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RICAM-Report 2021-43

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Abstract The study of powertrain multi-body systems in time-domain can be prohibitively expensive for systems with high rotational speeds. Solving the equations of motion in time-domain can provide orders of magnitude faster results when omitting non-linear coupling by decoupling the problem into independent equations for each load frequency, but this feature is lost when accounting for non-linearities, e.g. from gear meshing. We present an iterative algorithm, that avoids coupling of frequency components by switching between frequency- and time-domain for describing the non-linear terms. Utility of the algorithm is demonstrated by studying a two-shaft model system, comparing solution by time-domain integration and by the iterative algorithm.

1 Introduction

Typical powertrain multi-body system simulations represent the mechanical system by components (e.g. rotating shafts) and joints (e.g. gear contacts) which describe the forces coupling their motion [1, 2, 8]. In automotive applications these systems often have thousands of degrees of freedom [5], such that obtaining the steady-state

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motion of fast-moving components by solving the equations of component motion in time-domain can take hours, as non-periodic deviations from the steady state motion (e.g. transient oscillations) may decay slowly relative to the system cycle rate. This problem is exacerbated for turbo chargers, which may rotate at as much as 350,000 rpm [6].

Frequency domain solution of the motion on the other hand can yield the steady-state motion in a matter of minutes or seconds for the same systems [9]. In this approach non-periodic deviations are inherently suppressed and linearization of the equations of motion decouples them into independent equation systems for each frequency of the applied external load – but only under the assumption, that forces can be represented by *time-independent* linearization coefficients.

Some joints (e.g. gear contacts) are modeled by force laws exhibiting stiffness fluctuations, which cannot be represented within this assumption. Naive extension to time-dependent stiffness coefficients would result in coupling across all frequencies and potentially the need for a denser frequency grid, increasing the computational cost by orders of magnitude again. Potentially this negates the performance gains over a time-based solution.

In this paper we discuss approaches for extending frequency-domain simulations of powertrain systems beyond the constant linear approximation while maintaining its performance advantage. In sec. 2 we discuss the mathematical description of the problem. Refs. [3, 4, 6] can be used for further reading. In sec. 3 we develop the iterative solver algorithm by means of a perturbation approach. In sec. 4 we demonstrate the application of the algorithm to a two-shaft model by means of a prototype implementation.

2 Background

The dynamics of a powertrain system modeled as rigid bodies and finite-element discretization of flexible bodies are represented by equations of motion of the form $M(z) \cdot \ddot{z} = f(t, z, \dot{z})$ [3] with a mass matrix $M(z)$ representing inertia effects, $z(t)$ the trajectory of the system, and f describing forces within and across bodies.

Given a decomposition $z(t) = z_0(t) + q(t)$ where $z_0(t)$ is an approximation of the real trajectory and $q(t)$ assumed to be small, and given a suitable choice of coordinate systems [2, 7, 8] or limiting the allowed models sufficiently, $M(z_0(t))$ becomes constant, and the force equation can be linearized into

$$M \cdot \ddot{q} + D(t) \cdot \dot{q} + K(t) \cdot q = f(t) \tag{1}$$

where $f(t) = f(t, z_0, \dot{z}_0) - M \cdot \ddot{z}_0$ contains external forces, internal stiffness and joint forces along the reference trajectory $z_0(t)$ and inertia forces for nodes described in accelerated coordinate systems. For sufficiently simple models and suitable $z_0(t)$, the matrices D, K become time independent along $z_0(t)$, resulting in a frequency domain problem[1]

$$\left(-\omega^2 M + i\omega D + K\right) \cdot q_\omega = f_\omega. \quad (2)$$

This equation can be solved separately for each frequency in $\mathscr{O}(N_\omega N_q^2)$, where N_ω, N_q are size of the frequency grid and of the vector q respectively. However, in such a framework meshing effects cannot be represented. Taking into account the time-dependence $D(t), K(t)$ results in an equation system, where the frequencies are coupled. The time complexity increases by a factor of N_ω.

3 Iterative linear solver

We thus study whether it is possible to enhance the results by iteratively applying the frequency solver and evaluating forces in time domain in between.

