Pricing of electricity futures – The risk premium –

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Advanced Modelling in Finance and Insurance, RICAM Linz, September 22–26, 2008

Introduction

- Problem: what is the connection between spot and forward prices in electricity?
- Electricity is a non-storable commodity
- How to explain the risk premium?
  - Empirical and economical evidence: Sign varies with time to delivery
- Propose two approaches:
  1. Information approach
  2. Equilibrium approach
- Purpose: try to explain the risk premium for electricity

Outline of talk

1. Example of an electricity market: NordPool
2. The “classical” spot-forward relation
3. The information approach
4. The equilibrium approach
5. Conclusions

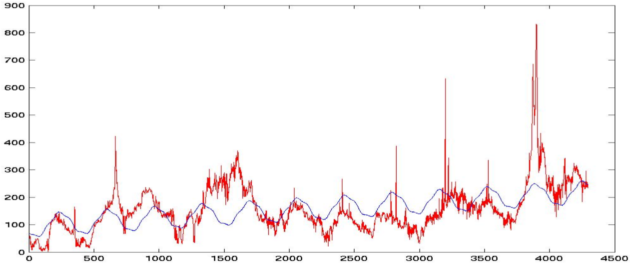
Example of an electricity market: NordPool

- The NordPool market organizes trade in
  - Hourly spot electricity, next-day delivery
  - Financial forward contracts
    - In reality mostly futures, but we make no distinction here
  - European options on forwards
- Difference from "classical" forwards:
  - Delivery over a period rather than at a fixed point in time

Elspot: the spot market

- A (non-mandatory) hourly market with physical delivery of electricity
- Participants hand in bids before noon *the day ahead*
  - Volume and price for each of the 24 hours next day
  - Maximum of 64 bids within technical volume and price limits
- NordPool creates demand and production curves for the next day before 1.30 pm

- The *system price* is the equilibrium
  - Reference price for the forward market
- Historical system price from the beginning in 1992
  - note the spikes....



The forward market

- Forward with delivery over a period
- Financial market
- Settlement with respect to system price in the delivery period
- Delivery periods
  - Next day, week or month
  - Quarterly (earlier seasons)
  - Yearly
- Overlapping settlement periods (!)
- Contracts also called *swaps*: Fixed for floating price

The option market

- European call and put options on electricity forwards
  - Quarterly and yearly electricity forwards
- Low activity on the exchange
- OTC market for electricity derivatives huge
  - Average-type (Asian) options, swing options ....

The spot-forward relation

The spot-forward relation: some "classical" theory

- The no-arbitrage forward price (based on the buy-and-hold strategy)

$$F(t,T) = S(t)e^{r(T-t)}$$

- A risk-neutral expression of the price as

$$F(t,T) = \mathbb{E}_Q\left[S(T)\,|\,\mathcal{F}_t\right]$$

- The risk premium is defined as

$$R(t,T) = F(t,T) - \mathbb{E}\left[S(T)\,|\,\mathcal{F}_t\right]$$

- In the case of electricity:
  - Storage of spot is *not* possible (only indirectly in water reservoirs)
  - Buy-and-hold strategy fails
  - No foundation for the "classical" spot-forward relation
  - ...and hence no rule for what Q should be!
- Thus: What is the link between $F(t, T)$ and $S(t)$?

Economical "intuition" for electricity

- Short-term *positive* risk premium
  - Retailers (consumers) hedge "spike risk"
  - Spikes lead to expensive electricity
  - Accept to pay a premium for locking in prices in the short-term
- Long-term *negative* risk premium
  - Producers hedge their future production
  - Long-term contracts (quarters/years)
- The market may have a change in the sign of the risk premium

Empirical evidence for electricity

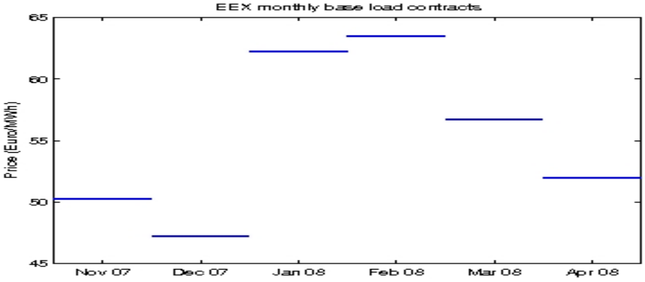
- Longstaff & Wang (2004), Geman & Vasicek : PJM market
  - Positive premium in the short-term market
- Diko, Lawford & Limpens (2006)
  - Study of EEX, PWN, APX, based on multi-factor models
  - Changing sign of the risk premium
- Kolos & Ronn (2008)
  - Market price of risk: expected risk-adjusted return
  - Multi-factor models
  - Negative on the short-term, positive on the long term

