BSS Processes and Intermittency/Volatility

Turbulence Stochastics

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- BSS models
- Turbulence background
- BSS models (cont.)
- Inference on intermittency/volatility
- Further ongoing work
- Some open questions

Brownian semistationary ($\mathcal{BSS}$) processes:

$$Y_t = \int_{-\infty}^{t} g(t-s)\sigma_s \mathrm{d}B_s + \int_{-\infty}^{t} q(t-s)a_s \mathrm{d}s \qquad (1)$$

where B is Brownian motion, g and q are square integrable functions on $\mathbb{R}$, with $g(t) = q(t) = 0$ for $t < 0$, and σ and a are cadlag processes.

When σ and a are stationary, as will be assumed throughout this talk, then so is Y.

It is sometimes convenient to indicate the formula for Y as

$$Y = g * \sigma \bullet B + q * a \bullet Leb. \qquad (2)$$

We consider the $\mathcal{BSS}$ processes to be the natural analogue, in stationarity related settings, of the class $\mathcal{BSM}$ of Brownian semimartingales.

- The $\mathcal{BSS}$ processes are not in general semimartingales

The key object of interest is the *integrated variance* (IV)

$$\sigma_t^{2+} = \int_0^t \sigma_s^2 \mathrm{d}s$$

We shall discuss to what extent *realised quadratic variation* of Y can be used to estimate σ_t^{2+}.

- Note that the relevant question here is whether a suitably normalised version of the realised quadratic variation, and not necessarily the realised quadratic variation itself, converges in probability/law.

In semimartingale theory the quadratic variation $[Y]$ of Y is defined in terms of the Ito integral $Y \bullet Y$, as $[Y] = Y^2 - 2Y \bullet Y$. In that setting $[Y]$ equals the limit in probability as $\delta \to 0$ of the *realised quadratic variation* $[Y_\delta]$ of Y defined by

$$[Y_\delta]_t = \sum_{j=1}^{\lfloor t/\delta \rfloor} \left(Y_{j\delta} - Y_{(j-1)\delta} \right)^2 \tag{3}$$

where $\lfloor t/\delta \rfloor$ is the largest integer smaller than or equal to t/δ. It is this latter definition of quadratic variation that we will use here.

White noise case:

$$\begin{aligned} Y_t(x) &= \mu + \int_{A_t(x)} g(t-s, |\xi - x|)\, \sigma_s(\xi)\, W(\mathrm{d}\xi, \mathrm{d}s) \\ &\quad + \int_{D_t(x)} q(t-s, |\xi - x|)\, a_s(\xi)\, \mathrm{d}\xi \mathrm{d}s. \end{aligned}$$

Here $A_t(\sigma)$ and $D_t(\sigma)$ are termed *ambit sets* .

Lévy case:

$$\sigma_t^2(x) = \int_{C_t(x)} h(t-s, |\xi - x|)\, L(\mathrm{d}\xi, \mathrm{d}s)$$

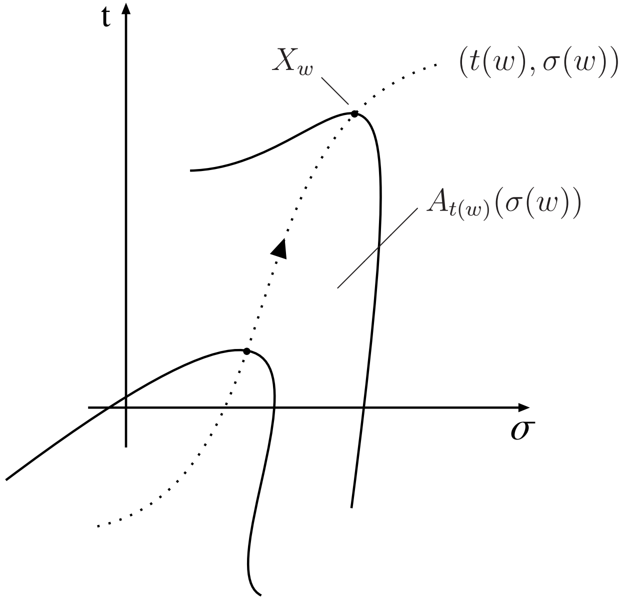

Figure: Ambit processes

In turbulence the basic notion of *intermittency* refers to the fact that the energy in a turbulent field is unevenly distributed in space and time.

