Workshop on Function Approximation

December 1, 2016 – December 2, 2016

Cooperation workshop of
Johann Radon Institute for Computational and Applied Mathematics (RICAM)
and
Johannes Kepler University Linz (JKU Linz)

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Schedule

Thursday, December 1st	
9:00 – 10:30 SP2 416-2	**Lecture 1: Spline approximation** S. Takacs & T. Takacs
10:30 – 11:00	Coffee break
11:00 – 12:30 SP2 416-2	**Lecture 2: Multivariate approximation** C. Irrgeher & P. Kritzer
12:30 – 14:00	Lunch break
Thursday-Session 1, Chair: G. Leobacher	
14:00 – 14:45 SP2 416-2	**$\mathbb{L}_p$-approximation in Korobov spaces with exponential weights** F. Pillichshammer
14:45 – 15:30 SP2 416-2	**∞-Variate integration** G. Wasilkowski
15:30 – 16:00	Coffee break
Thursday-Session 2, Chair: T. Takacs	
16:00 – 16:45 SP2 416-2	**Convergence of orthonormal spline series** M. Passenbrunner
16:45 – 17:30 SP2 416-2	**Automatic refinement for partially nested hierarchical B-splines** N. Engleitner
Friday, December 2nd	
Friday-Session 1, Chair: M. Hauer	
9:00 – 9:45 SP2 416-2	**Low-rank function approximation and applications in isogeometric analysis** A. Mantzaflaris
9:45 – 10:30 SP2 416-2	**Fast approximation of bivariate functions using low-rank splines** C. Hofreither
10:30 – 11:00	Coffee break
Friday-Session 2, Chair: C. Irrgeher	
11:00 – 11:45 SP2 416-2	**Szegő-Widom asymptotics of Chebyshev polynomials on circular arcs** B. Eichinger
11:45 – 12:30 SP2 416-2	**Function approximation by aggregation in the context of supervised learning** S. Pereverzyev

Abstracts – Lectures

Thursday, December 1, 9:00-10:30

Lecture 1: Spline approximation

Stefan Takacs
RICAM, Computational Methods for PDEs
Thomas Takacs
JKU Linz, Institute of Applied Geometry

In the first part of the lecture we discuss standard approximation properties for B-spline and NURBS spaces. B-splines are piecewise polynomial functions of a given prescribed smoothness. NURBS stands for *non-uniform rational B-splines* and denotes rational functions, where both the numerator and the denominator are B-splines. We consider approximation error bounds for functions in Sobolev spaces.

We start with classical approximation properties for univariate functions and then extend the results to tensor-product spaces as well as to mapped NUBRS spaces, so called *isogeometric function spaces*. The presented results determine the convergence properties of isogeometric methods, a family of numerical methods which use isogeometric functions for the solution of partial differential equations.

In the second part of the lecture we present improved approximation error estimates as well as corresponding inverse inequalities for B-splines of maximum smoothness given in standard Sobolev norms and semi-norms. The presented estimates do not depend on the polynomial degree of the splines but only on the grid size.

We will see that the approximation lives in a subspace of the classical B-spline space. We show that for this subspace, there is an inverse inequality which is also independent of the polynomial degree. As the approximation error estimate and the inverse inequality show complementary behavior, the results can be used to construct fast iterative methods for solving problems arising from isogeometric discretizations of partial differential equations.

Thursday, December 1, 11:00-12:30

Lecture 2: Multivariate approximation

Christian Irrgeher
RICAM, Multivariate Algorithms and Quasi-Monte Carlo Methods
Peter Kritzer
RICAM, Multivariate Algorithms and Quasi-Monte Carlo Methods

We start the lecture by introducing the general setting, which is considered in the field of information-based complexity. That is, we consider operators $S : F \to G$ between two (function) spaces F, G and we study algorithms to approximate such operators S. These algorithms can be categorized with respect to the sort of information which they use. To analyze the “quality” of an algorithm we can consider different settings, e.g. the worst-case setting or the average-case setting. In this lecture we mainly focus on the worst-case setting. To this end, we give the definitions of the relevant errors as well as of the information complexity. Furthermore, we introduce various notions of tractability which help to figure out the dependence on the dimension.

Then we present some general properties of linear problems which show that it is sufficient to restrict oneself to a simple class of algorithms to achieve optimal convergence results. Moreover, we give a general result on optimal algorithms for the Hilbert space case.

In the second part of the lecture we turn to multivariate L_2-approximation in the Korobov space. The Korobov space is a Sobolev-type space of periodic functions. We show that, in general, the approximation problem suffers from the curse of dimensionality. Then we move on to the weighted approximation problem, i.e. we introduce weights to the norm of the Korobov space such that successive variables may have diminishing importance. In this case we can give conditions on the weights to achieve tractability of the approximation problem.

