Multigrid optimization for higher order accurate space-time DG methods for compressible flow

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Space-time discontinuous Galerkin (DG) methods combine the efficiency of dealing with deforming grids, in which the geometric conservation law is automatically satisfied, with all the benefits of standard DG methods, i.e., their ability to efficiently deal with unstructured grids, local mesh refinement (h-adaptation), adjustment of the polynomial order (p-refinement) and parallel computation. These benefits stem from the use of discontinuous basis functions in both space and time, resulting in a compact stencil of the discretization. Furthermore, (space-time) DG methods easily deal with shocks and other discontinuities in the solution.

Space-time DG discretizations of partial differential equations result in large systems of (non-) linear algebraic equations for the polynomial expansions in each element. Second order accurate space-time DG discretizations of the Euler and Navier-Stokes equations can be found in [1, 3]. Extending the space-time discretizations to higher order accuracy is straightforward, but current solvers are inefficient in solving these discretizations.

In this presentation we discuss efficient solvers for higher order accurate space-time DG methods. To solve the systems of (non)linear algebraic equations, we augment the space-time DG discretization with a pseudo-time derivative and efficiently march the system to steady-state in pseudo-time using optimized multigrid methods. As a smoother in the multigrid method we introduce new Singly Implicit Runge-Kutta (SIRK) methods [2]. These SIRK methods have been optimized for the multigrid method using 3-level Fourier analysis of the multigrid error transformation operator. We will compare the performance of SIRK with optimized explicit Runge-Kutta methods.

References

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