Pricing of electricity futures
– The risk premium –

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Introduction

- Problem: what is the connection between spot and forward prices in electricity?
- Electricity is a non-storable commodity
- How to explain the risk premium?
  - Empirical and economical evidence: Sign varies with time to delivery
- Propose two approaches:
  1. Information approach
  2. Equilibrium approach
- Purpose: try to explain the risk premium for electricity
Outline of talk

1. Example of an electricity market: NordPool
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Example of an electricity market: NordPool
• The NordPool market organizes trade in
  • Hourly spot electricity, next-day delivery
  • Financial forward contracts
    • In reality mostly futures, but we make no distinction here
  • European options on forwards

• Difference from “classical” forwards:
  • Delivery over a period rather than at a fixed point in time
Elspot: the spot market

- A (non-mandatory) hourly market with physical delivery of electricity
- Participants hand in bids before noon *the day ahead*
  - Volume and price for each of the 24 hours next day
  - Maximum of 64 bids within technical volume and price limits
- NordPool creates demand and production curves for the next day before 1.30 pm
• The *system price* is the equilibrium
  • Reference price for the forward market
• Historical system price from the beginning in 1992
  • note the spikes....
The forward market

- Forward with delivery over a period
- Financial market
- Settlement with respect to system price in the delivery period
- Delivery periods
  - Next day, week or month
  - Quarterly (earlier seasons)
  - Yearly
- Overlapping settlement periods (!)
- Contracts also called *swaps*: Fixed for floating price
The option market

- European call and put options on electricity forwards
  - Quarterly and yearly electricity forwards
- Low activity on the exchange
- OTC market for electricity derivatives huge
  - Average-type (Asian) options, swing options ....
The spot-forward relation
The spot-forward relation: some “classical” theory

• The no-arbitrage forward price (based on the buy-and-hold strategy)

\[ F(t, T) = S(t)e^{r(T-t)} \]

• A risk-neutral expression of the price as

\[ F(t, T) = \mathbb{E}_Q [S(T) | \mathcal{F}_t] \]

• The risk premium is defined as

\[ R(t, T) = F(t, T) - \mathbb{E} [S(T) | \mathcal{F}_t] \]
• In the case of electricity:
  • Storage of spot is *not* possible (only indirectly in water reservoirs)
  • Buy-and-hold strategy fails
  • No foundation for the “classical” spot-forward relation
  • …and hence no rule for what \( Q \) should be!

• Thus: What is the link between \( F(t, T) \) and \( S(t) \)?
Economical “intuition” for electricity

- **Short-term positive risk premium**
  - Retailers (consumers) hedge “spike risk”
  - Spikes lead to expensive electricity
  - Accept to pay a premium for locking in prices in the short-term

- **Long-term negative risk premium**
  - Producers hedge their future production
  - Long-term contracts (quarters/years)

- The market may have a change in the sign of the risk premium
Empirical evidence for electricity

• Longstaff & Wang (2004), Geman & Vasicek: PJM market
  • Positive premium in the short-term market

• Diko, Lawford & Limpens (2006)
  • Study of EEX, PWN, APX, based on multi-factor models
  • Changing sign of the risk premium

• Kolos & Ronn (2008)
  • Market price of risk: expected risk-adjusted return
  • Multi-factor models
  • Negative on the short-term, positive on the long term
• Explore two possible approaches to price electricity futures
  1. The information approach based on market forecasts
  2. An equilibrium approach based on market power of the consumers and producers

• For simplicity we first restrict our attention to $F(t, T)$
  • Electricity forwards deliver over a time period
  • Creates technical difficulties for most spot models
  • Ignore this here
  • In the equilibrium approach we consider delivery periods
The information approach
The information approach: idea

• Idea is the following:
  • Electricity is non-storable
  • Future predictions about market will not affect current spot
  • However, it will affect forward prices

• Stylized example:
  • Planned outage of a power plant in one month
  • Will affect forwards delivering in one month
  • But not spot today

• Market example
  • In 2007 market knew that in 2008 CO2 emission costs will be introduced
  • No effect on spot prices in the EEX market in 2007
  • However, clear effect on the forward prices around New Year
Introduction  NordPool  The spot-forward relation  The information approach  The equilibrium approach  Conclusions

![EEX monthly base load contracts](image-url)
The information approach: definition

- Define the forward price as

\[ F_G(t, T) = \mathbb{E} [S(T) | G_t] \]

- \( G_t \) includes spot information up to current time (\( F_t \)) and forward-looking information.