The present presentation is part of a project that aims to construct a stochastic process model of the field of velocity vectors representing the fluid motion, conceiving of the intermittency as a positive random field with random values $\sigma_t(x)$ at positions (x, t) in space-time.

However, most extensive data sets on turbulent velocities only provide the time series of the main component (i.e. the component in the main direction of the fluid flow) of the velocity vector at a single location in space.

In the present talk the focus is on this latter case, but in the concluding Section some discussion will be given on the further intriguing issues that arise when addressing tempo-spatial settings.

For simplicity we now assume that $\sigma \perp\!\!\!\perp B$ and that $q = 0$, i.e. there is no drift term in Y and

$$Y_t = \int_0^t g\,(t-s)\,\sigma_s \mathrm{d}B_s.$$

However, at the end of the talk some discussion will be given on more general settings.

Let $Z = \{Z_t\}_{t\in\mathbb{R}}$ denote a second order stationary stochastic process, possibly complex valued, of mean 0 and continuous in quadratic mean. Recall that Z is said to be a *moving average process* if it is of the form

$$Z_t = \int_{-\infty}^{\infty} \phi^* (t-s)\, \mathrm{d}\Xi^*_s \qquad (4)$$

where ϕ^* is an, in general complex, deterministic and square integrable function and where the process Ξ^* has orthogonal increments with $\mathrm{E}\left\{|\mathrm{d}\Xi^*_t|^2\right\} = \varpi^* \mathrm{d}t$ for some constant $\varpi^* > 0$; finally, the integral in (4) is defined in the quadratic mean sense.

A second order stationary process (of mean 0 and continuous in quadratic mean) is called *regular* (or linearly regular) provided its future values cannot be predicted by linear operations on past values without error. Such processes can be written in the continuous time Wold decomposition form

$$Z_t = \int_{-\infty}^{t} \phi\left(t-s\right) \mathrm{d}\Xi_s + V_t \tag{5}$$

where ϕ is an, in general complex, deterministic square integrable function satisfying $\phi\left(s\right) = 0$ for $s < 0$ and where the process Ξ has orthogonal increments with $\mathrm{E}\left\{\left|\mathrm{d}\Xi_t\right|^2\right\} = \varpi \mathrm{d}t$ for some constant $\varpi > 0$. Finally, the process V is *nonregular* (i.e. predictable by linear operations on past values without error).

Given Z and neglecting sets of measure 0, both ϕ and Ξ are uniquely determined, ϕ up to a factor of modulus 1 and the driving process Ξ in the L^2 sense and up to an additive constant.

- A slightly stronger condition than regularity is the requirement that Z be *completely nondeterministic*, meaning that $\cap_{t\in\mathbb{R}}\overline{\mathrm{sp}}\left\{Z_s : s \leq t\right\} = \{0\}$ In this case

$$Z = \phi * \Xi$$

with ϕ real and uniquely determined up to a real constant of proportionality; and the same is therefore true of Ξ.

We begin by recalling a classical necesssary and sufficient condition, due to [Kni92], for the semimartingale property of a process X of the form

$$X_t = \int_{-\infty}^{t} g(t-s)\,\mathrm{d}B_s. \tag{6}$$

Knight's Theorem says that $(X_t)_{t\geq 0}$ is a semimartingale in the $\left(\mathcal{F}_t^{B,\infty}\right)_{t\geq 0}$ filtration if and only if

$$g(t) = c + \int_0^t b(s)\,\mathrm{d}s \tag{7}$$

for some $c \in \mathbb{R}$ and a square integrable function b.

Example An example of some particular interest is where

$$g(t) = t^{\alpha} e^{-\lambda t} \quad \text{for} \quad t \in (0, \infty)$$

and some $\lambda > 0$. In order for the integral $g * \sigma \bullet B$ to exist α is required to be greater than $-\frac{1}{2}$, and for g to be of the form (7) we must have $\alpha > \frac{1}{2}$. In other words, the nonsemimartingale cases are $\alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$. □

Recall that the *integrated variance* (IV)

$$\sigma_t^{2+} = \int_0^t \sigma_s^2 \mathrm{d}s \tag{8}$$

is the main object of interest and that we wish to discuss the extent to which realised quadratic variation of Y can be used to estimate σ_t^{2+}.