Abstracts – Talks

Thursday, December 1, 14:00-14:45

$\mathbb{L}_p$-approximation in Korobov spaces with exponential weights

Friedrich Pillichshammer
JKU Linz, Institute of Financial Mathematics and Applied Number Theory

We study multivariate $\mathbb{L}_p$-approximation ($p \in [2,\infty]$) for a weighted Korobov space of periodic functions for which the Fourier coefficients decay exponentially fast. The weights are defined, in particular, in terms of two sequences $\boldsymbol{a} = \{a_j\}$ and $\boldsymbol{b} = \{b_j\}$ of positive real numbers bounded away from zero. We study the minimal worst-case error $e^{\mathbb{L}_p-\mathrm{app},\Lambda}(n,s)$ of all algorithms that use n information evaluations from a class Λ in the s-variate case where we consider two classes Λ: the class Λ^{all} of all linear functionals and the class Λ^{std} of only function evaluations. We study exponential convergence of the minimal worst-case error, which means that $e^{\mathbb{L}_p-\mathrm{app},\Lambda}(n,s)$ converges to zero exponentially fast with increasing n. Furthermore, we consider how the error depends on the dimension s. To this end, we define the notions of κ-EC-weak, EC-polynomial and EC-strong polynomial tractability, where EC stands for "exponential convergence". In particular, EC-polynomial tractability means that we need a polynomial number of information evaluations in s and $1+\log\varepsilon^{-1}$ to compute an ε-approximation. We derive necessary and sufficient conditions on the sequences $\boldsymbol{a}$ and $\boldsymbol{b}$ for obtaining exponential error convergence, and also for obtaining the various notions of tractability.

The presentation summarizes joint works with Josef Dick (UNSW Sydney), Peter Kritzer (RICAM Linz) and Henryk Woźniakowski (Columbia University and University of Warsaw).

Thursday, December 1, 14:45-15:30

∞-Variate integration

Greg W. Wasilkowski
University of Kentucky, Department of Computer Science

We discuss recent results on efficient numerical approximation of integrals with infinitely many variables. For functions from weighted tensor product spaces, we show that the truncation and superposition dimensions are very small under modest error demands. We also present a new and efficient Multivariate Decomposition Method that is almost as efficient as quadratures for univariate problems.

Thursday, December 1, 16:00-16:45

Convergence of orthonormal spline series

Markus Passenbrunner
JKU Linz, Institute of Analysis

We discuss results on the convergence of orthonormal spline series corresponding to arbitrary spline degree and arbitrary knot sequences. The simplest and most well studied example of such systems corresponding to degree zero and the dyadic knot sequence is the Haar system.

We give a survey of classical results related to such systems and as well present some more recent results (see for instance [1, 2, 3]). In doing this we introduce a refinement [1] of Shadrin's theorem on the boundedness (in the sup-norm) of orthogonal projection operators onto spline spaces for any spline order by a constant that does not depend on the grid point sequence. A vital tool in these investigations are B-splines, especially sharp decay inequalities on the inverse to the B-spline Gram matrix are of interest and can be related to boundedness questions of spline orthoprojectors.

References:
[1] M. Passenbrunner and A. Shadrin: On almost everywhere convergence of orthogonal spline projections with arbitrary knots, *J. Approx. Theory* 180 (2014) 77–89.
[2] M. Passenbrunner: Unconditionality of orthogonal spline systems in L^p, *Studia Math.* 222 (2014), no. 1, 51–86.
[3] G. Gevorkyan, A. Kamont, K. Keryan, M. Passenbrunner: Unconditionality of orthogonal spline systems in H^1, *Studia Math.* 226 (2015), no. 2, 123–154.

Thursday, December 1, 16:45-17:30

Automatic refinement for partially nested hierarchical B-splines

Nora Engleitner
JKU Linz, Institute of Applied Geometry

The established construction of hierarchical B-splines starts from a given sequence of nested spline spaces, and hence it is not possible to pursue independent refinement strategies in different parts of a model. However, this possibility would be highly useful for designing surfaces with creases or similar features and in isogeometric analysis for using independent refinement techniques (such as h- and p-refinement) in different areas of the computational domain. In order to overcome the limitation of the hierarchical B-splines, we generalized Kraft's selection mechanism to obtain sequences of partially nested bivariate spline spaces of uniform degree and maximum order smoothness [1]. Now we provide a generalization of this earlier work to the case of higher dimensions, non-uniform degrees and varying order of smoothness. We introduce assumptions that enable us to define a hierarchical spline basis, to establish a truncation operation, to obtain the partition of unity property, and to derive a completeness result. In addition we discuss an automatic refinement algorithm for such spline spaces and present its application to a least-squares approximation problem. This is joint work with B. Jüttler and U. Zore.

