

# Implicit and $p$ -multigrid solution strategies for compressible RANS DG solvers

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Higher-order accurate discontinuous Galerkin methods are among the most promising numerical techniques for applications where accuracy is of primary importance, such as for example in large eddy simulation, direct numerical simulation, computational aeroacoustic and, more generally, in the simulation of flows characterized by significant vortex structures. However, a widespread use of this class of methods is somewhat hampered by their high computational cost and storage requirement.

In order to improve the computational efficiency of DG methods, implicit time discretization schemes,  $h$ - and  $p$ -multigrid solution strategies have been considered, see e.g. [1, 2, 3]. Implicit methods can compute steady state solutions in very few nonlinear iterations, but require considerable computational resources for the solution of the linear system arising at each time step, both in terms of CPU time and in the memory required to store the Jacobian matrix, which may be prohibitive for large scale problems and high-order accurate approximations. Multigrid methods are potentially more effective than implicit methods in this respect.

This presentation is focused on efficient implicit and  $p$ -multigrid solvers for the high-order accurate steady state DG solution of the Reynolds-averaged compressible Navier-Stokes equations coupled with a realizable  $k$ - $\omega$  turbulence model. The performance of the two solution strategies will be presented and compared by showing the results obtained in the numerical simulation of several 2D and 3D applications of aeronautical interest.

## References

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- [3] C.M. Klaij, M.H. van Raalte, H. van der Ven, J.J.W. van der Vegt,  $h$ -Multigrid for space-time discontinuous Galerkin discretizations of the compressible Navier-Stokes equations, *J. Comput. Phys.* **227** (2), pp. 1024-1045 (2007).