On the stability and error structure of BDF schemes applied to sectorial problems

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We consider BDF schemes applied to linear evolution equations $u' = Au + f$ and study their stability and damping behavior. The relevant $A(\alpha)$-stable BDF methods are uniformly strongly stable; in particular, explicit inclusion sets for the parasitic roots of the characteristic polynomial $\rho(\zeta) - \mu\sigma(\zeta)$ have recently been specified which are valid for any μ in the stability domain. This can be used to extend the work of former authors by deriving quantitative stability estimates for sectorial operators A. The two-step BDF scheme is considered in particular detail, and the structure of its global error is described by an asymptotic expansion with a precise, uniform estimate for its remainder. This expansion involves non-smooth perturbations which rapidly decay for $t > 0$ due to the damping properties of the coefficients of the discrete resolvent.