Remark It is convenient to define σ_t^{2+} also for $t < 0$ by the same expression.

Note that, as regards inference on σ^{2+}, there may be notable differences between cases where g is positive on all of $(0, \infty)$ and those where $g(t) = 0$ for $t > l$ for some $l \in (0, \infty)$.

- Recall that $\sigma \perp\!\!\!\perp B$.

Suppose $g(t) = 0$ for $t > 1$.
For any $t \in \mathbb{R}$ and $u < t$,

$$Y_t - Y_{t-u} = \int_{t-u}^{t} g(t-s)\sigma_s \mathrm{d}B_s + \int_{-\infty}^{t-u} \{g(t-s) - g(t-s$$

It is illuminating to rewrite this as

$$Y_t - Y_{t-u} = \int_{-\infty}^{0} \phi_u(-v)\sigma_{v+t}\mathrm{d}B_{v+t} \tag{9}$$

defining ϕ_u by

$$\phi_u(v) = \begin{cases} g(v) & \text{for} \quad 0 \le v < u \\ g(v) - g(v-u) & \text{for} \quad u \le v < \infty \end{cases}.$$

This implies in particular that the conditional variance of $Y_t - Y_{t-u}$ given the process σ takes the form

$$\mathrm{E}\left\{\left(Y_t - Y_{t-u}\right)^2 \middle| \sigma\right\} = \int_0^\infty \psi_u(v)\,\sigma_{t-s}^2 \mathrm{d}s \qquad (10)$$

where

$$\psi_u(v) = \begin{cases} g^2(v) & \text{for} \quad 0 \le v < u \\ \{g(v) - g(v-u)\}^2 & \text{for} \quad u \le v < \infty \end{cases}.$$

And unconditionally

$$\mathrm{E}\left\{\left(Y_t - Y_{t-u}\right)^2\right\} = \mathrm{E}\left\{\sigma_0^2\right\} \int_0^\infty \psi_u(v)\,\mathrm{d}v.$$

Hence

$$\frac{\mathrm{E}\left\{ \left(Y_t - Y_{t-u} \right)^2 | \sigma \right\}}{\mathrm{E}\left\{ \left(Y_t - Y_{t-u} \right)^2 \right\}} = \frac{\int_0^\infty \psi_u(v)\, \sigma_{t-v}^2 \mathrm{d}v}{\mathrm{E}\{\sigma_0^2\} \int_0^\infty \psi_u(v)\, \mathrm{d}v} = \int_0^\infty \bar{\sigma}_{t-v}^2 \pi_u (\mathrm{d}$$

where

$$\bar{\sigma}^2 = \sigma^2 / \mathrm{E}\left\{\sigma_0^2\right\}.$$

and π_u is the probability measure on $[0,1]$ having pdf $\psi_u(v)/c(\delta)$ with $c(\delta) = 2\|g\|^2 (1 - r(u))$ with r being the autocorrelation function of Y.

Example $g(t) = e^{-\lambda t} 1_{(0,1)}$ (non-semimartingale case). Then

$$\psi_\delta(v) = e^{-2\lambda v} \begin{cases} 1 & \text{for} \quad 0 \le v < \delta \\ \left(e^{\lambda\delta} - 1\right)^2 & \text{for} \quad \delta \le v < 1 \\ e^{2\lambda\delta} & \text{for} \quad 1 \le v < 1+\delta \\ 0 & \text{for} \quad 1+\delta \le v \end{cases}.$$

It follows that $\pi_\delta \to \pi$ where

$$\pi = \frac{1}{1+e^{-2\lambda}}\delta_0 + \frac{e^{-2\lambda}}{1+e^{-2\lambda}}\delta_1$$

where δ_x denotes the delta measure at x. □

Example $g(t) = t^{\alpha}(1-t)^{\beta}$ with $-\frac{1}{2} < \alpha$ and $\beta \geq 1$. The first inequality ensures existence of the stochastic integral $g * \sigma \bullet B$, and if α is less than $\frac{1}{2}$ then we are in the nonsemimartingale situation In this case $\pi = \delta_0$. □