- Explore two possible approaches to price electricity futures
  1. The information approach based on market forecasts
  2. An equilibrium approach based on market power of the consumers and producers
- For simplicity we first restrict our attention to $F(t, T)$
  - Electricity forwards deliver over a time period
  - Creates technical difficulties for most spot models
  - Ignore this here
  - In the equilibrium approach we consider delivery periods

The information approach

The information approach: idea

- Idea is the following:
  - Electricity is non-storable
  - Future predicitions about market will not affect current spot
  - However, it will affect forward prices
- Stylized example:
  - Planned outage of a power plant in one month
  - Will affect forwards delivering in one month
  - But *not* spot today
- Market example
  - In 2007 market knew that in 2008 CO2 emission costs will be introduced
  - No effect on spot prices in the EEX market in 2007
  - However, clear effect on the forward prices around New Year


EEX monthly base load contracts
Price (Euro/MWh)
65
60
55
50
45
Nov 07
Dec 07
Jan 08
Feb 08
Mar 08
Apr 08

The information approach: definition

- Define the forward price as

$$F_{\mathcal{G}}(t,T) = \mathbb{E}\left[S(T)\,|\,\mathcal{G}_t\right]$$

- $\mathcal{G}_t$ includes spot information up to current time ($\mathcal{F}_t$) *and* forward-looking information
- The information premium

$$I_{\mathcal{G}}(t,T) = F_{\mathcal{G}}(t,T) - \mathbb{E}\left[S(T)\,|\,\mathcal{F}_t\right]$$

- Rewrite the information premium using double conditioning and $\mathcal{F}_t \subset \mathcal{G}_t$

$$I_{\mathcal{G}}(t,T) = \mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right] - \mathbb{E}\left[\mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right] \,|\, \mathcal{F}_t\right]$$

- The information premium is the residual random variable after projecting $F_{\mathcal{G}}(t,T)$ onto $L^2(\mathcal{F}_t, P)$
  - $I_{\mathcal{G}}$ measures how much more information is contained in $\mathcal{G}_t$ compared to $\mathcal{F}_t$

- Note that

$$\mathbb{E}\left[I_{\mathcal{G}}(t,T)\,|\,\mathcal{F}_t\right]=0$$

- $I_{\mathcal{G}}(t,T)$ is orthogonal to $R(t,T)$
  - The risk premium $R(t,T)$ is $\mathcal{F}_t$-adapted
- Thus, impossible to obtain a given $I_{\mathcal{G}}(t,T)$ from an appropriate choice of Q in $R(t,T)$
  - Including future information creates new ways of explaining risk premia

Example: temperature predictions

- Temperature dynamics

$$dY(t) = \gamma(\mu(t) - Y(t))\,dt + \eta\,dB(t)$$

- Spot price dynamics

$$dS(t) = \alpha(\lambda(t) - S(t))\,dt + \sigma\rho\,dB(t) + \sigma\sqrt{1-\rho^2}\,dW(t)$$

- ρ is the correlation between temperature and spot price
  - NordPool: $\rho < 0$, since high temperature implies low prices, and vice versa

- Suppose we have some temperature forecast at time T_1
  - Full, or at least some, knowledge of $Y(T_1)$

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t \triangleq \mathcal{F}_t \vee \sigma(Y(T_1))$$

- We want to compute (for $T \leq T_1$)

$$F_{\mathcal{G}}(t, T) = \mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right]$$

- Program:
  1. Find a Brownian motion wrt $\mathcal{G}_t$
  2. Compute the conditional expectation

- From the theory of "enlargement of filtrations":
  - There exists a $\mathcal{G}_t$-adapted drift θ_1 such that $\widetilde{B}$ is a $\mathcal{G}_t$-Brownian motion,

$$d\widetilde{B}(t) = dB(t) - \theta_1(t)\,dt$$

- The drift is expressed as

$$\theta_1(t) = a_1(t)\left(e^{\gamma T_1}\mathbb{E}[Y(T_1)\,|\,\mathcal{G}_t] - e^{\gamma t}Y(t) - \gamma\int_t^{T_1}\mu(u)e^{\gamma u}\,du\right)$$

$$a_1(t) = \frac{2\gamma e^{\gamma t}}{\eta(e^{2\gamma T_1} - e^{2\gamma t})}$$

- Dynamics of S in terms of $\widetilde{B}$:

$$dS(t) = \alpha\left(\rho\frac{\sigma}{\alpha}\theta_1(t) + \lambda(t) - S(t)\right)\,dt + \sigma\rho\,d\widetilde{B}(t) + \sigma\sqrt{1-\rho^2}\,dW(t)$$

