Reduce to the max…

Efficient solutions for mid- size problems in interest rate derivative pricing and risk management at RLB OOE

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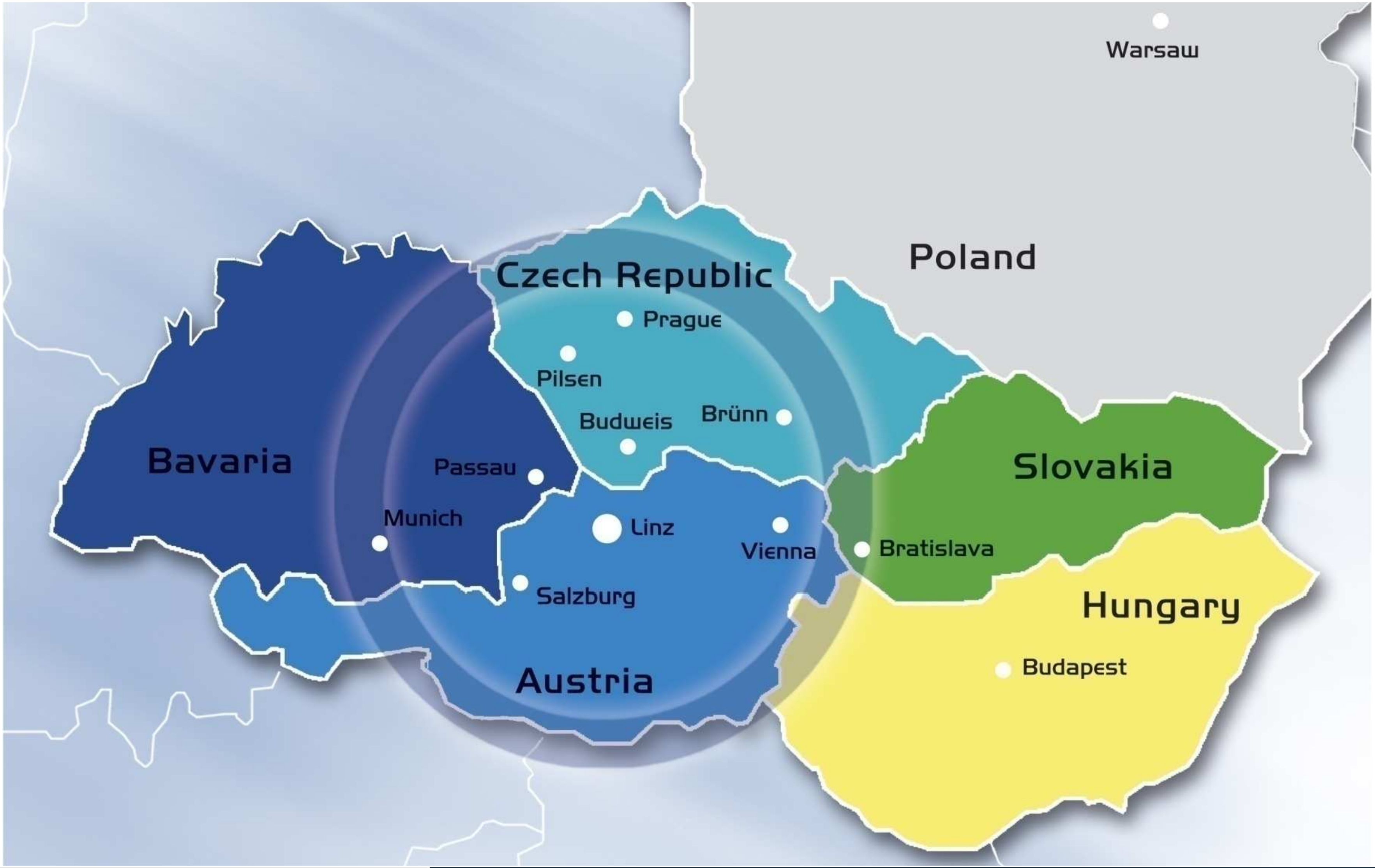
Outline

- Introduction
- Motivation for Structured Products
- Stages of Implementation in Trading, Mid Office and RC
- Pricing problems and challenges
  - Model Risk
  - Data Restrictions
- VaR Calculation for Structured Products
- Conclusion

- *Credits:*
- *Most of the computational work presented here was done by MathConsult / the UnRisk Consortium*

RLB Upper Austria - Domestic market

Focus: 300 km



RLB Upper Austria - Focus

- Strategic positioning as trading oriented bank:

  Customer focused direct trading services
  Active participation in interbank-vanilla trading
  Syndication and origination activities

- Market Maker and Primary Dealer for Austrian Capital Market Products

- Turnover p.a.: ~Euro 160 bln.

- 27 Treasury Specialists in 3 trading areas:

  Equity and Bond Trading
  Foreign Exchange/Money Market
  Treasury/Syndication and Origination

Before 2005: Market-Making for Standard IR-Derivatives [only]

Interest Rate Swaps

- Plain vanilla
- structured (e.g. step ups, single callable) spot start / forward start

Caps & Floors

- Plain vanilla
- structured (e.g. step ups, amortising nominal)

Swaptions

- Receivers / Payers Options plain vanilla
- structured (e.g. amortising nominal)

Market conditions that time made life with plain vanilla/single callable IR Derivatives only a hard thing...

From 2005:
Need for structured IR Products

- **Motivation:**
  - Profits from plain vanilla products → 0 due to high liquidity
  - Clients demanded for structured off-market coupons (Asset & Liability side)
  - Individual risk profiles required tailor made IR structures
  - Range of exotic products on the market had become widespread, volume was rapidly increasing

→ Taking part in the structured rates market was a must

- **Solution:**
  - RLB started to provide a market for small- to mid- level structured products in order to enable yield enhancement by cumulated option premiums from exotic options

Participation in Structured IR Market – Phase 1

- Offering products to clients without capabilities of pricing or risk handling of path dependent risks
  - RLB before 2002
  - Product ideas from partner investment banks only - no innovative capability
  - Each pricing has to be outsourced
  - Delays in servicing clients (from pricing to regular valuation)
  - Huge minimum transaction sizes → k.o. for many clients & ideas
  - Expensive secondary market for institutional sizes
  - No secondary market for retail sizes possible
  - No idea of "mid market" – no check for plausibility

Participation in Structured IR Market – Phase 2 (2002 – 2005)

- Pricing Tool for Treasury Front Office only
  - Start of Cooperation with Mathconsult / Implementing UnRisk Pricing Engine
  - Independent generation of structured ideas
  - Tailor-making strategies for individual clients
  - Mid market pricing
  - No need to verify each pricing indication → increases product pool
  - Scenario analysis for clients – improved servicing

  - Still no non- hedged positions
  - Problems providing secondary market liquidity
  - “Feeling” for mid market, but bid offer spreads still lost…
  - Problems with minimum sizes of the deals

Participation in Structured IR Market – Phase 3

- Entering the Structured IR market by applying UnRisk Pricing Engine as a Pricing Tool

- Requirements :
  - *Needed* easy-to-use & flexible Pricing Engine & GUI for Front-, Mid-, Back Office and Risk controlling
  - *Needed* regular product updates with latest structured innovations
  - *Needed* fast computation for daily valuation tasks and risk scenario analysis

  … in order to …

  - *Enable* continuous and consistent valuation
  - *Enable* individual (IR and volatility) curve shift scenarios
  - *Enable* flexibility (size & frequency) in providing secondary market liquidity
  - *Enable* profit optimization (macro hedges ought to be sufficient)

- Implementation: Challenges for Pricing and Risk Controlling

Challenges for Pricing (I)

- We started offering a Standard Product Pool:
  - Callable/Putable CMS linked Products
  - Callable/Putable Range Accruals
  - Callable/Putable TARN's
  - Callable/Putable (varying) Fixed Rate Products

  …priced with a Hull White 1F/2F IR model, swaption- calibrated with available ATM market data
- But we also did…
  - Callable IR Spread Structures (Leveraged "Steepeners")
  - Callable Snowballs
  - …priced the same way

- … and we learned…

Challenges for Pricing (II)

- **Cornerstones of the learning process:**
- **Problems:**
  - Suitability of (normal) HW models for different product categories is restricted
  - Even for moderately structured instruments, there were clear signs of severe model dependence, depending on the model class (e.g. lognormal Range accruals vs. normal Bermudans)
  - For “feedback loop” products (Snowballs) and leveraged correlation trades (Steepeners), our prices were far away from tradeable prices, but
  - … the tradeable prices themselves differed up to ~300 bps in terms of PV
- **Conclusions:**
  - In order to come up to the pricing tasks and to limit model risk, we expanded our toolbox by adding NumeriX as a pricing Engine (supplying a n-factor LMM, including StochVol) and using the new BK 1F Model in UnRisk

Sustainable Model Risk (I)

