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A linear frequency domain solver workflow for fast simulation of transmission systems

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Abstract The contribution presents a complete frequency solver workflow applied to automotive transmissions modeled as multi-body systems consisting of mechanical components like rotating rigid and elastic shafts interconnected by gear contacts and supported by bearing joints. Solving the equations of motion in frequency domain based on a linearized model yields the periodic steady-state results very fast compared to transient time integration methods, where fast oscillating components may decay slowly. The frequency domain solver workflow is described in detail from getting a loaded state of the model, which is used for linearization, up to solving linear equation systems for each non-negligible frequency load component. The presented solver workflow is applied to a simple gearbox model. Resulting vibrations from the linear frequency domain solution are compared against results of a transient time domain solution, where the frequency domain solution matches well the time domain results, but is obtained within a fraction of CPU time.

Keywords frequency domain · transmission systems · linearized equation of motion

1 Introduction

When key design parameters are varied in the early design phase of transmission systems, a fast computation of the dynamic properties is essential,

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e.g. [15], [14]. This is especially true for automotive transmissions modeled as multi-body systems consisting of mechanical components like rotating rigid and elastic shafts interconnected by joints like gear contacts and supported by bearing joints.

A standard transient time integration for such systems based on the BDF integration method for a floating frame of reference approach is described in [3]. In case of simulating typical application cases like periodic steady-state motion, the computation may take minutes up to several hours depending on the model complexity, as transient oscillations from fast moving components may decay slowly. Solving the equations of motion in frequency domain based on a linearized model, yields the steady-state results within seconds.

The main motivation for a frequency domain solver (FDS) is to get a fast solution compared to the standard time domain solver (TDS). An FDS operates on the same model depth as is used for the regular transient calculation in time domain. As the FDS is operating on a linearized version of the model, this method is best suited for models, where nonlinear effects do not play a dominant role. Otherwise, the FDS may not yield meaningful results.

In general, the linearization of the nonlinear equations of motion of a multibody model leads to a second order ODE with time-varying coefficient matrices [12], [6]. Here, the focus is on the description of the workflow how to obtain a fast solution: it is assumed that the first order Taylor approximation leads to a second order ODE with constant, time-independent coefficient matrices [4], [5]. For instance in transmission applications, time-dependent variable meshing effects in gear meshes are not represented by the linear FDS as presented here. They have to be taken into account as external loads by, e.g., an additional tooth contact analysis (TCA).

This paper is organized as follows: in Sect. 2 the basic equations of motion of multi-body systems, their linearization, and their solution is summarized from the theoretical point of view. Sect. 3 presents the complete frequency solver workflow: starting from the setup of the multi-body model, which is analogous to the one in a TDS, calculation of initial velocities, kinetostatics, linearization of the model around the kinetostatics configuration to the solution of the linear FDS equations for determining frequency result components. The presented solver workflow is applied to a simple gearbox model in Sect. 4. As the assumption on the coefficient matrices are rather strictly ones, an outlook how to extend the FDS workflow to a more general class of models is given in Sect. 5.

2 Basic equations of motion, their linearization, and their solution

2.1 Nonlinear equations of motion in floating frame of reference

The time-dependent description of the motion of the multi-body system is based on the floating frame of reference approach. This approach leads to the standard system of nonlinear second order differential-algebraic equations as used in the multi-body simulation software AVL EXCITE^{TM} [3]. The differential equation of this system for body i is given as:

$$\begin{aligned} &\boldsymbol{M}_i^{FEM}\boldsymbol{q}_i'' + \boldsymbol{D}_i^{FEM}\boldsymbol{q}_i' + \boldsymbol{K}_i^{FEM}\boldsymbol{q}_i + \\ &\quad + \; \boldsymbol{f}_i^{inertia}(\boldsymbol{y}_i, \boldsymbol{y}_i', \boldsymbol{y}_i'') + \sum_m \boldsymbol{f}_i^{joint_m}(\tilde{\boldsymbol{y}}, \tilde{\boldsymbol{y}}') = \boldsymbol{f}_i^{load}(\boldsymbol{y_i}, \boldsymbol{y}_i') \end{aligned} \tag{1}$$