Now, define the normed realised quadratic variation $\overline{[Y_\delta]}$ of Y as

$$\overline{[Y_\delta]} = \frac{\delta}{c(\delta)} [Y_\delta] .$$

Then

$$\mathrm{E}\left\{\overline{[Y_\delta]}_t | \sigma\right\} = \int_0^\infty \left\{\delta \sum_{j=1}^{\lfloor t/\delta \rfloor} \sigma^2_{j\delta - v}\right\} \pi_\delta(\mathrm{d}v) .$$

Suppose that π_δ converges weakly, as $\delta \to 0$, to a probability measure π on $[0, l]$, i.e.

$$\pi_\delta \xrightarrow{w} \pi. \tag{11}$$

Then we have

$$\mathrm{E}\left\{\overline{[Y_\delta]}_t|\sigma\right\} \to \int_0^1 \left(\sigma^{2+}_{t-v} - \sigma^{2+}_{-v}\right) \pi\left(\mathrm{d}v\right). \qquad (12)$$

In particular, if $\pi = \delta_0$, the delta measure at 0, then

$$\mathrm{E}\left\{\overline{[Y_\delta]}_t\right\} \to \sigma^{2+}_t.$$

We will refer to this case by saying that the model for Y is *volatility memoryless*.

(Under a mild restriction the same holds for general g and σ provided the upper limit of the integral is taken as ∞.)

On the other hand, if π is a convex combination of the delta measure at 0 and l, i.e. $\pi = \theta\delta_0 + (1-\theta)\,\delta_l$ for some $\theta \in (0,1)$, then

$$\mathrm{E}\left\{\overline{[Y_\delta]}\right\} \to \theta\sigma_t^{2+} + (1-\theta)\left(\sigma_{t-l}^{2+} - \sigma_{-l}^{2+}\right).$$

Suppose that $l = 1$ and that $g > 0$ is a square integrable continuously differentiable function on $(0, 1)$. If, as $\delta \to 0$, we have $c(\delta)^{-1} \delta^2 = o(1)$ and if the probability measure π_δ converges weakly to a probability measure π on $[0, 1]$ then π is concentrated on the endpoints 0 and 1.

To formulate conditions ensuring that π exists and equals δ_0, let

$$\Psi_\delta(u) = \int_0^u \psi_\delta(v)\,\mathrm{d}v \quad \text{and} \quad \bar{\Psi}_\delta(u) = \int_{1+\delta-u}^{1+\delta} \psi_\delta(v)\,\mathrm{d}v,$$

so that $c(\delta)^{-1}\Psi_\delta$ is the distribution function of π_δ.

Proposition If $(\delta)^{-1}\delta^2 = o(1)$ and if for some $\varepsilon_0 \in (0,1)$

$$\frac{\bar{\Psi}_\delta(\varepsilon)}{\Psi_\delta(\varepsilon)} \to 0 \quad \text{for all } \varepsilon \in (0,\varepsilon_0)$$

then $\pi_\delta \xrightarrow{w} \delta_0$. □

Remark In case $c(\delta) \sim \delta^2$ it may happen that π is absolutely continuous on $[0, 1]$.

Above it was assumed that $l < \infty$ (in fact, we took $l = 1$). The following Example has $l = \infty$.

Example For $g(t) = t^\alpha e^{-\lambda t}$ ($\alpha > -\frac{1}{2}$) it can be shown, using detailed calculations for this special case given in [BNCP08], that $\pi = \delta_0$. □

We now seek conditions under which the conditional variance of the normalised realised quadratic variation tends to 0 as $\delta \to 0$, i.e.

$$\mathrm{Var}\{\overline{[Y_\delta]}|\sigma\} \to 0. \tag{13}$$

In that case and provided

$$\mathrm{E}\left\{\overline{[Y_\delta]}|\sigma\right\} \to \int_0^\infty \left(\sigma^{2+}_{t-v} - \sigma^{2+}_{-v}\right)\pi\left(\mathrm{d}v\right) \tag{14}$$

holds we have that, conditionally on σ,

$$\overline{[Y_\delta]}_t \xrightarrow{p} \int_0^\infty \left(\sigma^{2+}_{t-v} - \sigma^{2+}_{-v}\right)\pi\left(\mathrm{d}v\right). \tag{15}$$