- **…and the next Problems:**
- **1. Sustainable Model Risk or: *"It always depends…"***
  - **It is not enough to calibrate the models for individual products to "market pricing" once…**
  - **Even for a single and moderately structured product, the outcomes are far from being constant:**
  - **The following figure shows fair values of a *Callable Reverse Floater* with a nominal value of 100 EUR, maturing on Jan. 1 ,2021, and paying annual coupons of:**
    Max(16.5% - 2 x CMS 5y, 0%) set *in arrears* (at the end of each coupon period).
    The bond is early redeemable by the issuer for a price of 100 on every coupon data, starting in 2011.
    (Without this callability, the price curve of the bond would be the same for **all considered interest rate models**, but due to callability, there is also model risk associated with the price)

Sustainable Model Risk (II)

- [Example cont'd]
- **The models used for valuating this bond were Hull-White, Black-Karasinski and Libor market model (LMM).**
- **Hull-White (one factor)**

$$dr = (a(t) - b(t)r(t))dt + \sigma(t)dW$$

- **Black-Karasinski**

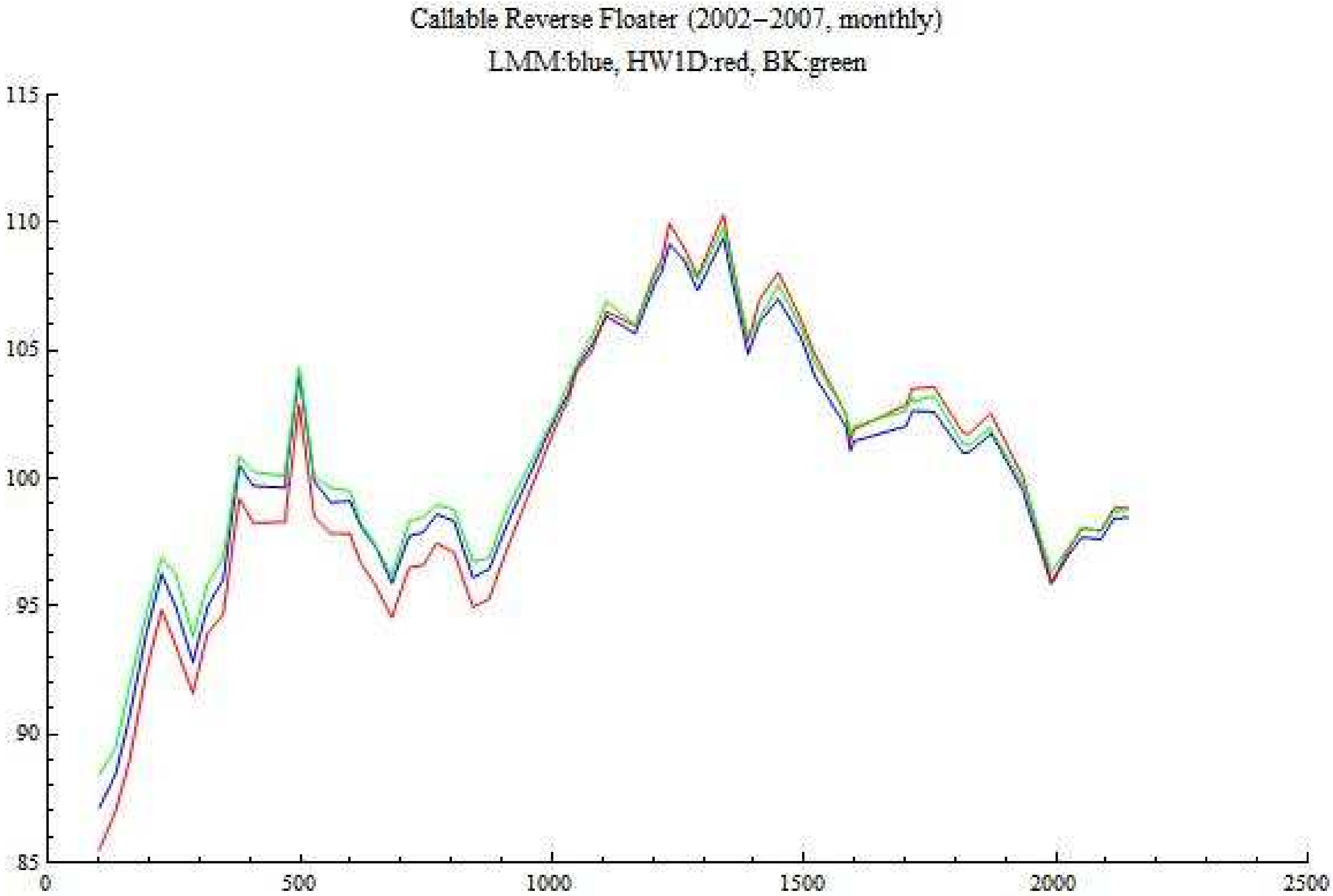
$$d\ln(r(t)) = (\eta(t) - \gamma\ln(r(t)))dt + \sigma dW$$

- **Libor Market Model for forward rates F_k**

$$dF_k(t) = \sigma_k(t)F_k(t)\sum_{j=\beta(t)}^{k}\frac{\tau_j\rho_{j,k}\sigma(t)F_j(t)}{1+\tau_jF_j(t)}dt + \sigma_k(t)F_k(t)dZ_k^d(t)$$

$$k = 1,\ldots,m$$

Sustainable Model Risk (III)



Sustainable Model Risk (IV)

- The example showed fair values of the callable reverse floater between May 2002 and Dec. 2007 under three different interest rate models calibrated to the same market
- There is a model risk of up to 4% associated with this (simple) sample floater.
- …and this model risk does not always display the same ranking!

Available vs. Necessary Data(I)

- …and the next Problems:
- 2. Available vs. (theoretically) Necessary Market Data or
  "If you don't have cannon food, don't use cannons"
  - Whereas the Unrisk Engine (so far) handles ATM data for calibration only, NumeriX is (theoretically) able to treat volatility Cubes
  - According to market practice (e.g. for bermudan swaps), the models shall be calibrated to co-terminal swaptions with the appropriate strikes – which are, for most of the products, some way from being ATM
  - We started to work using NumeriX modeling capabilities in LMM terms, trying to find the necessary data in the market *(as we don't get them from the traders):*
    - (Implied) Swaption Vol Cubes with volatility smile
    - Implied Correlations for Correlation products

Available vs. Necessary Data(II)

- …and we found some data in the market:

Quote: SMKR410

00:35 04SEP08 (c)2008 Tullett Prebon Information UK08509 SMKR410

EUR Swaption Volatility Smile

OPTION/ TENOR	(BASIS POINT DEVIATIONS FROM ATM) -200	-100	-50	-25	ATM	25	50	100	200	ATM STRIKE
1Y1Y	29.5	24.8	23.3	22.7	22.3	22.1	21.9	21.9	22.6	4.498
3M2Y	32.4	27.0	25.3	24.6	24.1	23.7	23.5	23.3	23.9	4.689
2Y2Y	25.9	21.4	20.0	19.5	19.1	18.8	18.6	18.5	18.9	4.456
1Y5Y	26.8	21.5	19.7	19.0	18.5	18.1	17.8	17.6	17.9	4.473
5Y5Y	19.6	15.7	14.4	13.9	13.5	13.2	13.0	12.8	13.1	4.708
3M10Y	26.3	20.3	18.2	17.4	16.7	16.2	15.9	15.7	16.3	4.636
1Y10Y	23.1	18.1	16.3	15.5	15.1	14.7	14.5	14.3	14.8	4.619
2Y10Y	21.4	16.9	15.3	14.7	14.2	13.8	13.6	13.4	13.7	4.671
5Y10Y	18.6	14.8	13.4	12.9	12.5	12.2	12.0	11.8	12.1	4.847
10Y10Y	17.0	13.4	12.1	11.6	11.2	11.0	10.8	10.6	11.1	4.930
15Y15Y	16.9	12.8	11.4	10.9	10.5	10.2	10.0	9.9	10.4	4.548
10Y20Y	17.7	13.7	12.3	11.7	11.3	11.0	10.8	10.7	11.1	4.711
5Y30Y	19.3	15.0	13.5	12.9	12.5	12.1	11.8	11.6	11.9	4.655
	-200	-100	-50	-25	ATM	25	50	100	200	

Available vs. Necessary Data(III)

- Although data is quoted on a permanent basis for a certain number of swaptions, 2 problems make working with them a real challenge:
  - The datapoints are not all equally liquid, therefore inconsistencies in the matrix arise frequently
  - As only some cornerstones of datapoints for the volatility cube are available, the inter-/extrapolation problem arises as well

	1Y	2Y	5Y	10Y	15Y	20Y	30Y
3M		X		X			
1Y	X		X	X			
2Y		X		X			
5Y				X	X		X
10Y						X	
15Y					X		

  - Implied Correlation data is not available on the market at all

Reduce to the MAX…

- The (preliminary) results of the learning process:
  - As Snowballs & Steepeners are extremely sensitive in terms of pricing (and the necessary market data is hardly available), we excluded them from our standard toolbox and reduce our product universe to moderately ("mid-level") structured products –
  - they are most of the time more suitable for the consumer as well…

Reduce to the MAX…

- **As even simple products exhibit time-varying and non-negliable model risk, we always price using different models and do "re-calibrate" our pricing for certain products with market practice on a regular basis –**
- **This enables us to come up not to "fair", but tradeable pricing levels.**
- ***A "fair" price only seems to exist for the trader who does effectively hedge the position…***

Reduce to the MAX…

- For this multi-model approach we use UnRisk, which is now cabable of pricing HW (1- and 2F), BK (1F) and LMM as well (restricted to ATM data), with a NumeriX "Security Back Up"

- For pricing purposes, we restrict ourselves to ATM data – a good model with liquid data in mid-level practice turned out to be better than a complex SV one with questionable data input

Challenges for Risk Controlling

- With an increasing number of (unhedged) IR structures on the book, computational time for doing historical Value at Risk – simulations for these products increased dramatically.
- (especially given a rather not too flexible VaR system)
- Although implementing the UnRisk "Factory" as a common product & pricing database and therefore simplifying Front-Mid-Office communication, the problem with VaR calculation remained.