with state variables $\boldsymbol{y}_i$, including global variables and the local, elastic variables $\boldsymbol{q}_i$. $\boldsymbol{M}_i^{FEM}$, $\boldsymbol{D}_i^{FEM}$ and $\boldsymbol{K}_i^{FEM}$ denote the mass, damping and stiffness matrix, resulting from spatial discretization. The spatial finite element discretization can be obtained by beam-mass-models or by optionally condensed 3D-volume models, see [13]. Nonlinear inertia forces - like Coriolis and gyroscopic forces and torques - are collected in $\boldsymbol{f}_i^{inertia}$. External forces and moments applied at a single body are covered by $\boldsymbol{f}_i^{load}$. Forces and moments resulting from coupling between the bodies are contained in $\boldsymbol{f}_i^{joint}$. The interaction between different bodies - including gear contact - is modeled by force-elements (joints) and not by kinematic constraints. The joint forces/moments do not only depend on coordinates of the body itself, but also on possibly all states of other bodies, which are summarized in the vector $\tilde{\boldsymbol{y}}$. The equations of motion are completed by algebraic constraints such as reference conditions for uniquely splitting global and local motion parts for an elastic body.

The equations of motion form an implicit second order differential-algebraic equations system (DAE) with differentiation index two [2]. For the computation of the dynamic behavior of the multi-body system the DAE system (1) has to be solved for each body for both global and local motion quantities. For the time integration of the DAE system a Backward Differentiation Formula (BDF) scheme is used. As the bodies are connected by joint force laws, iterative decoupling of the full multi-body system into a sequence of smaller units on body level is applied for efficiency reasons. In each iteration step, the joint forces are kept fixed. After the body iteration step is finished, the joint forces are updated and predicted for the next iteration step of the bodies. For further details see [3].

2.2 Linearization of equations of motion

In their most general form, the equations of motion of the overall multi-body system with n_b bodies can be summarized as:

$$\boldsymbol{f} = \left(\boldsymbol{f}_1^T, \boldsymbol{f}_2^T, \cdots, \boldsymbol{f}_{n_b}^T\right)^T = \boldsymbol{f}^{load} \tag{2}$$

where $\boldsymbol{f}$ contains all single body terms $\boldsymbol{f}_i$ on the left hand side of (1). Similarly $\boldsymbol{f}^{load}$ contains all the load components of all single bodies.

A standard linearization procedure (first order Taylor approximation) is used to linearize the equations of motion about a reference trajectory $\boldsymbol{y}_0(t)$ of the multi-body system such that the solution $\boldsymbol{y}(t) = \boldsymbol{y}_0(t) + \boldsymbol{\Delta y}$ is represented as small perturbations about the reference solution. The following equations are obtained:

$$\begin{aligned}\boldsymbol{f}(\boldsymbol{y}_0'' + \boldsymbol{\Delta y}'', \boldsymbol{y}_0' + \boldsymbol{\Delta y}', \boldsymbol{y}_0 + \boldsymbol{\Delta y}) \approx \boldsymbol{f}(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0) + \\ + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}''} \left.\right|_{(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0)} \boldsymbol{\Delta y}'' + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}'} \left.\right|_{(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0)} \boldsymbol{\Delta y}' + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}} \left.\right|_{(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0)} \boldsymbol{\Delta y}\end{aligned} \tag{3}$$

These equations are rewritten in the usual matrix form of the linearized equations of motion:

$$\boldsymbol{M}(t)\boldsymbol{\Delta y}'' + \boldsymbol{D}(t)\boldsymbol{\Delta y}' + \boldsymbol{K}(t)\boldsymbol{\Delta y} = \boldsymbol{f}^{load}(\boldsymbol{y}_0) - \boldsymbol{f}(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0) \tag{4}$$

where $\boldsymbol{M}, \boldsymbol{D}$ and $\boldsymbol{K}$ denote the - in general time-dependent - mass, damping and stiffness matrix of the linearized multi-body system with:

$$\boldsymbol{M} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}''}\Big|_{(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0)} \qquad \boldsymbol{D} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}'}\Big|_{(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0)} \qquad \boldsymbol{K} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}}\Big|_{(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0)} \tag{5}$$

Most parts of the linearization of the equations of motion (1) are computable analytically. While $\mathbf{M}$ describes bodies, $\mathbf{D}$ and $\mathbf{K}$ describe both bodies and joints. Linearizations include the FEM based body mass, damping and stiffness matrices in case of an elastic body. Additional gyroscopic terms may result from linearization of the inertia forces $\boldsymbol{f}^{inertia}$ such that

$$\boldsymbol{D} = \boldsymbol{D}_i^{FEM} + \boldsymbol{G} \tag{6}$$

with $\boldsymbol{G}$ representing the gyroscopic matrix depending on the angular velocities of the bodies. Analogously the linearized stiffness matrix $\boldsymbol{K}$ may contain beneath the elastic stiffness matrix $\boldsymbol{K}_i^{FEM}$ components that depend on angular velocities of bodies.