- Note that we have a mean-reversion level being *stochastic*
  - Explicitly dependent on the temperature prediction and todays temperature
- $\theta_1(t)$ is the market price of information, or information yield

- Calculate the forward price

$$
\begin{aligned}
F_{\mathcal{G}}(t,u) &= \mathbb{E}\left[S(u)\,|\,\mathcal{F}_t\right] + I_{\mathcal{G}}(t,T) \\
&= S(t)^{\exp(-\alpha(T-t))} + \alpha\int_t^T \lambda(s)\mathrm{e}^{-\alpha(T-s)}\,ds + I_{\mathcal{G}}(t,T)
\end{aligned}
$$

- The information premium is, by applying the definition

$$
I_{\mathcal{G}}(t,T) = \rho\sigma\mathbb{E}\left[\int_t^T \mathrm{e}^{-\alpha(T-s)}\,dB(s)\,|\,\mathcal{G}_t\right]
$$

- Use that $\widetilde{B}$ is a $\mathcal{G}_t$-Brownian motion

- Expression for the information premium

$$I_{\mathcal{G}}(t,T) = \rho A(t,T)\left(e^{\gamma T_1}\mathbb{E}\left[Y(T_1)\,|\,\mathcal{G}_t\right] - e^{\gamma t}Y(t) - \gamma\int_t^{T_1}\mu(s)e^{\gamma s}\,ds\right)$$

where

$$A(t,T) = \frac{2\gamma\sigma e^{\gamma T}(1-e^{-(\alpha+\gamma)(T-t)})}{\eta(\alpha+\gamma)(e^{2\gamma T_1}-e^{2\gamma t})}$$

- Observe that $A(t,T)$ is positive
- The sign of the information premium is determined by
  - The correlation ρ
  - The temperature prediction

Example with complete information

- Suppose we know the temperature at T_1
  - The information set is $\mathcal{H}_t$
  - Unlikely situation of perfect future knowledge....
- Assume we we expect a temperature drop

$$Y(T_1) < e^{-\gamma(T_1-t)} Y(t) + \gamma \int_t^{T_1} \mu(s) e^{-\gamma(T_1-s)} \, ds$$

- At NordPool, where $\rho < 0$:
  - The information premium is positive
- Drop in temperature will lead to increasing demand, and thus higher prices

The equilibrium approach

The equilibrium approach: idea

- Producers and consumers can trade in both spot and forward markets
  - No speculators in our set-up
- We suppose that the forwards deliver electricity over an agreed period
  - No fixed delivery time as in other commodity markets
  - Natural for electricity due to its nature
- Choice of an electricity producer
  - Sell production on spot market, *or* on the forward market

- Producer is indifferent when (U_{pr} is the utility function)

$$\mathbb{E}\left[U_{pr}\left(\int_{\tau_1}^{\tau_2} S(u)\,du\right)\right] = \mathbb{E}\left[U_{pr}\left((\tau_2-\tau_1)F_{pr}(t,\tau_1,\tau_2)\right)\right]$$

- The certainty equivalence principle
- F_{pr} is the lowest acceptable price for the producer can accept to be interested in entering a forward
    - Similarly, F_c is the highest acceptable price for the consumer, for a given utility function U_c
- We assume exponential utility $U(x) = 1 - \exp(-\gamma x)$, with respective risk aversion for producer and consumer γ_{pr} and γ_c

- By Jensen's inequality, the predicted average spot price is within the price bounds

$$F_{\mathrm{pr}}(t,\tau_1,\tau_2) \leq \mathbb{E}\left[\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} S(u)\,du \,|\, \mathcal{F}_t\right] \leq F_{\mathrm{c}}(t,\tau_1,\tau_2)$$

- Hypothesis: The settlement price of the forward will depend on the market power $p \in [0,1]$ of the producer

$$F^p(t,\tau_1,\tau_2) = pF_{\mathrm{c}}(t,\tau_1,\tau_2) + (1-p)F_{\mathrm{pr}}(t,\tau_1,\tau_2)$$