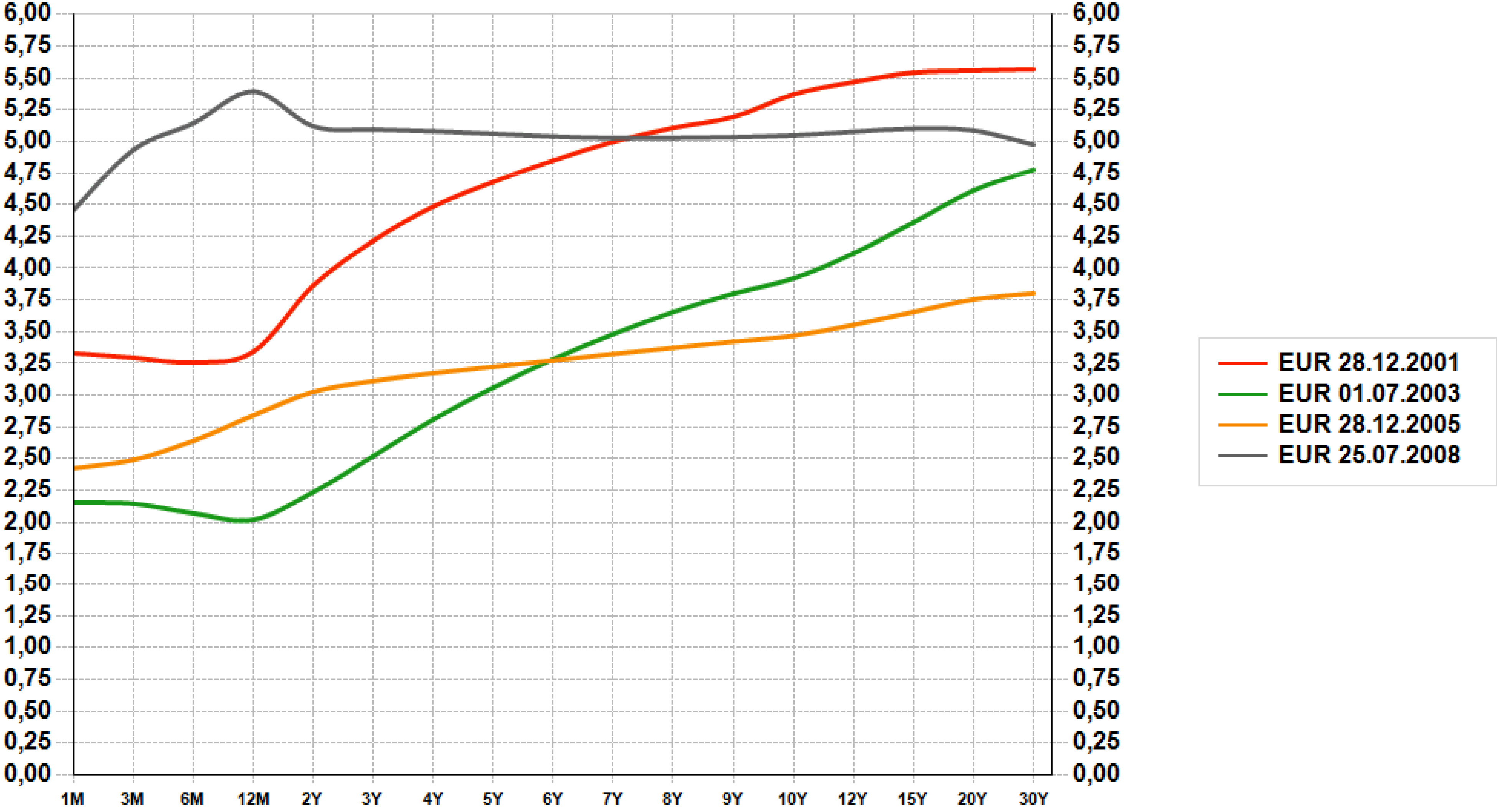
- Therefore we started a project together with MathConsult, trying to find an efficient and robust solution for these problem

Speeding up VaR Calc

- It is common knowledge in risk management that movements of interest rate curves can be mainly described by just a few factors (often named “shift”, “twist”, “butterfly”).
- If this common knowledge is supported by evidence, these factors could be used for approximating IR curve movements in order to speed up valuations.

- Analysis was started with daily EUR interest rate values (spot market, zero rates continuous compounding) between August 2000 and July 2007 (1766 data sets) given for the curve points {overnight, 1week, 3months, 6m, 9m, 1year, 2y, 3y, 4y, 5y, 7y, 10y, 15y, 20y, 25y, 30y, 50y}

