# Inverse problems for transport equations: stability and uniqueness

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**Abstract**

We consider a single transport equation of the first order and a symmetric system of transport equations in a bounded smooth domain $\Omega \subset \mathbb{R}^d$. The single case is described as :

$$\partial_t u(x,t) + (H(x,t) \cdot \nabla u(x,t)) + p(x,t)u(x,t) = F(x,t), \quad x \in \Omega,\ 0 < t < T, \tag{1}$$

where $H \in C([0,T]; C^1(\overline{\Omega})^d)$.

We discuss two topics.

- global Carleman estimates in $\Omega \times (0,T)$
- Applications to inverse problems: establish global estimates in the domain

Our approach is global in $x$ and see [2] as for a local approach.

In order to establish the stability in $L^2$-spaces for inverse problems of determining for example a spatial factor in (1) by boundary data, the Carleman estimate is relevant. The method of characteristics is not convenient for $L^2$-estimates of solutions and moreover the transport equation often appears coupled with parabolic systems in fluid dynamics, so that the characteristics does not work for such studies.

**Part I.**

A Carleman estimate can be established directly if the rotation angle $H(x,t)$ over $\Omega \times (0,T)$ is

smaller than $\pi$ (I will formulate in my talk). In the case where the rotational angle is greater than or equal to $\pi$, we make a suitable partition of $\Omega$ and $0 < t < T$ into smaller subdomains and construct suitable piecewise weight functions to establish a Carleman estimate in $\Omega \times (0, T)$. The partition of $\Omega$ requires us to make favorable estimation on interfaces, and we must assume some outgoing-incoming conditions associated with $H(x, t)$ on some interfaces. We illustrate such interface conditions of $H$ and prove a Carleman estimate.

**Part II.**

By Huang, Imanuvilov and Yamamoto [4] which modifies a classical methodology (Bukhgeim and Klibanov [1]), we demonstrate that a Carleman estimate yields the stability for inverse source problems and inverse coefficient problems and also the observability inequality.

This is a joint work with Professor Piermarco Cannarsa (University of Rome Tor Vergata) and Professor Giuseppe Floridia (Università Mediterranea di Reggio Calabria).

# References

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