Subspace correction method for discontinuous Galerkin discretizations of linear elasticity equations

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In this talk we will present a preconditioning techniques for certain classes of discontinuous Galerkin (DG) methods, so-called Interior Penalty (IP) Finite Element (FE) methods, for linear elasticity problems in primal (displacement) formulation. We will recall some of their stability and approximation properties and comment on their suitability as a discretization tool for problems with nearly incompressible materials.

Next we propose a natural splitting of the DG space, which gives rise to uniform preconditioners. The presented approach was recently introduced by B. Ayuso and L. Zikatanov (2009) in the context of designing subspace correction methods for scalar elliptic equations and is extended here to linear elasticity, i.e., a class of vector field problems. Similar to the scalar case the solution of the linear algebraic system corresponding to the IP DG method is reduced to a solution of a problem arising from discretization by nonconforming Crouzeix-Raviart elements plus the solution of a well-conditioned problem on the complementary space.

Regarding the sub-problem on the nonconforming FE space and considering the case of Dirichlet boundary conditions on the entire boundary –the so-called pure displacement problem– it is known how to construct optimal order multilevel preconditioners that are robust with respect to the Poisson ratio ν, i.e., when ν approaches $1/2$ in the incompressible limit. However, for mixed boundary conditions or pure Neumann boundary conditions (the traction free case), it is much more difficult to devise a robust optimal order method. The presented subspace correction method is robust and reduces the efficient solution of the original problem on the DG space to the solution of a problem on a much smaller nonconforming FE space.