Let $n = \lfloor t/\delta \rfloor$ and $\Delta_j^n Y = Y_{j\delta} - Y_{(j-1)\delta}$. Via the Cauchy-Schwarz inequality and using that for any pair X and Y of mean zero normal random variables we have

$$\mathrm{Cov}\{X^2, Y^2\} = 2\mathrm{Cov}\{X, Y\}^2. \tag{16}$$

it can be shown that

$$\mathrm{Var}\{\overline{[Y_\delta]_t}|\sigma\} \leq 2K(\sigma)^2 \left(\delta + 2\left(\delta \sum_{k=1}^{n-1} k\bar{c}_k(\delta) + \sum_{k=n}^{\infty} \bar{c}_k(\delta)\right)\right)$$

where $K(\sigma) = \sup_{0\leq s\leq t} \sigma_s^2$ (as σ is assumed càdlàg, $K(\sigma) < \infty$ a.s.) and

$$\bar{c}_k(\delta) = \frac{c_k(\delta)}{c(\delta)} \quad \text{with} \quad c_k(\delta) = \delta \int_0^1 \psi_\delta((k+u)\delta)\,\mathrm{d}u.$$

Thus, for $\mathrm{Var}\{\overline{[Y_\delta]}|\sigma\} \to 0$ to be valid it suffices to have

$$\delta \sum_{k=1}^{n-1} k\bar{c}_k(\delta) \to 0 \quad \text{and} \quad \sum_{k=n}^{\infty} \bar{c}_k(\delta) \to 0.$$

Example If $g(t) = t^\alpha (1-t)^\beta$ with $-\frac{1}{2} < \alpha$ and $\beta \geq 1$ then the above two conditions hold and $\overline{[Y_\delta]} \xrightarrow{p} \sigma^{2+}$. □

$$\frac{[Y_\delta]_t}{\lfloor t/\delta \rfloor \{\widehat{\mathrm{Var}\{Y_0\}}\}(1-\hat{r}(\delta))} \xrightarrow{p} \int_0^\infty \left(\sigma_{t-v}^{2+} - \sigma_{-v}^{2+}\right) \pi(\mathrm{d}v).$$

So far we have assumed that $q = 0$ and $\sigma \perp\!\!\!\perp B$. In joint work (near completion) with José Manuel Corcuera and Mark Podolski these conditions have been substantially weakened. This more refined analysis has shown that $\overline{[Y_\delta]} \xrightarrow{p} \sigma^{2+}$ in wider generality and using the theory of multipower variation and recent powerful results of Malliavin calculus, due to Nualart, Peccati et al, a feasible CLT for $\overline{[Y_\delta]}$ has been established.

The results are further extended to consistency and feasible CLTs for multipower variations, in particular for bipower variation (as is essential for inference on σ^{2+}).

Above only the case of time-wise behaviour at a single point in space was considered. In the general turbulence setting, space and the velocity vector are three dimensional. There the questions, analogous to those disussed above, are substantially more intricate, major differences occurring already for the case of a one-dimensional space component.

The *realised variation ratio* (RVR) is defined by

$$RVR_t = \frac{\pi}{2} \frac{[Y_\delta]_t^{[1,1]}}{[Y_\delta]_t}$$

where $[Y_\delta]^{[1,1]}$ is the bipower variation, i.e.

$$[Y_\delta]_t^{[1,1]} = \sum_{j=1}^{\lfloor t/\delta \rfloor - 1} \left| Y_{j\delta} - Y_{(j-1)\delta} \right| \left| Y_{(j+1)\delta} - Y_{j\delta} \right| .$$

The properties of RVR are presently under study in joint work with Neil Shephard.

- $r = g * g$?
- In non-semimartingale cases, can one give a natural meaning to $\mathrm{d}X_t$ such that $\mathrm{d}[X]_t = (\mathrm{d}X_t)^2$?
- Volatility modulated Volterra Processes ($VMVP$):

$$Y_t = \int_{-\infty}^{\infty} K_t(s)\,\sigma_s B(\mathrm{d}s)$$

$$Y_t(x) = \int_{-\infty}^{\infty} K_t(\xi, s; x)\,\sigma_s(\xi)\,W(\mathrm{d}\xi\mathrm{d}s)$$

- Relevance for Finance? Arbitrage?

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