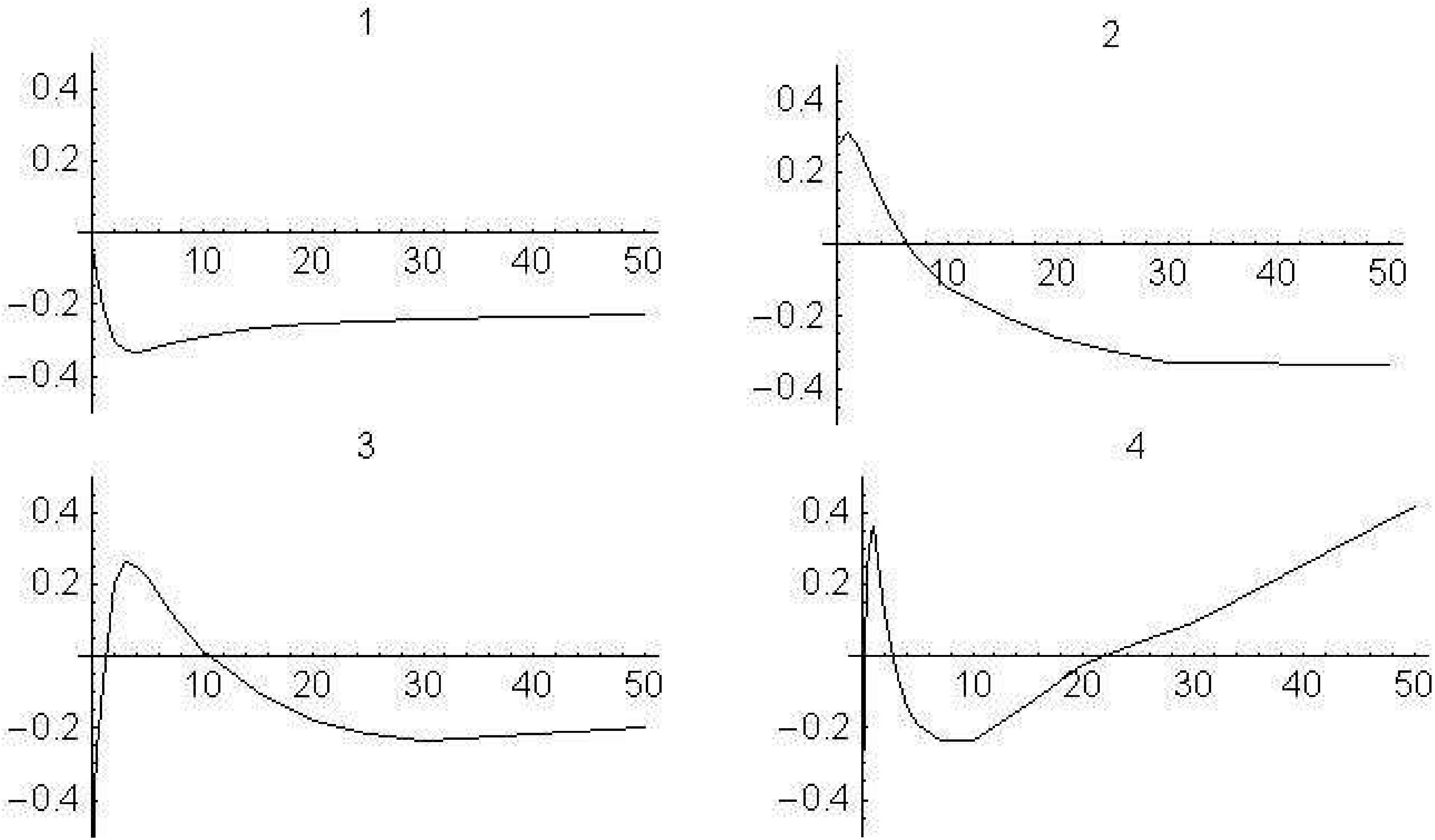
Speeding up VaR Calc



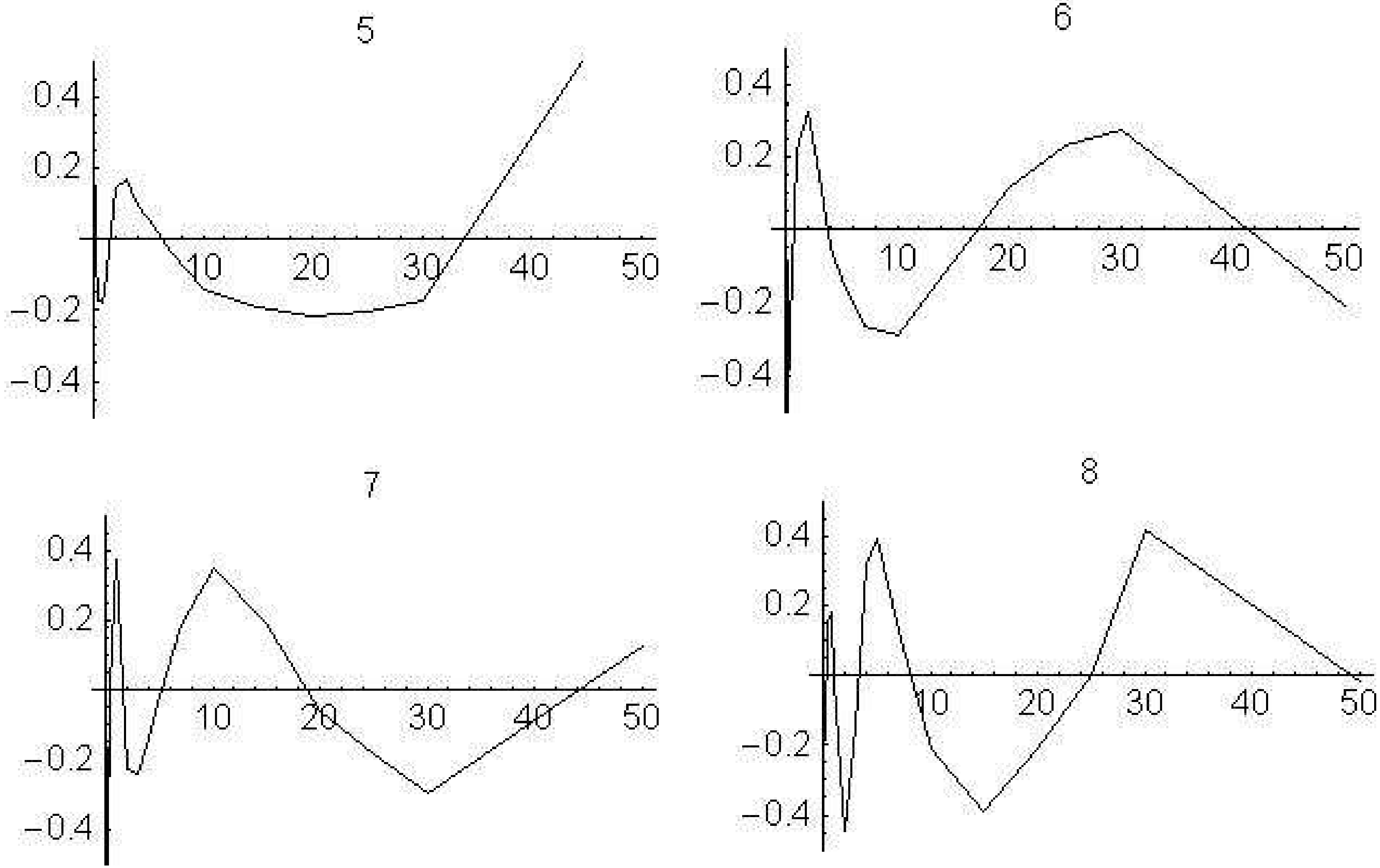
Interest changes and their Principal Components

- Using these datapoints and excluding the overnight rate (without any influence on PV calculation), interest rate curves reduce to points in a 16-dimensional space.
- The changes in the EUR curves were calculated on a weekly basis
- Then a plain Principal Component Analysis was applied to these changes
- In this analysis, all tenors for interest rate had equal weight, which means that, as the short end of the yield curve is more dense in terms of data points, this part of the curve was more important for the following analysis.
- The 16 principal components then had the following shapes

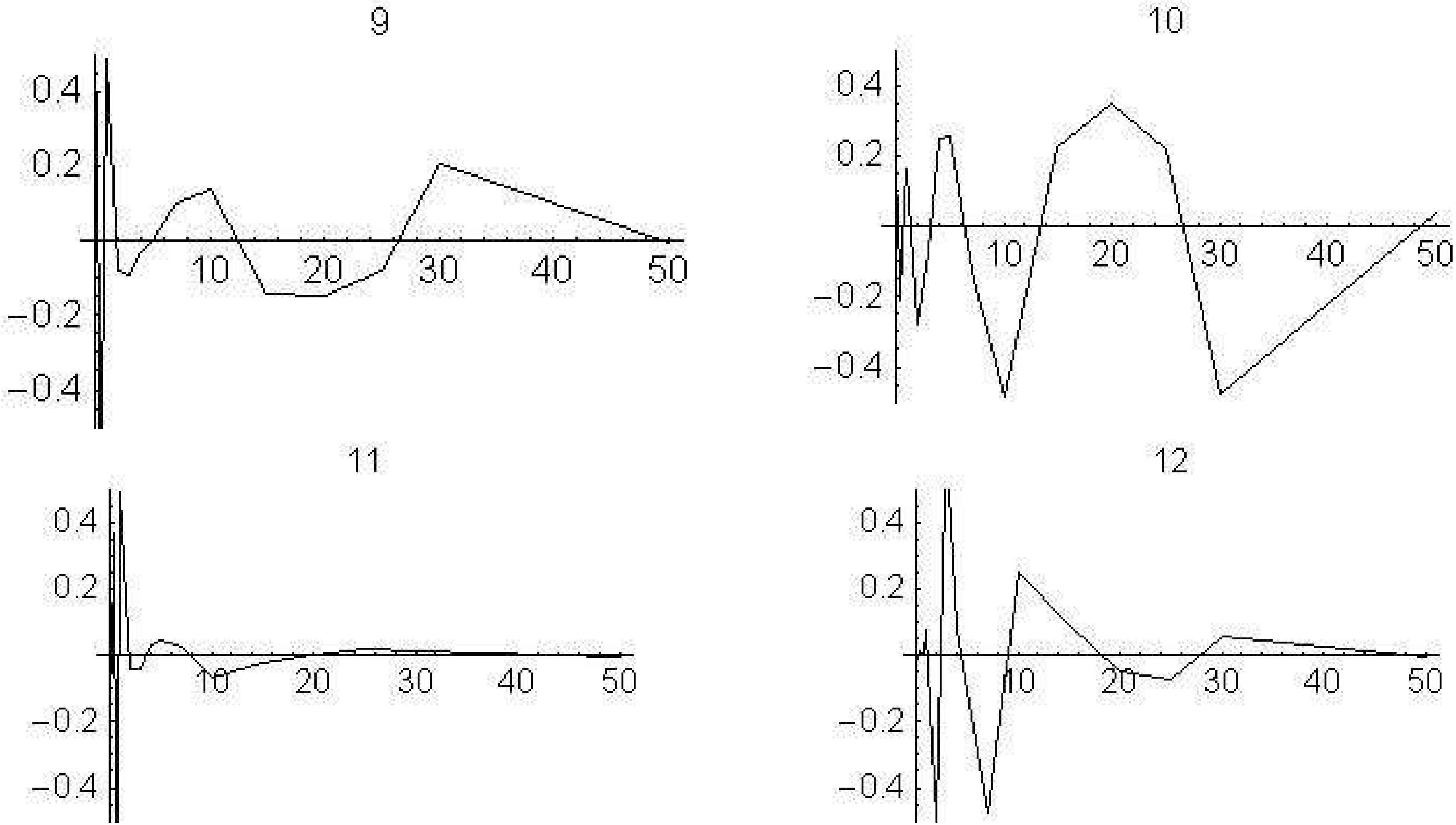
Interest changes and their Principal Components (PC 1 - 4)