The linearization of the joint forces/moments $\boldsymbol{f}^{joint}$ yields contributions to the linearized stiffness and damping matrices like in the force equilibrium approach in [8]. For a joint connecting one node on body i and one node on body j, the 12 x 12 joint stiffness matrix $\boldsymbol{K}^{joint}$ is assembled from four 6 x 6 blocks as [7]:

$$\boldsymbol{K}^{joint} = \begin{pmatrix} \boldsymbol{K}_{ii}^{joint} & \boldsymbol{K}_{ij}^{joint} \\ \boldsymbol{K}_{ji}^{joint} & \boldsymbol{K}_{jj}^{joint} \end{pmatrix}. \tag{7}$$

If the joint connects a flexible body with another flexible body, the partial derivatives are directly added to the body stiffness matrix at the positions of the connected nodes. The 6 x 6 blocks on the diagonal are added to the overall stiffness matrix at the diagonal positions corresponding to body i, node k and body j, node l. The off-diagonal 6 x 6 blocks are added at block positions (body i, node k / body j, node l) and vice versa to the overall stiffness matrix.

2.3 Solution of linearized equations of motion in frequency domain

The linearized equations (4) may be solved efficiently. In general, the matrices in (4) do depend on time; they form a linear time variant ordinary differential equation system. Most often the matrices are time periodic. If additionally the external loads are periodic, solution methods are proposed in e.g. [12], [6], [16] for modal analysis. An iterative algorithm, especially for powertrain simulation is proposed in [1].

In the following, we restrict ourselves to models where the linearization of the equations of motion do lead to a time invariant differential equation, i.e. the matrices $\boldsymbol{M}, \boldsymbol{D}$ and $\boldsymbol{K}$ do not depend on time:

$$\boldsymbol{M}\boldsymbol{\Delta y}'' + \boldsymbol{D}\boldsymbol{\Delta y}' + \boldsymbol{K}\boldsymbol{\Delta y} = \boldsymbol{f}^{load}(\boldsymbol{y}_0) - \boldsymbol{f}(\boldsymbol{y}_0'', \boldsymbol{y}_0', \boldsymbol{y}_0) \tag{8}$$

Furthermore, the external load along the reference trajectory $\boldsymbol{f}^{load}(\boldsymbol{y}_0(t))$ shall be periodic with primitive period T.

Any periodic function $f(t)$ can be represented as a complex-valued Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} f_n \mathrm{e}^{\mathrm{i}\omega_n t} \tag{9}$$

on a frequency grid $\omega_n = n\Delta\omega = n\frac{2\pi}{T}$ where T is the (primitive) period of the function. In practice, the amplitudes $f_n \in \mathbb{C}$ are obtained by some form of discrete Fourier transform (DFT), such as FFT.

All the described matrices as well as the vectors $\mathbf{q}$ (displacement vector, or vector of generalized coordinates) and $\mathbf{f}$ (active load vector, or harmonic excitation vector) contain real-valued elements. The constraint, that all functions are expected to be real-valued, corresponds to a symmetry

$$f_n = f_{-n}^* \tag{10}$$

between the positive- and negative-frequency components, which allows to skip the calculation of negative-frequency Fourier coefficients. The time-dependent function is then reconstructed as

$$f(t) = f_0 + \sum_{n=1}^{\infty}(f_n \mathrm{e}^{\mathrm{i}\omega_n t} + f_n^* \mathrm{e}^{\mathrm{i}\omega_{-n} t}) = f_0 + \sum_{n=1}^{\infty} 2Re(f_n \mathrm{e}^{\mathrm{i}\omega_n t}) \tag{11}$$

We are interested in steady-state solutions $\boldsymbol{\Delta y}(t)$ under application of a periodic load $\mathbf{f}(t)$. Substituting the ansatz

$$\boldsymbol{\Delta y}(t) = \sum_n \boldsymbol{\Delta y}_n \mathrm{e}^{\mathrm{i}\omega_n t} \tag{12}$$

in the equation of motion yields

$$\sum_n \left[-\omega_n^2 \mathbf{M} + \mathrm{i}\omega_n \mathbf{D} + \mathbf{K}\right] \boldsymbol{\Delta y}_n \mathrm{e}^{\mathrm{i}\omega_n t} = \sum_n \mathbf{f}_n \mathrm{e}^{\mathrm{i}\omega_n t} \tag{13}$$