- Assume a simple two-factor spot model with jump component

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

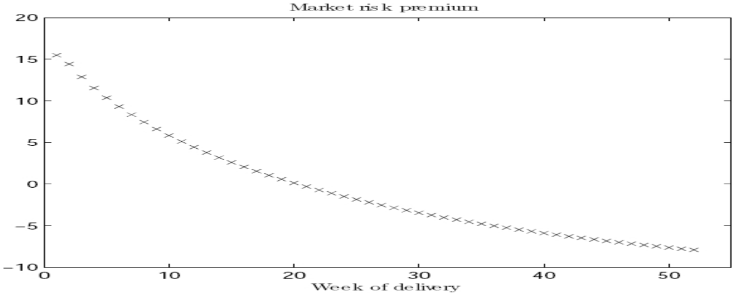
- $\Lambda(t)$ seasonal function

$$dY(t) = -\lambda Y(t)\,dt + Z\,dN(t)$$

- Jumps (accounting for spikes)
  - Z jump size
  - N Poisson process
- Slowly varying base component

$$dX(t) = -\alpha X(t)\,dt + \sigma\,dB(t)$$

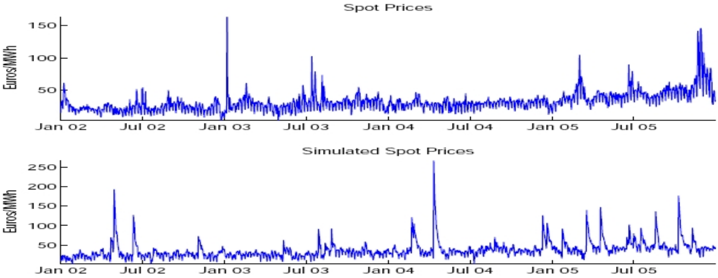
- Calculate prices for weekly contracts and compute the risk premium
  - The market power set to $p = 0.25$
  - Constant positive jumps at rate 2/year



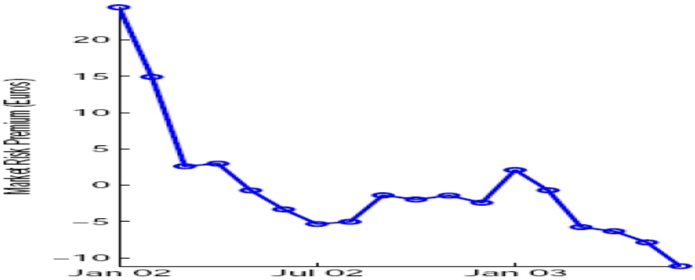
- Note the positive risk premium in the short end
  - Caused by the jump risk

Empirical example: EEX (Metka, Ulm)

- Fit two-factor model to daily EEX spot prices (Jan 02 – Dec 05)



- Using observed prices for 18 monthly forward contracts and fitted spot model
  - Calculate the risk premium,
  - Difference between forward price and predicted spot
  - Observe a positive premium in the short end, and negative in the long end



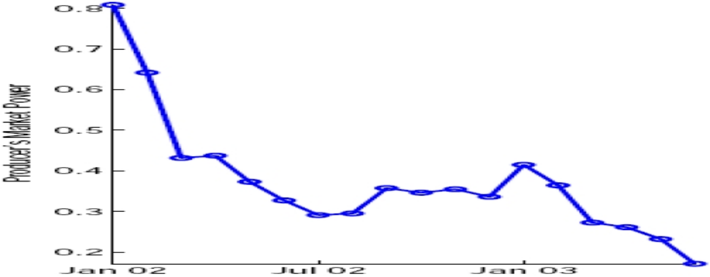
- Based on all available forward prices in the study, risk aversion parameters were determined
  - $\gamma_{\mathrm{pr}} \geq 0.421$ and $\gamma_{\mathrm{c}} \geq 0.701$ are such that

$$F_{\mathrm{pr}}(t,\tau_1,\tau_2) \leq F(t,\tau_1,\tau_2) \leq F_{\mathrm{c}}(t,\tau_1,\tau)$$

- Calculate the empirical market power

$$p(t,\tau_1,\tau_2) = \frac{F(t,\tau_1,\tau_2) - F_{\mathrm{pr}}(t,\tau_1,\tau_2)}{F_{\mathrm{c}}(t,\tau_1,\tau_2) - F_{\mathrm{pr}}(t,\tau_1,\tau_2)}$$

- Observe that producer's power is strong in the short end, while decreasing to be rather weak in the long end



Conclusions

- Discussed two potential ways to understand the link between spot and forward prices in electricity markets
- Information approach:
  - Include future information in pricing
- Equilbrium approach:
  - Certainty equivalence principle for upper and lower bounds of prices
  - Use market power as an explanantory variable for price formation

Coordinates

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