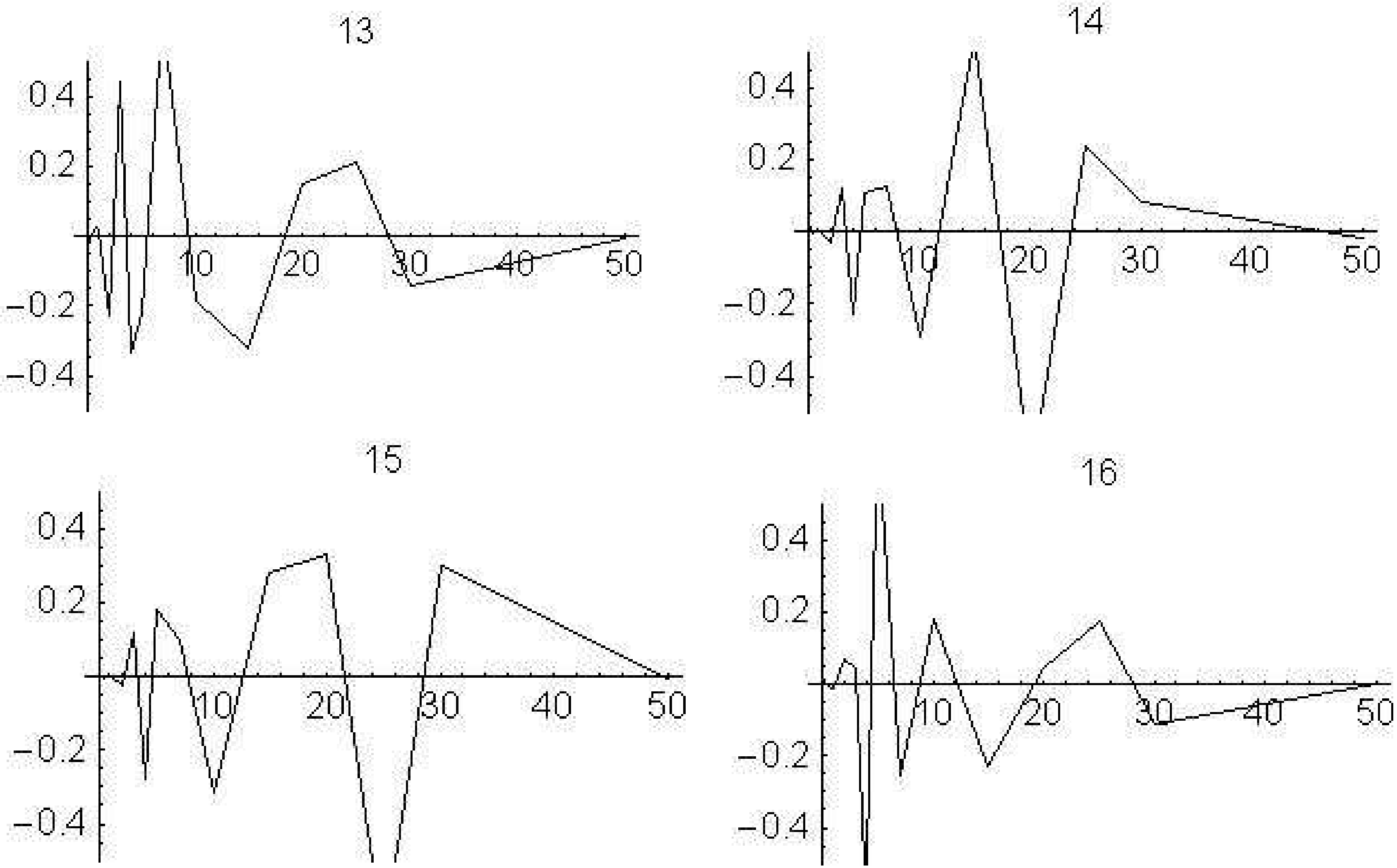
Interest changes and their Principal Components (PC 5 - 8)



Interest changes and their Principal Components (PC 9 - 12)



Interest changes and their Principal Components (PC 13 - 16)



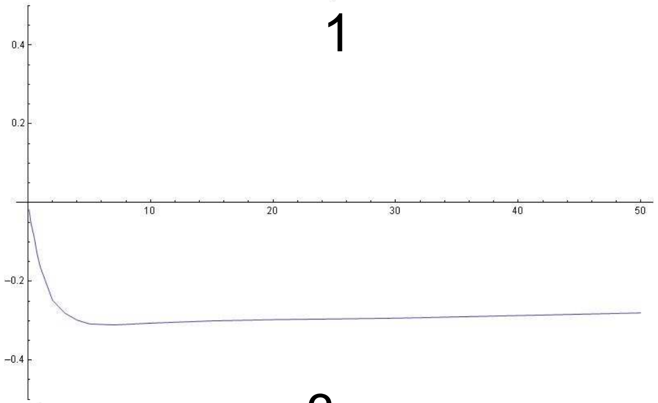
Interest changes and their Principal Components

- For the calculation of these principal components of the increments, all data sets (between 2000 and 2007) were used.
- The first three unit vectors exhibit the "shift, twist, butterly" behaviour.
- Unit vector 1 explains 77 percent of interest rate changes,
- 1 and 2 explain 92% ,
- and 1, 2, 3 explain 96,88% of the weekly interest rate changes.

Robustness of the PC's

- For the above analysis, all available data sets were used
- It turns out that, if the observation window is reasonably long, the shapes of the main components more or less always look the same:
- Observation window: 300 days

1



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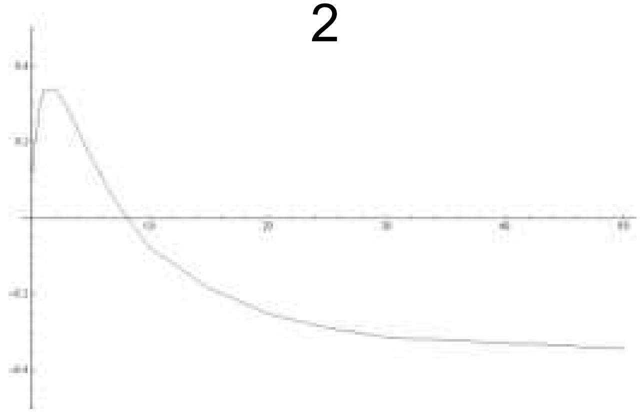

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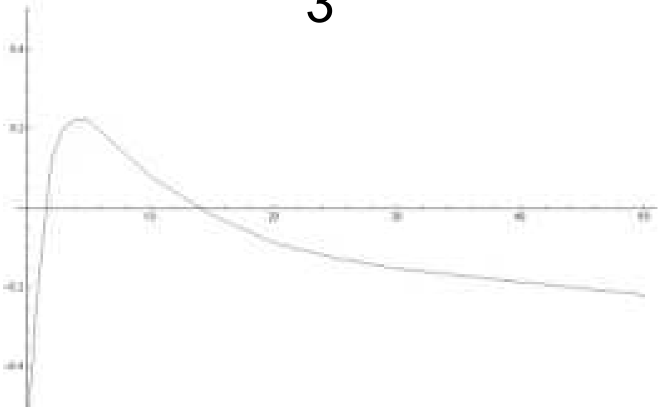

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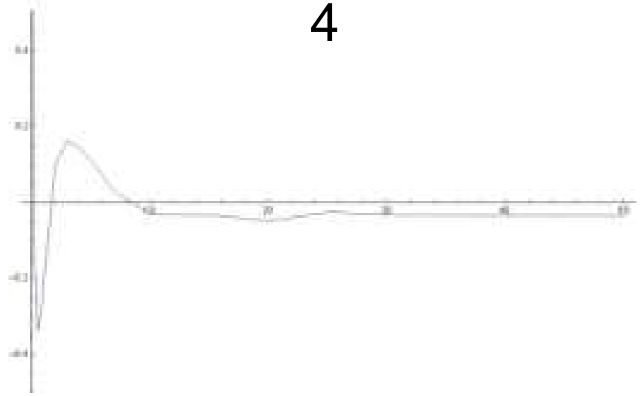
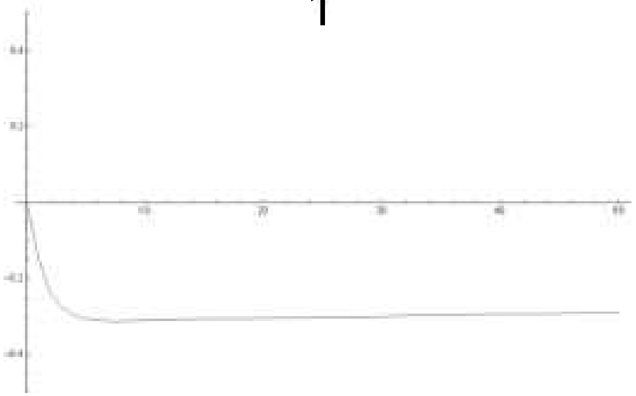


Robustness of the PC's

- **Observation window: 500 days**

1



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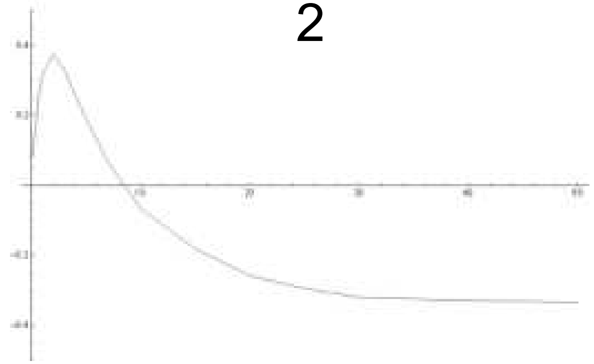