Since the functions $e^{i\omega_n t}$ are linearly independent, and since we do not need to calculate negative-frequency coefficients separately, we thus obtain the equations for $n \geq 0$:

$$\left[-\omega_n^2 \mathbf{M} + i\omega_n \mathbf{D} + \mathbf{K}\right] \boldsymbol{\Delta y}_n = \mathbf{f}_n \tag{14}$$

The linear complex-valued equation system (14) with the system matrix

$$-\omega_n^2 \mathbf{M} + i\omega_n \mathbf{D} + \mathbf{K} \tag{15}$$

often called *dynamic stiffness matrix*, is solved then separately for each of the excitation frequencies. The components of the excitation $\mathbf{f}_n$ are determined using the harmonic analysis: discrete Fourier transform (DFT) see, for example [15], or fast Fourier transform (FFT).

3 Workflow for a linear frequency domain solution

3.1 General FDS workflow

The multi-body model of the transmission system is set up in the same way as in a standard transient time integration. All rigid and flexible body types may be used. The class of joint types is restricted to the ones which may be linearized analytically or by using finite differences. As in standard time calculation, initial velocities of the bodies are calculated or corrected. Especially angular velocities of bodies are computed as the rotation of the bodies around a fixed axis constitutes the reference motion.

Then, using the kinetostatics solver [10], a loaded configuration of the model is sought for, i.e., external loads are applied and a static equilibrium position is computed resulting in pre-loaded bodies and joints, yielding in particular gear contacts being closed.

This equilibrium position is the starting point for the linearization of the model, where effects from the rotation are included. The joint types are linearized by the method of finite differences or - in cases of relatively simple force laws - analytically [7]. The usual differential equation (8) of second order with constant coefficient matrices resulting from the linearization describes the small motion of the model around the linearization point, where constant rotation of the individual bodies around a fixed axis is assumed. This differential equation is then solved in frequency domain. On equidistant frequency grids, the external loads are first transformed to frequency domain by FFT. The dynamic stiffness matrix is generated and the corresponding linear equation system is solved for each non-negligible frequency load component. The results can be represented in the frequency domain as well as recomputed in the time domain.

The workflow is schematically represented in Fig. 1.

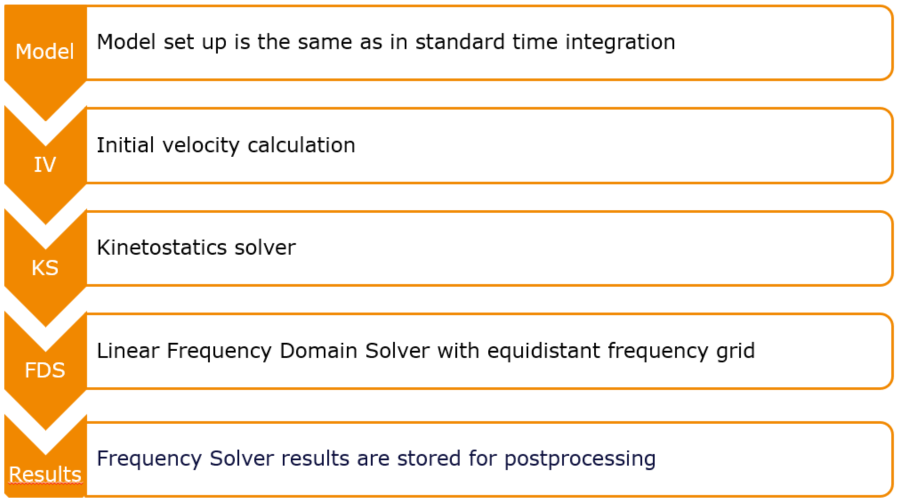

Fig. 1 Schematic representation of the workflow

3.2 Detailed description of FDS workflow

After the transmission model is created, the following steps are carried out consecutively in a linear FDS based solution procedure:

1. Calculation of initial velocities Initial global velocities of the bodies of the multi-body system are calculated (or corrected) in the same way as in standard transient integration [3]. The knowledge of consistent initial velocities is essential as the (uniform) rotational motion of the bodies with the calculated angular velocities is used as reference motion. In transmission systems, usually the angular velocity of one or some shafts are prescribed, but not all global initial velocities are known. Therefore, it is necessary to compute those missing taking into account constraints from joint interactions. Hence, the joint force laws are temporarily replaced by corresponding algebraic constraints and the velocities are computed by taking into account the Jacobian of the constraints.