- The information premium

\[ l_G(t, T) = F_G(t, T) - \mathbb{E} [S(T) | F_t] \]
• Rewrite the information premium using double conditioning and $\mathcal{F}_t \subset \mathcal{G}_t$

$$l_G(t, T) = \mathbb{E}[S(T) | \mathcal{G}_t] - \mathbb{E}[\mathbb{E}[S(T) | \mathcal{G}_t] | \mathcal{F}_t]$$

• The information premium is the residual random variable after projecting $F_G(t, T)$ onto $L^2(\mathcal{F}_t, P)$
  - $l_G$ measures how much more information is contained in $\mathcal{G}_t$ compared to $\mathcal{F}_t$
• Note that

\[ \mathbb{E} [ I_G(t, T) | \mathcal{F}_t ] = 0 \]

• \( I_G(t, T) \) is orthogonal to \( R(t, T) \)
  • The risk premium \( R(t, T) \) is \( \mathcal{F}_t \)-adapted

• Thus, impossible to obtain a given \( I_G(t, T) \) from an appropriate choice of \( Q \) in \( R(t, T) \)
  • Including future information creates new ways of explaining risk premia
Example: temperature predictions

- Temperature dynamics

\[ dY(t) = \gamma(\mu(t) - Y(t)) \, dt + \eta \, dB(t) \]

- Spot price dynamics

\[ dS(t) = \alpha(\lambda(t) - S(t)) \, dt + \sigma \rho \, dB(t) + \sigma \sqrt{1 - \rho^2} \, dW(t) \]

- \( \rho \) is the correlation between temperature and spot price
  - NordPool: \( \rho < 0 \), since high temperature implies low prices, and vice versa
• Suppose we have some temperature forecast at time $T_1$
  • Full, or at least some, knowledge of $Y(T_1)$

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t \triangleq \mathcal{F}_t \vee \sigma(Y(T_1))$$

• We want to compute (for $T \leq T_1$)

$$F_{\mathcal{G}}(t, T) = \mathbb{E}[S(T) | \mathcal{G}_t]$$

• Program:
  1. Find a Brownian motion wrt $\mathcal{G}_t$
  2. Compute the conditional expectation
• From the theory of “enlargement of filtrations”:
  • There exists a $G_t$-adapted drift $\theta_1$ such that $\tilde{B}$ is a $G_t$-Brownian motion,

\[
d\tilde{B}(t) = dB(t) - \theta_1(t) dt
\]

• The drift is expressed as

\[
\theta_1(t) = a_1(t) \left( e^{\gamma T_1} \mathbb{E}[Y(T_1)|G_t] - e^\gamma Y(t) - \gamma \int_t^{T_1} \mu(u)e^{\gamma u} du \right)
\]

\[
a_1(t) = \frac{2\gamma e^\gamma t}{\eta (e^{2\gamma T_1} - e^{2\gamma t})}
\]
• Dynamics of $S$ in terms of $\tilde{B}$:

$$dS(t) = \alpha \left( \frac{\sigma}{\alpha} \theta_1(t) + \lambda(t) - S(t) \right) \, dt + \sigma \rho \, d\tilde{B}(t)$$

$$+ \sigma \sqrt{1 - \rho^2} \, dW(t)$$

• Note that we have a mean-reversion level being *stochastic*
  - Explicitly dependent on the temperature prediction and today's temperature
  - $\theta_1(t)$ is the *market price of information, or information yield*
• Calculate the forward price

\[ F_G(t, u) = \mathbb{E} [S(u) | \mathcal{F}_t] + l_G(t, T) \]

\[ = S(t) \exp(-\alpha(T-t)) + \alpha \int_t^T \lambda(s) e^{-\alpha(T-s)} \, ds + l_G(t, T) \]

• The information premium is, by applying the definition

\[ l_G(t, T) = \rho \sigma \mathbb{E} \left[ \int_t^T e^{-\alpha(T-s)} \, dB(s) | \mathcal{G}_t \right] \]

• Use that \( \tilde{B} \) is a \( \mathcal{G}_t \)-Brownian motion
**Expression for the information premium**

\[ I_G(t, T) = \rho A(t, T) \left( e^{\gamma T_1} \mathbb{E}[Y(T_1)|G_t] - e^{\gamma t} Y(t) - \gamma \int_t^{T_1} \mu(s)e^{\gamma s} ds \right) \]

where

\[ A(t, T) = \frac{2\gamma \sigma e^{\gamma T}(1 - e^{-(\alpha+\gamma)(T-t)})}{\eta(\alpha + \gamma)(e^{2\gamma T_1} - e^{2\gamma t})} \]

- Observe that \( A(t, T) \) is positive
- The sign of the information premium is determined by
  - The correlation \( \rho \)
  - The temperature prediction
Example with complete information

- Suppose we know the temperature at $T_1$
  - The information set is $\mathcal{H}_t$
  - Unlikely situation of perfect future knowledge....