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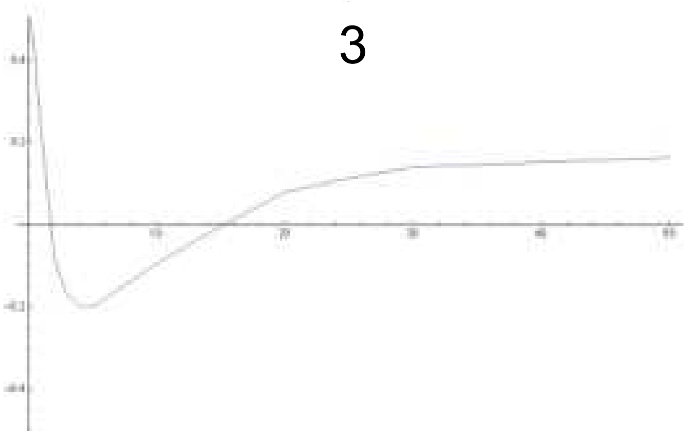

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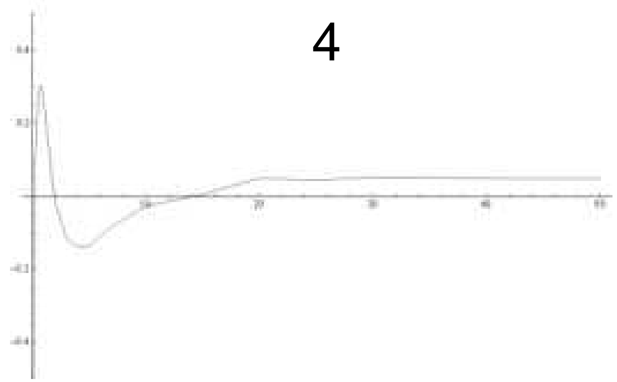
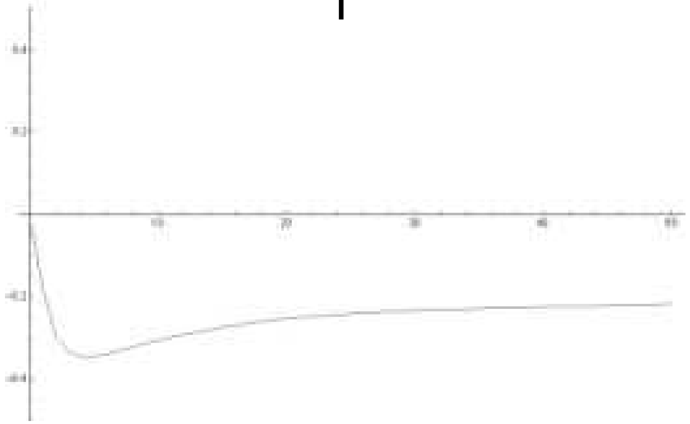


Robustness of the PC's

- **Observation window: 1000 days**

1



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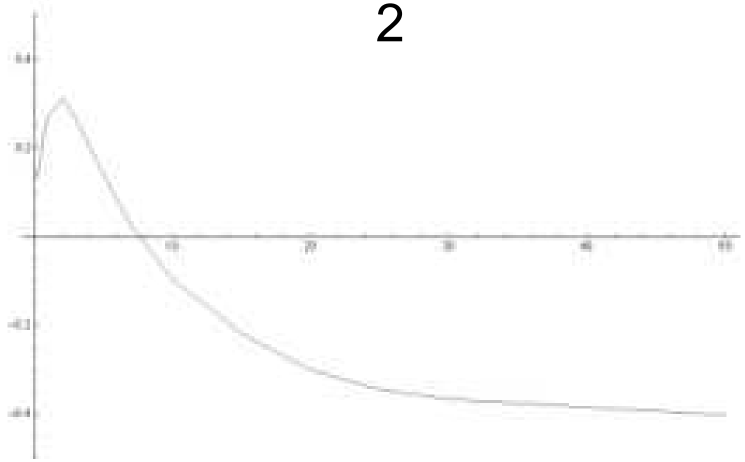

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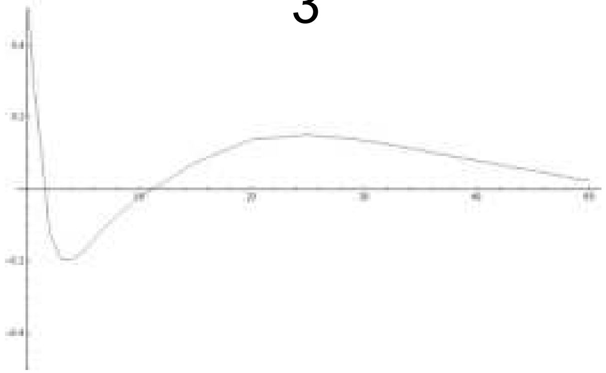

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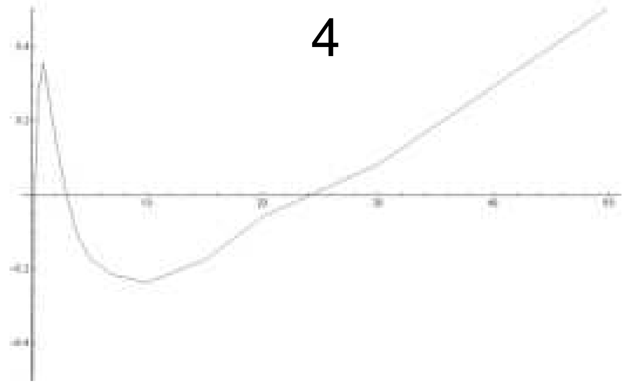
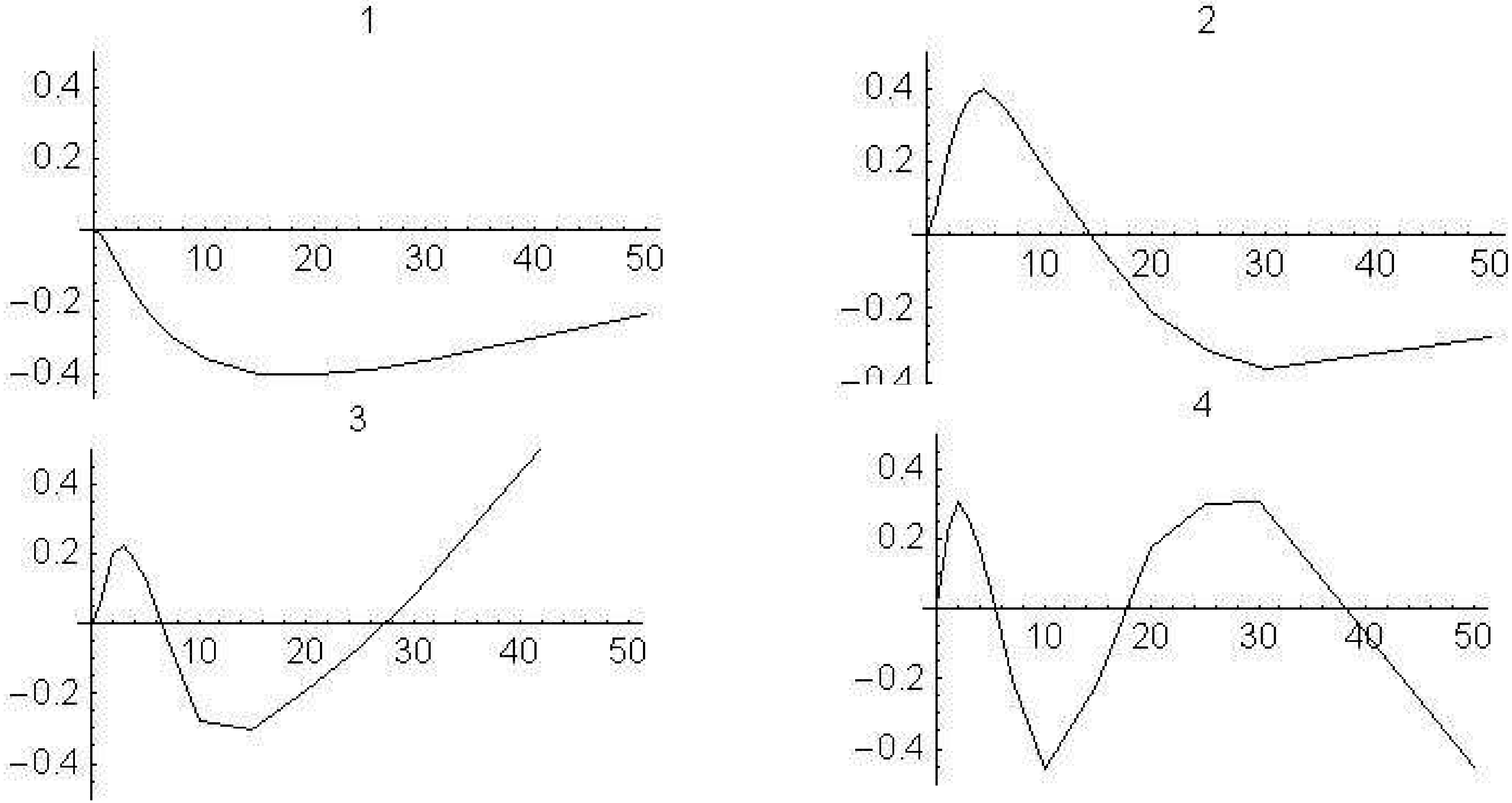


