

Arcs and MDS codes

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A *Maximum Distance Separable code* (or *MDS code*) is a code meeting the Singleton bound. This means that an MDS code corrects the maximum number of errors given its size and length. Examples of MDS codes include the (extended) Reed-Solomon codes which have many applications. MDS codes have been studied since the 1950's and, according to MacWilliams and Sloane [2], already in 1977, they formed “one of the most fascinating chapters in all of coding theory”. It is believed that a linear MDS code cannot have length larger than the extended Reed-Solomon code (except in some very special cases which are well-understood). This is known as the *MDS conjecture* (sometimes referred to as the *main conjecture*). Many mathematicians have contributed to a possible solution and several instances of the MDS conjecture have been solved in the last 50 years. However the conjecture in its full generality is still open. In 2012, Simeon Ball proved the conjecture for MDS codes which are linear over prime fields.

We will report on recent progress, in particular on the results from our paper [1].

Linear MDS codes are equivalent to arcs in projective spaces over finite fields, and as such, they have been a motivating application for many contributions in Galois geometry. In terms of arcs, the MDS conjecture states that an arc in a projective space over a finite field cannot have more points than a normal rational curve (ignoring a couple of well-understood special cases). Most previous contributions towards the MDS conjecture, obtained by Segre [1967], Hirschfeld and Korchmáros [1996, 1998], and Voloch [1990, 1991], rely on results on the number of points on algebraic curves over finite fields, in particular the Hasse-Weil theorem and the Stöhr-Voloch theorem, and are based on Segre's idea to associate an algebraic curve in the dual plane containing the tangents to an arc. In our work [1], we do not rely on such theorems, but use a new approach starting from a scaled coordinate-free version of Segre's lemma of tangents. We prove that in odd characteristic an arc in a projective plane over a finite field, which is not contained in a conic, is contained in the intersection of two curves, which do not share a common component, and have relatively small degree. This result then allows us to prove the MDS conjecture for a wider range of dimensions.

[1] S. Ball and M. Lavrauw: Planar arcs. *J. Combin. Theory Ser. A* 160 (2018), 261–287.

[2] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*, North-Holland, 1977.