EXACTLY EQUILIBRATED FIELDS, CAN THEY BE EFFICIENTLY USED IN A POSTERIORY ERROR ESTIMATION?

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The answer to the question in the title: yes, they definitely can. We advocate the approach to the a posteriori error estimation, which can be called "classical" and for the theory elasticity problems stems from the Lagrange and Castigliano variational principles, allowing to approximate the exact solution in the energy sense from above and from below. In it, the energy of the error of an approximate solution, satisfying geometrical restrictions and obtained from the Lagrange principle, is estimated by the difference of the energies corresponding to the approximate solution and to any stress tensor, satisfying the equations of equilibrium. Notwithstanding the sometimes pronounced point of view that the construction of equilibrated stress fields requires considerable computational effort, we show that in many cases it can be done for a number of arithmetic operations, which is asymptotically optimal. Two ways of constructing the equilibrated fields, suitable for error estimation of FEM solutions, are analyzed: α) on the basis of superconvergence properties of FEM solutions and β) by approximate solution of the dual problem. For the case β), we show that the use of special self-equilibrated coordinate fields allows us to obtain numerical methods with the computational properties similar to the properties of FEM for the primal problem. We derive also new general reliable and computable a posteriory estimates (in particular, without estimation induced constants), in which equilibrated fields are replaced by arbitrary fields of fluxes/stresses. Numerical experiments show that our a posteriori error estimators provide very good coefficients of effectiveness, which in many cases can be convergent to the unity. At the same time they have linear complexity and are robust.

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