2. Kinetostatics for a loaded configuration External loads are applied and a static equilibrium position is computed. In particular, an appropriate contact is established for e.g. gear joints.

For a transient time integration the kinetostatic state represents the preloaded conditions at the starting time point. The reduction of initial oscillations due to inaccurate initial conditions but also the determination of the correct start configuration, which avoids nonphysical penetrations of structural components, are main issues in time domain.

In FDS, the kinetostatic solution is used as the state where the model is linearized. Together with a possibly (uniform) rotation from the initial value calculation, it defines the reference trajectory of a body.

For the kinetostatic state, a solution $\boldsymbol{y}_{ks} = \boldsymbol{y}_0(0)$ of the right hand side of (8) is sought for [10] such that

$$\boldsymbol{f}^{load}(\boldsymbol{y}_{ks}) - \boldsymbol{f}(0, 0, \boldsymbol{y}_{ks}) = \mathbf{0}. \tag{16}$$

Inertia effects are neglected as the initial accelerations and velocities - especially of the flexible components of a body - are not known and are assumed to be zero. In principle, stiffness and joint forces/moments are brought into equilibrium with the applied external loads.

The numerical solution of (16) is based on a modified Quasi-Newton method. As the Jacobian of (16) is not regular in general, a regularization term is added based on the mass/inertia properties of the bodies.

3. Linearization At the kinetostatics equilibrium configuration $\boldsymbol{y}_{ks}$ the linearization of the model is carried out with the computation of the linearized matrices, i.e. mass-, damping-, stiffness matrices including gyroscopic parts from rotation - if requested. For details, see Sect. 2.2.

4. Computation of frequency components of external loads External loads $\boldsymbol{f}^{load}$ are specified in time domain, whether directly by the user, or precomputed by other components of the simulation suite. They are decomposed into their Fourier components by FFT, using separate frequency grids with the frequency resolution derived from the primitive period of each load item, and the upper frequency limit specified by the user. The results are then transferred to a global frequency grid with a frequency resolution specified by the user. In order to avoid introducing a beat in the signal, the frequency components are binned into the closest frequency of the global frequency grid, rather than distributed to the two closest frequencies.

This *multi-grid* sampling approach has arisen from the need to handle artifacts of the FFT. Originally, the user specified a single equidistant frequency grid, defined by an upper frequency limit and a frequency resolution. Frequency components that deviated from this grid by as little as 10^{-13} parts of the frequency resolution would produce windowing artifacts, that cause a full occupation of the grid, severely reducing the performance of the subsequent solution step, and introducing high-frequency error terms into calculated velocities. Avoiding this would require to find the smallest common period of all force terms, and using its inverse as the frequency resolution, which is unrealistically difficult for real-world models. The high sensitivity to small rounding errors proved to make internal adjustment of the frequency resolution unreliable.

Additionally, this would still leave the issue of aliasing artifacts, which caused occupation of unoccupied frequencies. While more benign than broadening of the spectrum from windowing artifacts, aliasing artifacts would still produce high-frequency error terms, that were enhanced in derivatives. The multi-grid load sampling approach removes windowing artifacts completely, and ensures that aliasing artifacts occur only as small contributions to already-occupied frequency components, rather than introducing *new* frequency components.

5. Linear frequency domain solution The linearized equations of motion are solved. The solution represents small amplitude periodic steady state motions around the reference motion including constant rotation of individual bodies around a fixed axis (in absolute coordinate frame).

The dynamic stiffness matrix (15) for each non-zero external load frequency component is set up and the corresponding complex-valued linear system (14) is solved. The result is the complex amplitude of motion at this frequency of each DOF of the system.

The solution of the linear systems (14) is carried out in $O(N_\omega N_{DOF}^2)$ operations, where N_{DOF} represents the number of DOFs in the model and N_ω the number of occupied frequencies in the global frequency grid. The solution is the cheaper the more load frequency components are zero on the global frequency grid.

6. Storage of results and postprocessing Native FDS results are stored for postprocessing. Additionally, the re-computation (synthesis) of FDS results in time domain may be requested.