We split $K(t)$ into its time average K_0 and the time-dependent part $K_1(t)$, and introduce a perturbative approach

$$\begin{aligned} K(t) &= K_0 + \lambda K_1(t) \text{ and likewise for } D \\ q(t) &= q_0(t) + \lambda q_1(t) + \lambda^2 q_2(t) + \cdots \end{aligned} \quad (3)$$

where K_0 is a suitable constant component of $K(t)$ such the time-average and $K_1(t)$ captures the time-dependence . Varying over λ and then setting $\lambda = 1$, the equations of motion (1) decompose into a sequence

$$\begin{aligned} M \cdot \ddot{q}_0 + D_0 \cdot \dot{q}_0 + K_0 \cdot q_0 &= f(t) \text{ for order } k = 0 \\ M \cdot \ddot{q}_k + D_0 \cdot \dot{q}_k + K_0 \cdot q_k &= -D_1(t) \cdot \dot{q}_{k-1} - K_1(t) \cdot q_{k-1} \text{ for } k \geq 1 \end{aligned} \quad (4)$$

with $f(t) = f(t, z_0, \dot{z}_0) - M \cdot \ddot{z}_0$ as before.

For implementation it is convenient to reformulate in terms of the cumulative solution up to order n, $q^{(n)}(t) = \sum_{k=0}^{n} q_k(t)$. By summing over the iteration equation (4) up to order $k = n$, we obtain

$$M \cdot \ddot{q}^{(n)} + D_0 \cdot \dot{q}^{(n)} + K_0 \cdot q^{(n)} = f(t) - D_1(t) \cdot \dot{q}^{(n-1)} - K_1(t) \cdot q^{(n-1)} \quad (5)$$

with an initialization condition $q^{(-1)}(t) = 0$. This form allows us to introduce a damping factor $\gamma \in (0, 1]$, by setting $q^{(n)} \to (1-\gamma)q^{(n)} + \gamma q^{(n-1)}$ after each solution step. The solution is then evaluated by:

[1] We assume that functions are decomposed into discrete Fourier coefficients according to $f(t) = \sum_\omega f_\omega e^{i\omega t}$.

1. Initialization
   a. Obtain some reference trajectory $z_0(t)$.
   b. Calculate $f(t)$, $K_0, K_1(t)$, $D_0, D_1(t)$ from $z_0(t)$.
   c. Initialize $q_\omega, q(t), \dot{q}(t)$ to 0.
2. Repeat for $n \geq 0$ until converged:
   a. Evaluate $f_{\text{rhs}}(t) = f(t) - M \cdot \ddot{z}_0 - D_1(t) \cdot \dot{q}(t) - K_1(t) \cdot q(t)$.
   b. Obtain $f_{\text{rhs},\omega}$ by Fourier analysis of $f_{\text{rhs}}(t)$.
   c. Solve $\left(-\omega^2 M + i\omega D_0 + K_0\right) q'_\omega = f_{\text{rhs},\omega}$.
   d. Update $q_\omega \to \gamma q_\omega + (1-\gamma) q'_\omega$.
   e. Fourier synthesis of $q(t), \dot{q}(t)$ from q_ω.

The result of step $n = 0$ corresponds to the linear frequency domain solution. The method of switching between time-domain and frequency-domain is intentionally left open. The easiest is to use Fast Fourier Transform (FFT), which requires equidistant frequency and time grids.

4 Application example

We consider a simple model system consisting of two shafts connected by gears, where the pinion is driven by a turbine at a constant angular velocity, and an angle-dependent load $L(\alpha)$ acting on the gear shaft (see fig. 1).

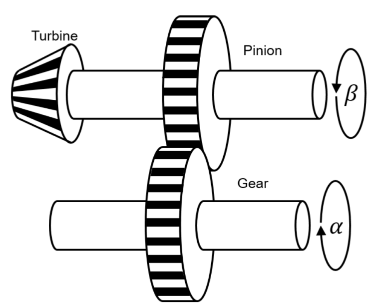

Fig. 1 Example model for demonstrating the iterative approach. The system is constrained to allow only rotations around the shaft axes and no translatory motion.

The depicted example model has a single degree of freedom α, the angular position of the gear shaft, while the trajectory of the pinion is assumed to be a uniform rotation $\beta(t) = \Omega t$. We assume a transmission ratio of 1 for simplicity, exerting a linear force $f_{\text{gear}}(\alpha) = -K(\beta)(\alpha - \beta) - D(\beta)\left(\dot{\alpha} - \dot{\beta}\right)$ with $K(\beta), D(\beta)$ varying periodically between two-tooth and one-tooth contact with 4 teeth per shaft cycle.[2]

[2] While 4 teeth are not particularly realistic, it produces more understandable results when showing a plot over a full shaft cycle.

