Tridiagonalizing Complex Symmetric Matrices

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We discuss a method for solving *complex symmetric* (*non-Hermitian*) eigenproblems $Ax = \lambda Bx$ arising in an application from optoelectronics, where reduced accuracy requirements provide an opportunity for increasing performance compared to standard approaches. In this specific setup, the objective is to exploit the structural symmetry. Consequently, we focus on the investigation of a non-Hermitian tridiagonalization process. In a first version, a variant of the Lanczos algorithm is used for solving the resulting complex symmetric tridiagonal problem. Experimental comparisons to the standard routine for non-Hermitian eigenproblems, `LAPACK/zgeev`, in terms of numerical accuracy and in terms of runtime performance are based on Fortran implementations. Both random data as well as data from the optoelectronics application are used.

Although the performance results reveal that more work is needed in terms of increasing the fraction of Level 3 Blas in our tridiagonalization routine, the numerical accuracy achieved with the non-Hermitian tridiagonalization process is very encouraging and indicates important research directions for this class of eigenproblems.

Our current work addresses two more central aspects: After tridiagonalization, the resulting tridiagonal complex symmetric eigenproblem has to be solved. We are currently comparing different solvers available for this task: the methods of Cullum and Willoughby (1986), Bar-On and Ryaboy (1997), and Hernández et al. (2007). Finally, parallelization strategies for the tridiagonalization process are investigated. While the main difficulty for parallelizing tridiagonal eigensolvers traditionally stems from (re-)orthogonalization of eigenvectors, in the tridiagonalization process the main challenge is to increase the computation to communication ratio.