Hybridizing Raviart-Thomas elements for the Helmholtz equation

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We start from the mixed formulation of the Helmholtz problem for the scalar field u and the flux field $\sigma = -\frac{1}{i\kappa}u$ with impedance boundary conditions

$$\begin{aligned} -i\kappa u &= \operatorname{div}\sigma \quad \text{in } \Omega \\ -i\kappa\sigma &= \nabla u \quad \text{in } \Omega \\ -\sigma_n - u &= g \quad \text{on } \Gamma \end{aligned}$$

We aim at finding a method for discretizing and subsequently solving the corresponding discrete system, which is suitable for large wave numbers κ. We start from a mixed variational formulation, where we use high order Raviart-Thomas finite elements for the fluxes, and piecewise polynomials for the scalar field. We then break the required normal-continuity of the fluxes, and enforce it back by a hybridization technique. There we introduce additional unknowns on element interfaces, which correspond to the traces of u and the normal flux σ_n respectively. Up to a transformation of variables, the method obtained is equivalent to an ultra-weak variational formulation (UWVF) based on the mixed finite element spaces. As for the original UWVF by Cessenat and Despres, we find that the operator $F : L^2(\partial T) \to L^2(\partial T)$, which maps incoming to outgoing impedance traces on the element, is an isometry.

We obtain a complex symmetric system, which we solve by a preconditioned cg iteration. We propose an additive-Schwarz block-preconditioner, for which we observe convergence of the cg method. Numerical tests show good behavior of the iterative solver when the wave number κ becomes large.