- Assume we we expect a temperature drop

$$Y(T_1) < e^{-\gamma(T_1-t)}Y(t) + \gamma \int_t^{T_1} \mu(s)e^{-\gamma(T_1-s)} ds$$

- At NordPool, where $\rho < 0$:
  - The information premium is positive

- Drop in temperature will lead to increasing demand, and thus higher prices
The equilibrium approach
The equilibrium approach: idea

- Producers and consumers can trade in both spot and forward markets
  - No speculators in our set-up
- We suppose that the forwards deliver electricity over an agreed period
  - No fixed delivery time as in other commodity markets
  - Natural for electricity due to its nature
- Choice of an electricity producer
  - Sell production on spot market, or on the forward market
• Producer is indifferent when \((U_{pr} \text{ is the utility function})\)

\[
\mathbb{E} \left[ U_{pr} \left( \int_{\tau_1}^{\tau_2} S(u) \, du \right) \right] = \mathbb{E} \left[ U_{pr} \left( (\tau_2 - \tau_1) F_{pr}(t, \tau_1, \tau_2) \right) \right]
\]

• The certainty equivalence principle

• \(F_{pr}\) is the lowest acceptable price for the producer can accept to be interested in entering a forward
  • Similarly, \(F_c\) is the highest acceptable price for the consumer, for a given utility function \(U_c\)

• We assume exponential utility \(U(x) = 1 - \exp(-\gamma x)\), with respective risk aversion for producer and consumer \(\gamma_{pr}\) and \(\gamma_c\)
• By Jensen’s inequality, the predicted average spot price is within the price bounds

\[
F_{pr}(t, \tau_1, \tau_2) \leq \mathbb{E} \left[ \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) \, du \mid \mathcal{F}_t \right] \leq F_c(t, \tau_1, \tau_2)
\]

• Hypothesis: The settlement price of the forward will depend on the market power \( p \in [0, 1] \) of the producer

\[
F^p(t, \tau_1, \tau_2) = pF_c(t, \tau_1, \tau_2) + (1 - p)F_{pr}(t, \tau_1, \tau_2)
\]
Assume a simple two-factor spot model with jump component

\[ S(t) = \Lambda(t) + X(t) + Y(t) \]

\( \Lambda(t) \) seasonal function

\[ dY(t) = -\lambda Y(t) \, dt + Z \, dN(t) \]

Jumps (accounting for spikes)
- \( Z \) jump size
- \( N \) Poisson process

Slowly varying base component

\[ dX(t) = -\alpha X(t) \, dt + \sigma \, dB(t) \]
• Calculate prices for weekly contracts and compute the risk premium
  • The market power set to \( p = 0.25 \)
  • Constant positive jumps at rate 2/year

![Graph showing market risk premium over weeks of delivery]

• Note the **positive** risk premium in the short end
  • Caused by the jump risk
Empirical example: EEX (Metka, Ulm)

- Fit two-factor model to daily EEX spot prices (Jan 02 – Dec 05)
• Using observed prices for 18 monthly forward contracts and fitted spot model
  • Calculate the risk premium,
  • Difference between forward price and predicted spot
  • Observe a positive premium in the short end, and negative in the long end
• Based on all available forward prices in the study, risk aversion parameters were determined
  • \( \gamma_{pr} \geq 0.421 \) and \( \gamma_{c} \geq 0.701 \) are such that
    \[
    F_{pr}(t, \tau_1, \tau_2) \leq F(t, \tau_1, \tau_2) \leq F_{c}(t, \tau_1, \tau)
    \]

• Calculate the empirical market power
  \[
  p(t, \tau_1, \tau_2) = \frac{F(t, \tau_1, \tau_2) - F_{pr}(t, \tau_1, \tau_2)}{F_{c}(t, \tau_1, \tau_2) - F_{pr}(t, \tau_1, \tau_2)}
  \]
Observe that producer’s power is strong in the short end, while decreasing to be rather weak in the long end.
Conclusions

• Discussed two potential ways to understand the link between spot and forward prices in electricity markets

• Information approach:
  • Include future information in pricing

• Equilibrium approach:
  • Certainty equivalence principle for upper and lower bounds of prices
  • Use market power as an explanatory variable for price formation
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