

Group Seminar

Inverse Problems and Mathematical Imaging & Transfer Group

Wednesday, March 6, 2024, 13:00

RICAM, SP2 416-2

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Optimisation algorithms in non-standard Banach spaces for inverse problems in imaging

In this talk, I will present optimisation algorithms for imaging inverse problems in non-standard Banach spaces. It is divided into two parts: in the former, the setting of Lebesgue spaces with a variable exponent map $L^{p(\cdot)}$ is considered to improve adaptivity of the solution with respect to standard Hilbert reconstructions; in the latter a modelling in the space of Radon measures is used to avoid the biases observed in sparse regularisation methods due to discretisation.

In more detail, the first part explores both smooth and non-smooth optimisation algorithms in reflexive $L^{p(\cdot)}$ spaces, which are Banach spaces endowed with the so-called Luxemburg norm. As a first result, we provide an expression of the duality maps in those spaces, which are an essential ingredient for the design of effective iterative algorithms. To overcome the non-separability of the underlying norm and the consequent heavy computation times, we then study the class of modular functionals which directly extend the (non-homogeneous) p -power of L^p -norms to the general $L^{p(\cdot)}$. In terms of the modular functions, we formulate handy analogues of duality maps, which are amenable for both smooth and non-smooth optimisation algorithms due to their separability. We thus study modular-based gradient descent (both in deterministic and in a stochastic setting) and modular-based proximal gradient algorithms in $L^{p(\cdot)}$, and prove their convergence in function values. The spatial flexibility of such spaces proves to be particularly advantageous in addressing sparsity, edge-preserving and heterogeneous signal/noise statistics, while remaining efficient and stable from an optimisation perspective. We numerically validate this extensively on 1D/2D exemplar inverse problems (deconvolution, mixed denoising, CT reconstruction).

The second part focuses on off-the-grid Poisson inverse problems formulated within the space of Radon measures. We consider a variational model which couples a Kullback-Leibler data term with the Total Variation regularisation of the desired measure (that is, a weighted sum of Diracs) together with a non-negativity constraint. We study optimality conditions and the corresponding dual problem is carried out and an improved version of the Sliding Frank-Wolfe algorithm is used for computing the numerical solution

efficiently. To mitigate the dependence of the results on the choice of the regularisation parameter, an homotopy strategy is proposed for its automatic tuning, where, at each algorithmic iteration checks whether an informed stopping criterion defined in terms of the noise level is verified and updates the regularisation parameter accordingly. Several numerical experiments are reported on both simulated 2D and real 3D fluorescence microscopy data.

Zoom (Wed, Mar 6, 2024, 01:00 PM (CET))

<https://oeaw-ac-at.zoom.us/j/95359054042?pwd=aThjUjZnNXdpckhLMEZ4dko2WnNlZz09>

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