# SIMPLE A POSTERIORI ERROR ESTIMATORS FOR THE $h$-VERSION OF THE BOUNDARY ELEMENT METHOD IN 3D

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ABSTRACT

The $h$-$h/2$-strategy is one very basic and well-known technique for the a posteriori error estimation for Galerkin discretizations of energy minimization problems. Let $\phi$ denote the exact solution. One then considers

$$\eta_H := \|\phi_h - \phi_{h/2}\|$$

to estimate the error $\|\phi - \phi_h\|$, where $\phi_h$ is a Galerkin solution with respect to a mesh $\mathcal{T}_h$ and $\phi_{h/2}$ is a Galerkin solution for a mesh $\mathcal{T}_{h/2}$ obtained from uniform refinement of $\mathcal{T}_h$. We stress that $\eta_H$ is always efficient – even with known efficiency constant $C_{\text{eff}} = 1$, i.e.

$$\eta_H \leq \|\phi - \phi_h\|.$$

Reliability of $\eta_H$ follows immediately from the assumption $\|\phi - \phi_{h/2}\| \leq q_S \, \|\phi - \phi_h\|$ with some saturation constant $q_S \in (0, 1)$. Under this assumption, there holds

$$\|\phi - \phi_h\| \leq \frac{1}{\sqrt{1 - q_S^2}} \, \eta_H.$$

However, for boundary element methods, the energy norm $\|\cdot\|$ is non-local and thus the error estimator $\eta_H$ does not provide information for a local mesh-refinement. Recent localization techniques from [1] for $\widetilde{H}^{-\alpha}$-norms allow one to replace the energy norm in the case of isotropic mesh-sequences by mesh-size weighted $L^2$-norms. In particular, this very basic error estimation strategy can be used to steer an $h$-adaptive mesh-refinement. For instance, for Symm's integral equation, the $L^2$-norm based estimator

$$\mu_H := \|\rho^{1/2}(\phi_h - \phi_{h/2})\|_{L^2(\Gamma)}$$

is equivalent to $\eta_H$. We thus may use $\mu_H$ to steer the mesh and $\eta_H$ to estimate the error.

Further simplifications of the proposed error estimators $\eta_H$ and $\mu_H$ consist in replacing $\phi_h$ by some appropriate projection $\Pi_h \phi_{h/2}$, for instance, by use of the $L^2$-projection onto the discrete space corresponding to $\mathcal{T}_h$. Moreover, the error estimator $\eta_H$ is proven to be equivalent to the averaging estimator in [1, 3] and the two-level estimator from [4].

However, the analytical results only cover the isotropic case, which in 3D doesn't reveal the optimal order of convergence. Numerical examples using a heuristic to steer anisotropic refinements conclude the talk, suggesting that the analysis could be improved to work in the anisotropic case as well.

## References


[1] C. Carstensen, D. Praetorius: Averaging techniques for the effective numerical solution of Symm's integral equation of the first kind. *SIAM J.Sci.Comp.* 27 (2006), 1226–1260.

[2] S. Ferraz-Leite, D. Praetorius: Simple a posteriori error estimators for the $h$-version of the boundary element method. *ASC Report* 01/2007, Institute for Analysis and Scientific Computing, Vienna University of Technology, Wien, 2007

[3] C. Erath, S. Ferraz-Leite, S. Funken, D. Praetorius: Energy norm based a posteriori error estimation for boundary element methods in two dimensions. *ASC Report* 07/2007, Institute for Analysis and Scientific Computing, Vienna University of Technology, Wien, 2007

[4] P. Mund, E. P. Stephan, J. Weisse: Two-level methods for the single layer potential in $\mathbb{R}^3$. *Computing* 60 (1998), 243–266



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