One level Dirichlet-Neumann method for DG discretization of elliptic problems with discontinuous coefficients

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In the talk a discontinuous Galerkin (DG) approximation of elliptic problems with discontinuous coefficients will be discussed. The problem is considered in polygonal region Ω which is a union of disjoint polygonal subregions Ω_i. The discontinuities of the coefficients occur across $\partial\Omega_i$. The problem is approximated by a conforming finite element method (FEM) on matching triangulation in each Ω_i and nonmatching one across $\partial\Omega_i$. This kind of triangulation and composite discretization are motivated first of all by the regularity of solution of the problem being discussed. The discrete problem is formulated using DG method with interior penalty terms on $\partial\Omega_i$.

In the talk one level Dirichlet-Neumann method for the resulting discrete problem will be designed and analyzed. It is proved that the method is almost optimal and its rate of convergence is independent of the parameters of triangulation, the number of substructures and the jumps of coefficients. An implementation of the method in each iterative step reduces to solve local independent Dirichlet problems and Neumann ones which are weakly coupled through the unknowns at the vertices of substructures. These couplings can be removed using additionally some coarse spaces. In the case of small number of substructures we can get the independent local problems by simple modification of the preconditioner. It gives only one additional log factor in the main estimate.

The method can be straightforwardly extended to the 3-D case.