4 Application to a gearbox model

The presented solver workflow is applied to a simple gearbox model as depicted in Fig. 2. In the application example three elastic gear shafts (the green input shaft, the blue layshaft, and the yellow output shaft) supported by in total five deep groove ball bearing joints in a rigid housing are connected by two cylindrical gear joints. Additionally, the input shaft supports the output shaft through a radial bearing joint. In total the model consists of 426 elastic DOFs (input shaft: 168 DOFs, layshaft: 204 DOFs, output shaft: 54 DOFs).

In general, the principal tasks which need to be processed by a gear joint evaluation are: contact detection, constitution of the deformation field in the gear mesh, force distribution in the mesh and if requested calculation of friction and damping forces. The modeling of the gear meshing process within a multi-body system reveals a wide variety of possible approaches [11]. In this model, the two cylindrical gear joints are parameterized in such a way that they are linear: both mesh stiffness and damping values of the cylindrical gear joints assumed to be constant, and tooth separation is not considered. The ratio of teeth of the gear joint connecting input shaft and layshaft is 38:19 and that of the gear joint connecting layshaft and output shaft is 27:39.

Deep groove ball bearings are typically applied for the bearings of gear-shafts. They show non-linear behavior, as the bearings are configured such that periodic variations of the resultant bearing stiffness are taken into account. The radial bearing joint connecting input and output shaft is a linear one.

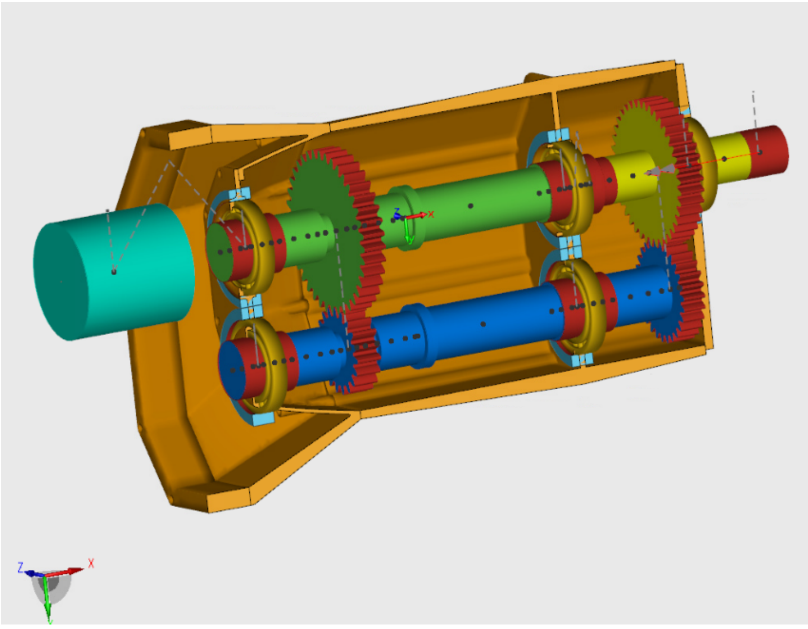

Fig. 2 The model of a simple gearbox with three elastic gear shafts: input shaft (green), layshaft (blue), output shaft (yellow) supported by ball bearings in a rigid housing and connected by cylindrical gears

Angular motion of the input shaft is realized through predefined motion of the cylinder (turquoise) on the left side connected by a linear rotational coupling joint. An oscillating moment is applied to the output shaft where the period of the oscillation coincides with one rotation of the output shaft.

The FDS workflow is applied to the model with a predefined motion of the cylinder of 3000 rotations per minute (rpm):

1. The initial velocity calculation gives rotational speeds of 3000 rpm for the input shaft, 6000 rpm for the layshaft (in negative direction of rotation of the input shaft), and about 4153.846 rpm (= 69.231 rps) for the output shaft. Clearly, the rotation ratio of the shafts is determined by the properties of the two gear joints.
2. Next step is the kinetostatic solution which converges within 75 iterations. The angular moment applied at the output shaft leads to slightly distorted shafts. Additionally, the nodes of the shafts are translationally displaced according to force equilibrium conditions which are mainly determined by the two gear joints.
3. The resulting kinetostatic state is used as configuration for the linearization of the model. All components of the model are linearized analytically except for the five ball bearings and the two gear joints. The partial derivatives of the joint forces with regard to node positions and velocities are approximated by the method of finite differences. As the gear joints are linear,

the results of numerical linearization may be checked against analytical derivation in literature, e.g. [9].