Switching to Discount Factor Curves

- For valuation purposes, the „heavier“ weighting of short maturities is not the best solution, discount factor curves were used as an alternative approach in order to emphasize longer tenors of interest rates more than the shorter rates

Switching to Discount Factor Curves

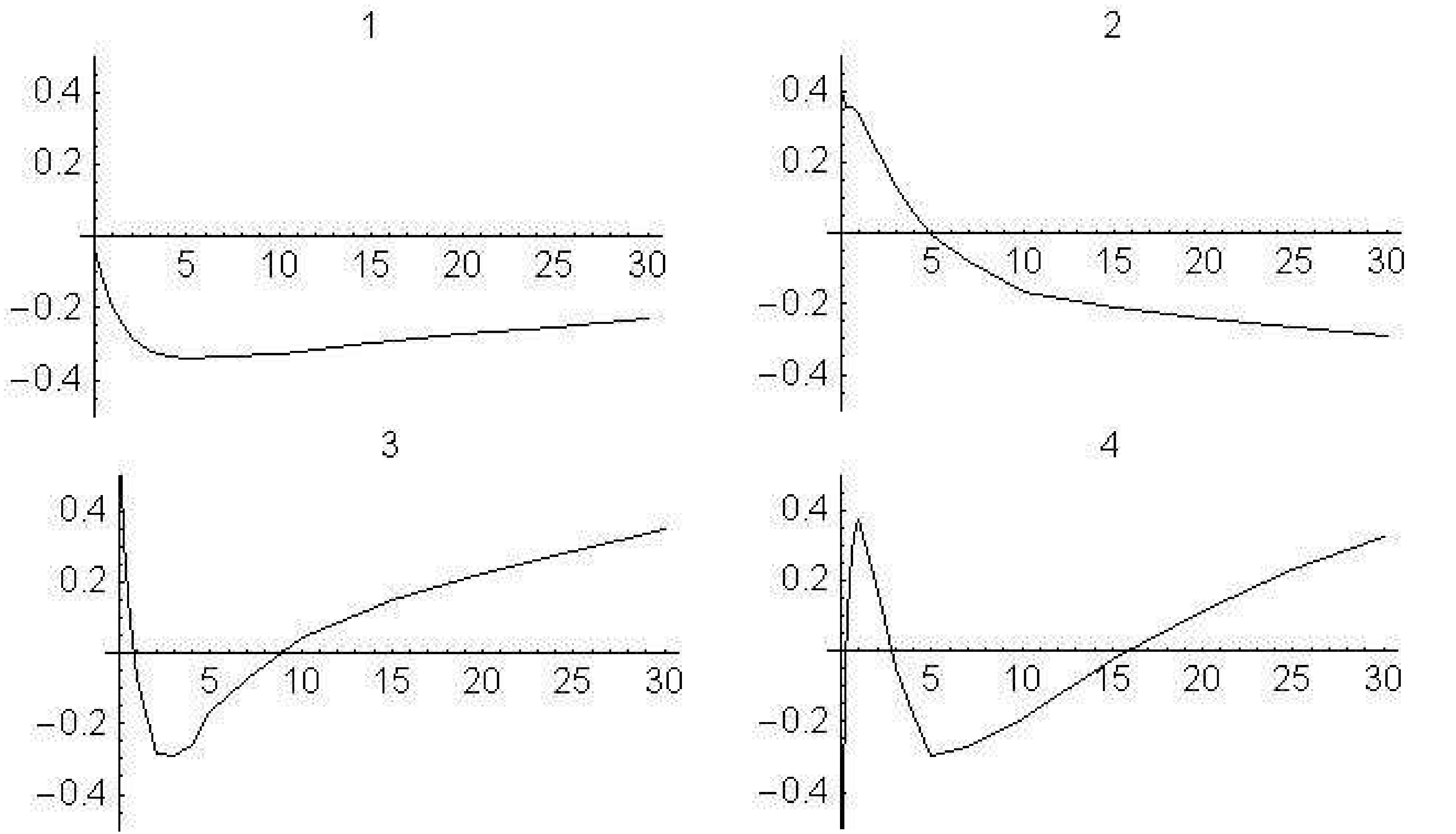
- **The discount factors on the short end of the curve cannot change too much, and therefore the first principal components of the discount shifts start close to the origin.**



EUR only?

- The project members analysed the interest rate shifts for different currencies (USD, GBP, CHF, JPY).
- It turns out that the principal components of the interest rate changes exhibit the same qualitative behaviour for all these currencies.
- As an example, here are the USD results.

USD Principal Components

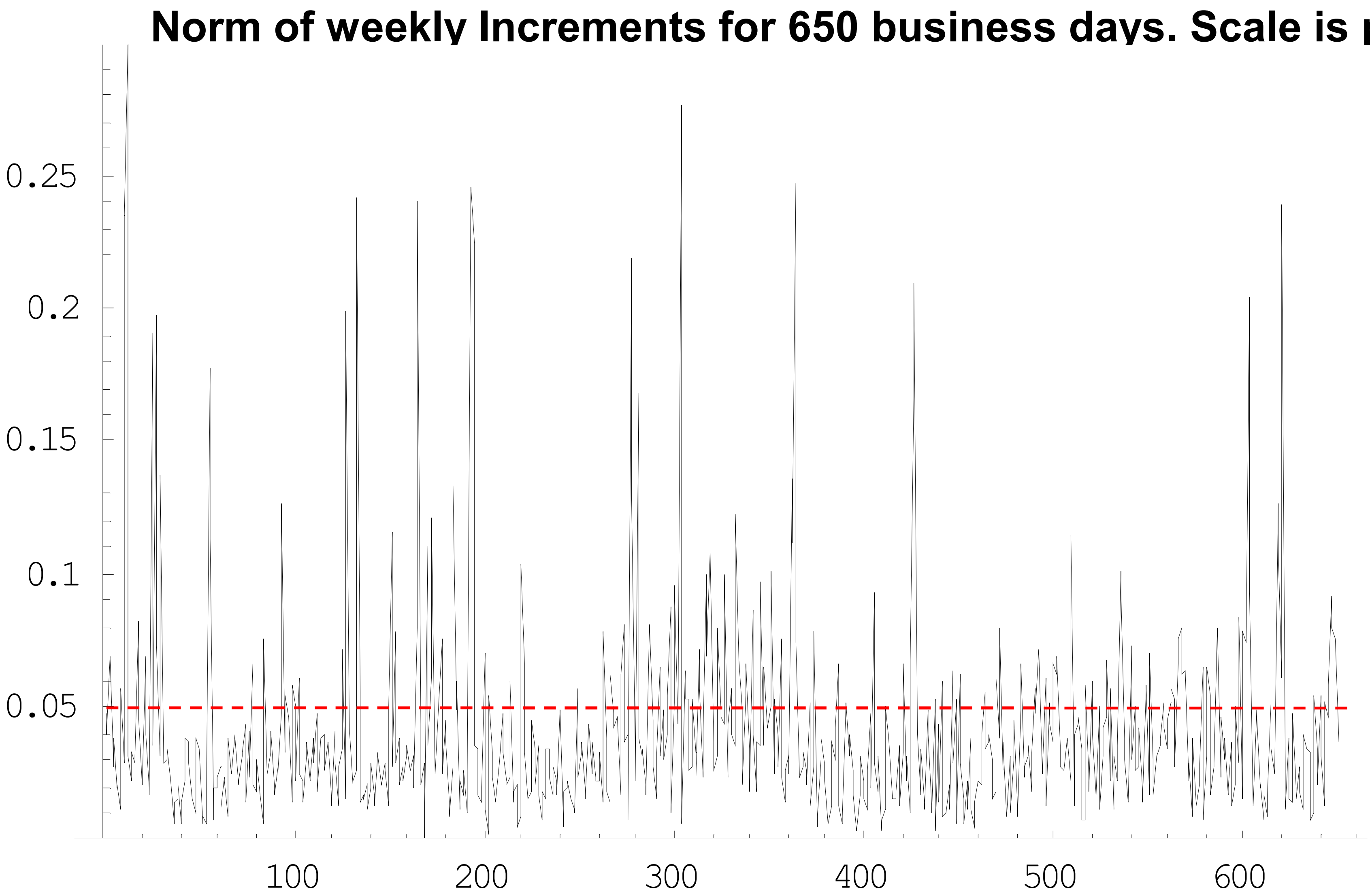


Quality of the approximation

- In order to analyze the quality of the projection to principal components, PCA was applied to the EUR yield curve increments to the first 1000 data sets (2001-2004)
- The resulting PC's were used as a basis for the increments of the dates 2005 and later.
- The norm of an increment was measured by:

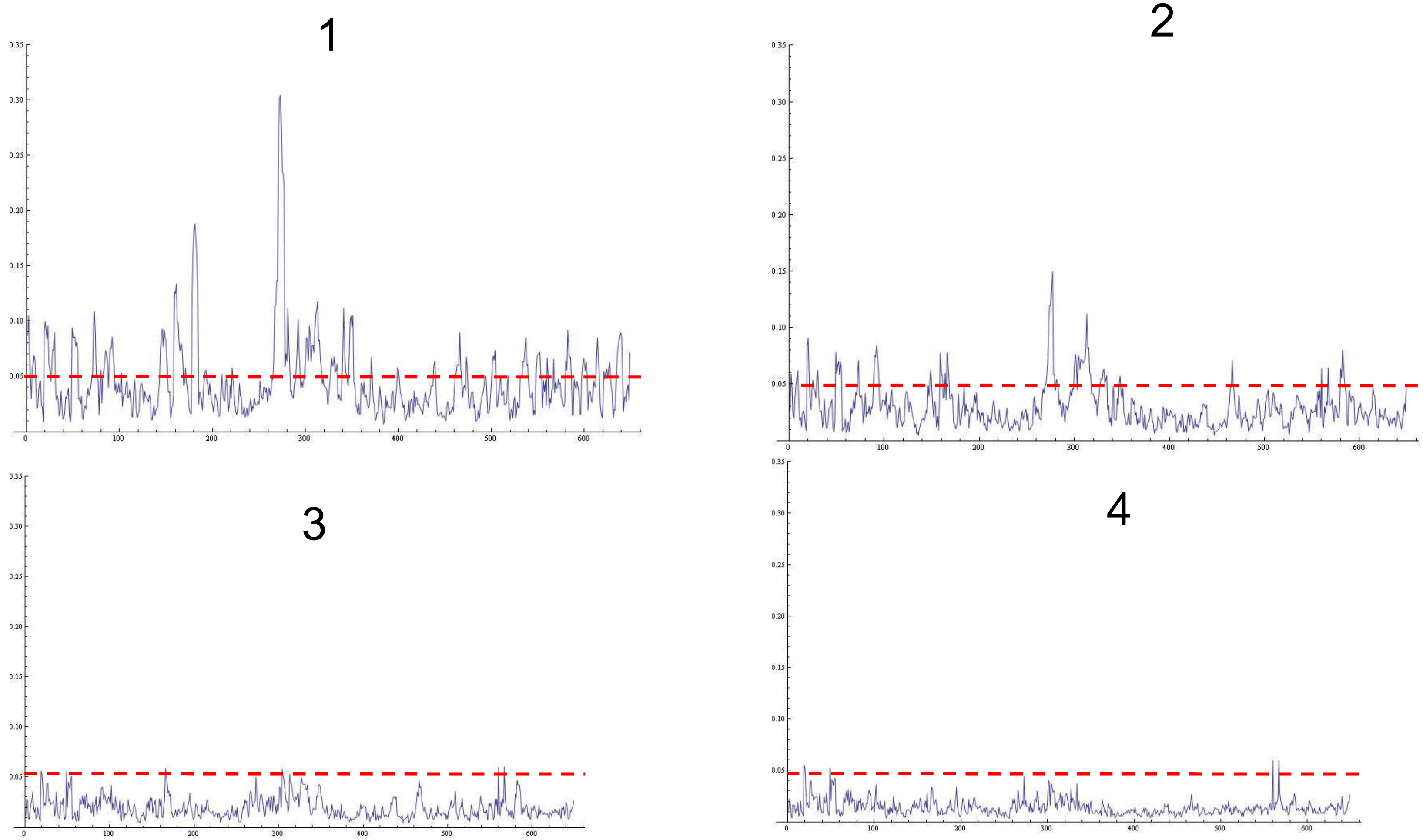
$$\|\Delta r\| = \sqrt{\frac{1}{16}\sum_{i=1}^{16}(\Delta r_i)^2}$$

Quality of the approximation



Quality of the approximation

- **Norm of approximation error after filtering 1, 2, 3, and 4 principal components. Scale is percent.**



Application to VaR Calculation

- RLB Risk controlling calculates historical VaR numbers at the 95% and 99% level, given historical data from 4 years (1000 data points).

- Therefore, the straightforward way to a historical VaR in our context consists of the following steps:

  - Apply 1000 historical changes (4 years – or more) of the interest rate curve to today's yield curve.
  - Calibrate the parameters of the interest rate model in use to the shifted yield curve data (in our case HW 1F)
  - Valuate all relevant structured instruments under these 1000 scenarios
  - Hence, if the portfolio once consists of 1000 instruments, this means that you have to carry out 1.000.000 valuations, which may definitely cause suicide of all computational systems in use.

Application to VaR Calculation

- Speeding up by implementing the following basic idea:
- V (r+Dr) = V(r) + grad V . Dr + higher order terms,

- 95% and 99% historical VaRs (1 week horizon) were then calculated by applying one factor Hull white models to 1000 weekly interest rate shifts.
- This was done either by exact calculation (applying 1000 curve fitting and valuation routines)
- and by Taylor expansion for the first 4, 5, and 6 principal components.

- Several structured products were used for the calculation:

Application to VaR Calculation

Structures for the test:

1. **Multicallable Step up Swap:** 2Y, quarterly callable
2. **Multicallable Step up Swap**: 6Y, quarterly callable
3. **Multicallable Step up Swap**: 10Y, quarterly callable
4. **Multicallable CMS:** 30Y, annually Callable, ISDA 5Y
5. **CMS deal**: 21Y, ISDA 8Y EUR
6. **Reverse Floater 1**: 10Y, Coupon: 15%-2*12M Euribor
7. **Reverse Floater 2:** 10Y, Coupon:15%-2*5Y CMS

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8. **Digital Range Accrual:** 7Y, ann.callable
9. **Snowball:** 7Y,ann. callable

Test results – Instrument Valuation

Average Error - 1000 hist. Scenarios using weekly IR shifts:

Testinstumente	Approx 4 PC's	Approx 5 PC's	Approx 6 PC's
Step up swap2y	0.79 bp	0.76 bp	0.76 bp
Step up swap6y	1.1 bp	0.8 bp	0.8 bp
Step up swap10y	1.8 bp	1.6 bp	1.5 bp
Multicallable CMS	0.8 bp	0.8 bp	0.8 bp
langer CMS deal	2.4 bp	1.7 bp	1.9 bp
Reverse Floater 1	10 bp	9 bp	9 bp
Reverse Floater 2	11 bp	11 bp	11 bp
Range Accrual	15 bp	15 bp	15 bp
Snowball	1.7 bp	1.3 bp	1.2 bp

Test results – VaR calculation

95 % VaR, [using Discount Curves]

10 MM EUR Notional per Structure

Structures	VaR 95% exact	VaR 95% approx	Approximation Error
Step up swap2y	32,699 €	30,860 €	1,839 €
Step up swap6y	73,337 €	71,218 €	2,119 €
Step up swap10y	88,539 €	87,616 €	923 €
Multicallable CMS	33,192 €	32,924 €	268 €
langer CMS deal	68,894 €	67,004 €	1,890 €
Reverse Floater 1	398,187 €	399,415 €	-1,228 €
Reverse Floater 2	360,071 €	356,016 €	4,055 €
Range Accrual	158,210 €	189,341 €	-31,131 €
Snowball	127,858 €	129,535 €	-1,677 €

Test results – VaR calculation

99 % VaR, [using Discount Curves]

10 MM EUR Notional per structure

Structures	VaR 99% exact	VaR 99% approx	Approximation Error
Step up swap2y	53,953 €	55,487 €	-1,534 €
Step up swap6y	120,614 €	117,566 €	3,048 €
Step up swap10y	138,629 €	133,461 €	5,168 €
Multicallable CMS	57,232 €	56,512 €	720 €
langer CMS deal	107,514 €	105,584 €	1,930 €
Reverse Floater 1	637,253 €	643,069 €	-5,816 €
Reverse Floater 2	579,391 €	613,291 €	-33,900 €
Range Accrual	266,725 €	277,212 €	-10,487 €
Snowball	204,618 €	210,545 €	-5,927 €

Test Results - Summary

- The results of the comparison can be summarised as follows:
  - Typical errors between full historical 95%VaR and 95%VaR based on 4 principal components was between less than 1 basis point and up to 10 basis points, for the 99% VaR up to 30 basis points.
  - The quality of the approximation for the digital range accrual VaR was lower due to the poorer quality of the Taylor approximation for the embedded digital options.
  - There was no systematic increase in accuracy when applying 5 or 6 principal components instead of 4.

  - So we can again reduce to the MAX!

Reduce to the MAX, Part II

- **Conclusions for Risk Management's VaR Calc:**

- **The hypothesis that principal directions of interest rate movements are shift, twist and butterfly was confirmed in the project**
- **These principal components can be used as unit directions in models reduced in dimensionality.**
- **For the fast calculation of the historical Value at Risk of moderately structured instruments which are in RLB's focus, the approximation properties are promising and sufficient.**
- **The project is now fully implemented and in use in RLB's Value at Risk system**