A time-dependent load of the form $f_{\text{load}}(t) = L_0 + L_1 \cos(4\Omega t)$ acts on the gear shaft. For this system, the obvious reference trajectory is $\alpha_0(t) = \Omega t$ and the displacement coordinate $q(t) = \alpha(t) - \Omega t$, resulting in an exact equation of motion

$$M\ddot{q} = f_{\text{load}}(t) - D(t)\dot{q} - K(t)q \tag{6}$$

which we solve in time domain domain (fig. 2) and by applying the algorithm described in sec. 3 (fig. 3), with no algorithmic damping ($\gamma = 0$).

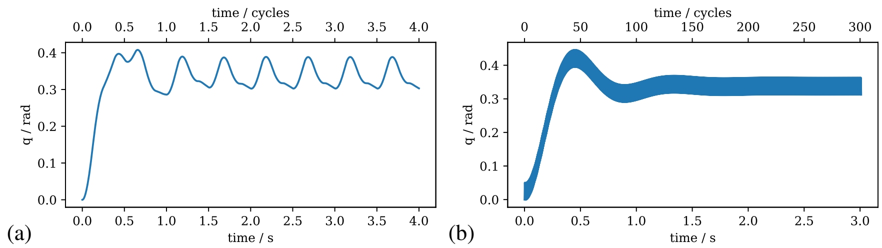

Fig. 2 Displacement $q(t)$ for the test model at (a) $\Omega = 60$rpm and (b) $\Omega = 6,000$rpm respectively, with the load amplitude scaled as $L_1 \propto \Omega^2$ to produce similar displacement amplitudes. Deviation of the converged oscillations from a cosine-shape visible in (a) is caused by meshing of the gears. Since the decay time of transient terms is constant, at higher rotation speeds it takes proportionally more cycles and thus computation time to reach the steady-state behavior.

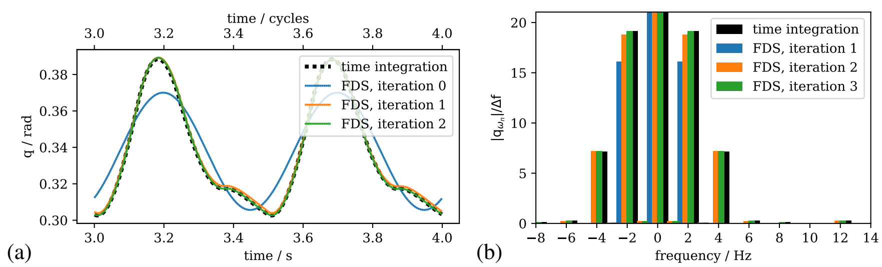

Fig. 3 (a) Time domain result and (b) frequency spectrum obtained by a time-domain solver and the iterative frequency domain solver (FDS) algorithm from sec. 3 for up to three iterations. The large static components ($\omega = 0$) are truncated. After one iteration ("linear solver") meshing effects are ignored entirely, visible in the spectrum (b) as presence of only the 2Hz component present in the load. After only two iterations the result nearly matches the time domain solver, with only small corrections in further iterations.

We see that the frequency-domain algorithm reproduces the meshing effects, in this example already after one iteration beyond the linear solver, and is mostly converged with one more iteration. Repeating the simulation with 60, 600 and 6000 rpm respectively demonstrates increasingly slow convergence of the time-domain solver, with the prototype simulations taking 0.05, 0.44 and 3.94 seconds respectively, while the same time resolution is achieved with three iterations of the iterative frequency domain solver within a constant 0.006 seconds.

5 Conclusion and Outlook

We have demonstrated an iterative frequency domain solver, that provides fast solutions compared to direct time-domain integration especially at high rotation speeds, while mapping non-linear contributions to iterative solution of a linear frequency-domain problem, thus maintaining the high efficiency of a linear frequency domain solver. More studies are needed to formulate formal convergence criteria and to verify convergence for more complex models. Moreover, the algorithm should be applied to real-world application models and integrated with industrial simulation software.

Acknowledgments Part of the work of Klaus-Dieter Bauer and Josef Haslinger was supported by the Government of Upper Austria within the FTI Struktur framework.

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