4. Next step is the computation of frequency components of the applied external moment. FFT is carried out on a load specific frequency grid. The only significant frequency component of the external moment is calculated to be 69.231 Hz (equal to rotation velocity of output shaft). Very small numerical artifacts appear at 28 additional multiples of the base frequency of 69.231 Hz. And a non-zero frequency component is computed for the constant part of the moment at 0 Hz.
5. The global equidistant frequency grid for the frequency domain solver itself is defined with lower frequency limit 0 Hz, upper limit 5000 Hz and frequency resolution of 0.833333 Hz. The main external load component of the applied moment is binned to the closest frequency, which is 69.167 Hz on the global grid. Similarly, the components resulting from numerical FFT artifacts are binned to their closest frequencies. Out of a number of 6001 possible frequency grid points, the linear equation system (14) for determining solution components is solved for 30 of them.

The resulting vibrations from a linear frequency domain solution are compared against results of a complete time domain solution up to 2000 deg reference angle, see Fig. 3 and for a detailed view Fig. 4. As the gear joints couple both transverse and torsional motion of the three shafts, the position of one transverse component of the gear joint connection node at the left side of the layshaft is compared as an example. The additional oscillations in the TDS solution in Fig. 4 are due to nonlinear behavior of deep groove ball bearings, but the basic behavior of the solution is well reproduced.

The time domain solution is accepted to be reliable, but it takes much more simulation time before a stationary state is reached: the time domain solution takes 115.4 seconds CPU time, compared to only 7.9 seconds for the presented frequency domain solver workflow.

5 Conclusion and Outlook

A linear frequency domain solver workflow has been presented and applied to a simple transmission system. The restriction to linear ordinary differential equations of second order with time-independent coefficient matrices limits the approach to systems with nearly negligible nonlinear effects. However, the method is very efficient to get a fast overview of the dynamic properties of simple systems.

In a next step, the method will be extended to include e.g. gear meshing effects. In a preprocessing step - the so-called “tooth contact analysis” (TCA) - the contact forces in the mesh are calculated. Then, these forces are applied as external loads in the linear frequency domain solver workflow.

Alternatively, gear meshing effects may be taken into account as stiffness fluctuations contributing to a time varying stiffness matrix. An iterative fre-

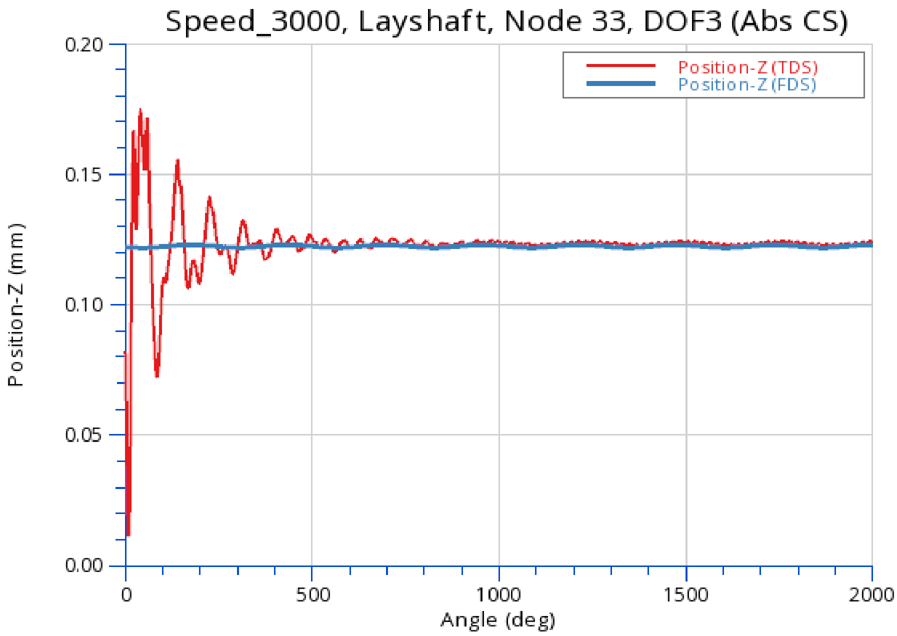

Fig. 3 Comparison of motion results obtained from a time domain (TDS, red) and frequency domain (FDS, blue) solver. The steady-state motion is calculated immediately by FDS.