References:
[1] N. Engleitner, B. Jüttler, U. Zore: Partially Nested Hierarchical Refinement of Bivariate Tensor-product Splines with Highest Order Smoothness, NFN Report No. 49.

Friday, December 2, 9:00-9:45

Low-rank function approximation and applications in isogeometric analysis

Angelos Mantzaflaris
RICAM, Geometry in Simulations

Low-rank approximation techniques have proven to be the solution of choice for reducing the complexity and restoring computational tractability in areas such as optimization, sensitivity analysis, inverse problems, biology and chemistry. In essence, these methods provide ways to recover structure in the parameters or carefully select a few, so that the evaluation of the, so called, reduced model, on large data grids becomes tractable. This is done by a suitable projection of the function on a lower-dimensional manifold of tensors, whose dimension is called the rank of the tensor. Different notions of ranks and the corresponding low-rank approximation formats have been introduced, having different approximation and computational complexity properties. In this talk we will review some popular low-rank formats and discuss applications of this approach to isogeometric analysis.

Friday, December 2, 9:45-10:30

Fast approximation of bivariate functions using low-rank splines

Clemens Hofreither
JKU Linz, Institute of Computational Mathematics

We present an algorithm for the approximation of bivariate functions by "low-rank splines", that is, sums of outer products of univariate splines. Our approach is motivated by the Adaptive Cross Approximation (ACA) algorithm for low-rank matrix approximation as well as the use of low-rank function approximation in the recent extension of the `chebfun` package to two dimensions. The resulting approximants lie in tensor product spline spaces, but typically require the storage of far fewer coefficients than tensor product interpolants. We analyze the complexity and show that our proposed algorithm can be efficiently implemented in terms of the cross approximation algorithm for matrices using either full or row pivoting.

We present some numerical examples and compare the performance of the algorithm to the in some sense best, but significantly slower, low-rank approximation using truncated singular value decomposition. We show that our approach leads to dramatic savings compared to full tensor product spline interpolation. We also make some remarks on the error analysis of the algorithm.

The presented algorithm has interesting applications in isogeometric analysis as a data compression scheme, as an efficient representation format for geometries, and in view of possible solution methods which operate on tensor approximations.

This talk is based on joint work with I. Georgieva (IMI-BAS, Sofia).

Friday, December 2, 11:00-11:45

Szegő-Widom asymptotics of Chebyshev polynomials on circular arcs

Benjamin Eichinger
JKU Linz, Institute of Analysis

We consider the Chebyshev polynomials T_n on a circular arc A_α, i.e., the monic polynomials of degree at most n that minimizes the sup-norm $\|T_n\|_{A_\alpha}$. Thiran and Detaille found an explicit formula for the asymptotics of $\|T_n\|_{A_\alpha}$. We give the Szegő-Widom asymptotics of the domain explicitly. That is, the limit of the properly normalized extremal functions T_n. Moreover, we solve a similar problem with respect to the upper envelope of a family of polynomials uniformly bounded on A_α. Our computations show that in the proper normalization the limit of the upper envelope is the diagonal of a reproducing kernel of a certain Hilbert space of analytic functions.

Friday, December 2, 11:45-12:30

Function approximation by aggregation in the context of supervised learning

Sergei Pereverzyev
RICAM, Inverse Problems and Mathematical Imaging

Supervised Learning consists in approximating an unknown function given a sample of its eventually corrupted values, which are interpreted as input-output responses of a system under consideration. There is a huge body of literature where the above mentioned approximation is performed by a regularization in a Reproducing Kernel Hilbert Space. This kernel based learning approach requires to define a suitable kernel and a regularization parameter. Moreover, the issue of dealing with big data samples has become a hot topic recently. In this talk we are going to demonstrate a possibility to address all these issues by an aggregation of different approximations constructed either for several regularization parameters and kernels, or for several shorter pieces of big data samples.

The talk is a dissemination of the results obtained in FWF-project I1669 and Horizon2020 project AMMODIT.

References:

[1] G, Kriukova, O. Panasiuk, S. Pereverzyev and P. Tkachenko. A linear functional strategy for regularized ranking. *Neural Networks*, 73, 26-35, 2016.
[2] P, Tkachenko, G. Kriukova, M. Aleksandrova, O. Chertov, E. Renard and S. Pereverzyev. Prediction of nocturnal hypoglycemia by an aggregation of previously known prediction approaches: proof of concept for clinical application. *Computer Methods and Programs in Biomedicine*, 134, 179-186, 2016.