^*[A_T^c]}_{\substack{= E[Z_T;\, Z_T < e^{-\gamma T}] \\ \leq e^{-\gamma T}}} - \frac{1}{T}\log P^*[A_T]$$

$$\Longrightarrow \quad \overline{\lim_T} \frac{1}{T} \log \alpha_T \leq -(\gamma - \gamma_1)$$

(even better!)

attainable at zero cost:

$$E^*[X_T] = e^{\gamma_1 T} P^*[A_T^c] - \alpha_T P^*[A_T]$$
$$= 0$$

Clearly,

$$\lim_{T\uparrow\infty} \frac{1}{T} \log P[\underbrace{X_T < e^{\gamma_1 T}}_{= A_T}] = -I(\cdot$$

This decay rate is optimal under the constraints

$\hat{X}_T$ attainable at zero cost,

$\hat{X}_T \geq -\alpha_T$:

$\hat{A}_T := \{\hat{X}_T < e^{\gamma_1 T}\}$ satisfies

$$P^*[\hat{A}_T^c](e^{\gamma_1 T} + \alpha_T) - \alpha_T$$
$$= e^{\gamma_1 T} P^*[\hat{A}_T^c] + (-\alpha_T) P^*[\hat{A}_T]$$
$$\leq E^*[\hat{X}_T] \leq 0 \, , \quad \text{i.e.}$$

$$P^*[\hat{A}_T^c] \leq \frac{\alpha_T}{\alpha_T + e^{\gamma_1 T}} = P^*[A_T]$$

$\Longrightarrow$ $P[\hat{A}_T] \geq P[A_T]$

Neyman-Pearson i.e. lower decay rate!

~ rates of growth for maximal expected utility

Bielecki, Pliska (1999)

Fleming, Sheu (1999, 2000, 2002)

Guasoni (2008)

Hata, Iida (2006)

Hata, Nagai, Sheu (2007)

Hata, Sekine (2006)

Pham (2003)

Sekine (2005)

$|\mathcal{Q}^*| \neq 1$?

robust utility ?

<u>Dynamics of</u>
<u>robust projection problem</u> ?
(Th. Knispel, HU Berlin)
~ robust large deviations

?

Sekine, Guasoni, ...

Thomas Knispel
Diss. HU Berlin (in progress):

some robust versions

in particular "complete" case studies which admit

Q_0 "least favorable in the long run",

i.e.

$$\lim_T \frac{1}{T} \log \inf_{Q \in \mathcal{Q}} Q\left[\frac{1}{T} \log \varphi_T^* \geq c\right]$$
$$= \lim_T \frac{1}{T} \log Q_0\left[\frac{1}{T} \log \varphi_T^* \geq c\right]$$

where $\varphi_T^* := \frac{dP^*}{dQ_0}\Big|_{\mathcal{F}_t}$

(e.g. logarithmic O-U with uncertain mean reversion) we hope $\Rightarrow$ examples of

$\longleftarrow$

H.F. and Thomas Knispel:

robust versions of
Sanov's theorem, ...

Shige Peng (2 weeks ago, Kyoto)

robust versions
of law of large numbers
and CLT

But how about

→

"cleaning up the mess"

?

new opportunities / obligations to contribute:

"good" regulation

(highly non-trivial: without arbitrage, "smart" ways out, ..)

= "systemic" risk management

point of view of supervising agencies ≠ banks (society)

challenging new "principal-agent" problems