A posteriori error estimates for nonconforming approximation of elliptic problems based on Helmholtz type decomposition of the error

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Abstract

In this talk a new type of a posteriori error estimates for nonconforming approximations of elliptic problems will be presented. The method is based on Helmholtz type decomposition of the error expressed in terms of fluxes. Such a decomposition of the nonconforming error has been suggested and studied in [1, 2] where it was shown that the error can be presented as the sum of two functions (a gradient term and a divergence-free term), which are the exact solutions of two auxiliary problems. A similar decomposition of the error is followed in this work. However, the procedure for obtaining computable two-sided bounds of the norms of these solutions is different, for which the method suggested in [4] for conforming approximations is used. A posteriori estimates obtained in this way differ from those which are based on a different type of decomposition of the error and the projection of the nonconforming approximation to the conforming space (see [3, 5]). Numerical experiments confirm that these new estimates provide very accurate error bounds and can be efficiently exploited in practical computations.

Key words: Discontinuous Galerkin FEM; a posteriori error estimates; Helmholtz decomposition.

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