Application of a multigrid-based solver in computational elastoplasticity and in functional a-posteriori estimates

Jan Valdman

SFB F013, Johannes-Kepler University of Linz, Altenberger Straße 69, 4040 Linz, Austria
e-mail: jan.valdman@sfb013.uni-linz.ac.at

ABSTRACT

Two fields, to which a multigrid-based solver is applied, are introduced. The first one is the computational plasticity, for which one can reformulate the problem of a minimization of a convex but not-smooth functional [2], where the unknowns are the displacement u and the plastic strain p. Our main interest is to develop a robust and fast solver working in both 2D and 3D cases. Due to the presence of the non-smooth norm term in the minimization functional, a regularization is applied. For the design of our algorithm it is usefull to study the minimization in displacement u and plastic strain p separately. Given local u one can compute local p exactly. Our solution algorithm is based on the reduction of the minimization functional to a quadratic functional. Then, the global Schur complement for u only is assembled for local contributions and the multigrid-preconditioned CG method is applied for solving. This is integrated in the package NETGEN/NGSOLVE [1] developed in Linz. The detailed implementation description for 3D case together with various numerical examples will be provided. Due to the design of NETGEN/NGSOLVE, the direct extension towards combined hp methods is feasible [2,3,4]. The second application involves an efficient computation of functional aposteriori estimates. This ongoing project [5] includes an implementation of Raviart-Thomas-elements and the block smoother known from works of Arnold, Falk and Winther.

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