

Optimal order AMLI preconditioning of the Navier-Stokes problem: Crouzeix-Raviart FE discretization of the velocity field

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The talk is devoted to the development and analysis of optimal computational complexity Algebraic MultiLevel Iteration (AMLI) algorithms in the case of composite time-stepping problems.

The Dirichlet initial-boundary value problem for the Navier-Stokes equation is considered. Two projection schemes are implemented for the efficient solution of the problem. The first one is based on first order Crouzeix-Raviart (C-R) Finite Element (FE) discretization of the velocity field and piece-wise constant (P_0) approximation of the pressure. Linear conforming FEs at the prediction step are alternatively used in the second scheme reducing the computational cost. Both schemes are locally conservative providing optimal order accuracy of the velocities.

The efficient implementation of the projection schemes is based on optimal order AMLI preconditioning of: (a) linear (conforming or non-conforming) Finite Element Method (FEM) Poisson problems at the prediction steps, and (b) (C-R, P_0) mixed FEM problems at the projection steps. Some recent results concerning iterative solution of the saddle-point problem (b) are discussed. The C-R mass matrix is diagonal which allows to eliminate locally the velocity unknowns. The reduced matrix for the pressure is referred to as weighted graph-Laplacian (WGL).

The presented complete analysis of WGL for the model 2-D problem is based on a novel parametric set of hierarchical splittings. The proposed approach allows for a local analysis of the constant in the strengthened Cauchy-Bunyakowski-Schwarz (CBS) inequality. Locally optimized uniform estimates of the CBS constant are derived to complete the construction of the related optimal order AMLI algorithm.