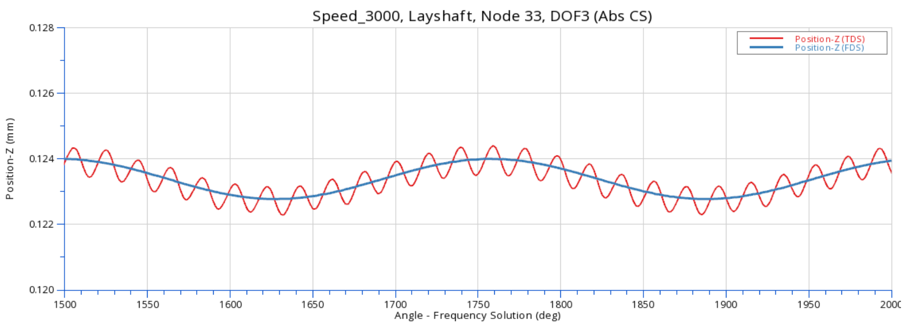

Fig. 4 Details of Fig. 3 from 1500 to 2000 deg rotation angle. The additional oscillations in the TDS solution are due to nonlinear behavior of the ball bearings, but the basic behavior of the solution is well reproduced.

quency domain approach with alternating computations in frequency and time domain is in development [1].

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References

1. Bauer, K.D., Haslinger, J., Offner, G., Parikyan, T.: Approaches for going beyond linear frequency domain powertrain simulation. In: Proceedings of ECMI 2021 (2022). Accepted.
2. Drab, C.B.: Dynamic simulation of power train related systems. Ph.D. thesis, Johannes Kepler Universität, Linz (2007).
3. Drab, C.B., Engl, H.W., Haslinger, J.R., Offner, G., Pfau, R.U., Zulehner, W.: Dynamic simulation of crankshaft multibody systems. Multibody System Dynamics **22**, 133–144 (2009).
4. Friswell, M., Penny, J., Garvey, S., Lees, A.: Dynamics of Rotating Machines. Cambridge Aerospace Series. Cambridge University Press (2010).
5. Genta, G.: Dynamics of Rotating Systems. Mechanical Engineering Series. Springer New York (2007).
6. Han, S., Bauchau, O.: Simulation and stability analysis of periodic flexible multibody systems. Multibody System Dynamics **50**(4), 381–413 (2020).
7. Haslinger, J., Offner, G., Sopouch, M., Zinkiewicz, B.: Modal analysis of planetary gearsets. In: Proceedings of the NAFEMS World Congress: Salzburg, Austria. NAFEMS (2021).
8. Kang, J.S., Bae, S., Lee, J.M., Tak, T.O.: Force equilibrium approach for linearization of constrained mechanical system dynamics. Journal of Mechanical Design **125**, 143 – 149 (2003).
9. Kaplan, J.A., Dousti, S., Allaire, P.E., Nichols, B.R., Dimond, T.W., Untaroiu, A.: Rotor dynamic modeling of gears and geared systems. In: Volume 7A: Structures and Dynamics. American Society of Mechanical Engineers (2013).
10. Lorenz, N., Offner, G., Colla, A.: Preloaded initial state evaluation of flexible multibody systems. In: ECCOMAS Thematic Conference on Multibody Dynamics 2013, 463–472. Zagreb (2013).
11. Neusser, Z., Sopouch, M., Schaffner, T., Priebsch, H.H.: Multi-body dynamics based gear mesh model for prediction of gear dynamics and transmission error. In: Transmission and Driveline 2010, SAE World Congress & Exhibition (2010).
12. Nguyen, V.K., Nguyen, P.D., Hoang, M.C.: Linearization and parametric vibration analysis of some applied problems in multibody systems. Multibody System Dynamics **22**(2), 163–180 (2009).
13. Offner, G.: Modelling of condensed flexible bodies considering non-linear inertia effects resulting from gross motions. Proc. IMechE Part K: J. Multi-body Dynamics **225**, 204–219 (2011).
14. Parikyan, T., Bukovnik, S.: Turbocharger dynamic analysis: concept-phase simulation in frequency domain. In: SIRM 2019, 893–902. Copenhagen (2019).
15. Parikyan, T., Resch, T.: Statically Indeterminate Main Bearing Load Calculation in Frequency Domain for Usage in Early Concept Phase. In: ASME 2012 Internal Combustion Engine Division Fall Technical Conference (2012).
16. Pedersen, R., Santos, I.F., Hede, I.A.: Advantages and drawbacks of applying periodic time-variant modal analysis to spur gear dynamics. Mechanical Systems and Signal Processing **24**(5), 1495–1508 (2010).