Domain Decomposition Algorithms for Mortar discretizations

Hyea Hyun Kim
Courant Institute (NYU)
Email: hhk2@cims.nyu.edu

July 4, 2006

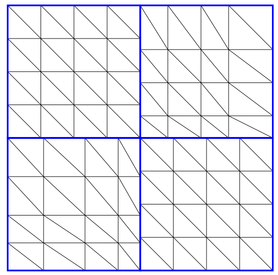
Outline

1. **Mortar discretizations**
   Nonmatching triangulations,
   Geometrically nonconforming partitions.
2. **Domain decomposition algorithms**
   Overlapping Schwarz algorithms,
   FETI-DP, BDDC algorithms.
3. **Additional applications to**
   Elasticity, Stokes,
   Inexact coarse problem.
4. **Numerical results**
5. **Conclusions**

Mortar element methods

(by Bernardi, Maday, and Patera (1994))

Coupling different approximations in N different subdomains



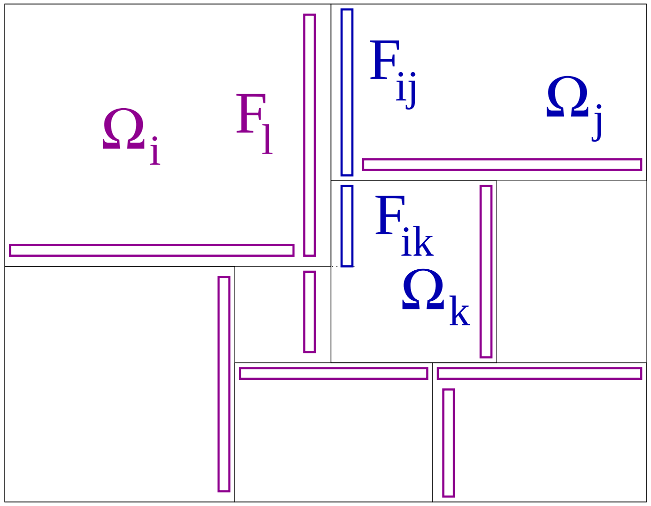
$X = \prod_{i=1}^{N} X_i$, finite element space

Glue $(v_1, \cdots, v_N) \in X$ across the interface F_{ij}

$$\int_{F_{ij}} (v_i - v_j)\psi \, ds = 0, \; \forall \psi \in M(F_{ij}) \qquad (1)$$

We call (1) **mortar matching condition**.

Geometrically Nonconforming partitions



$F_{ij} = \partial\Omega_i \cap \partial\Omega_j$: **interface**

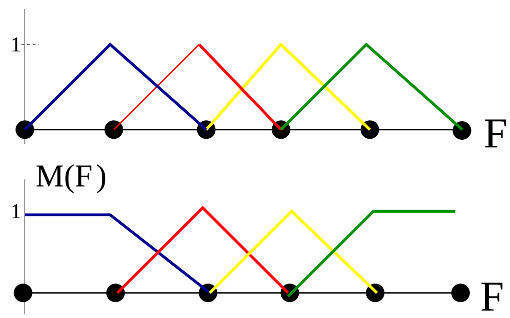
$\{F_l\}$ **: a collection of subdomain faces such that**

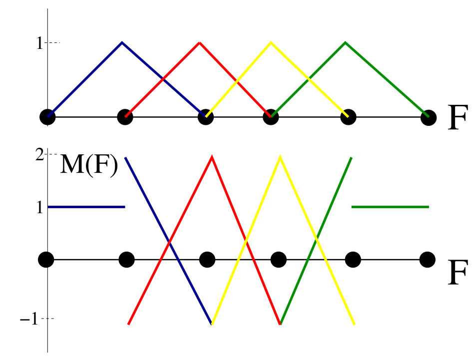
$$\bigcup_l F_l = \bigcup_{ij} F_{ij}, \quad F_l \cap F_k = \emptyset$$

F_l **: nonmortar side,** $\{F_{ij}, F_{ik}\}$ **: mortar sides.**

Lagrange multipliers spaces

- ✔ $M(F)$ on each nonmortar face F
  - ✘ the same dimension as that of finite element functions supported in F
  - ✘ contains constant functions
- ✔ Examples, standard (left) and **dual (right)**





(by Wohlmuth)
computationally more efficient,
easy to implement

Mortar Discretization

✔ **Mortar finite element space**

$$\widehat{X} \subset X = \prod_{i=1}^{N} X_i$$

satisfying mortar matching condition

✔ **Error estimate** for a mortar discretization:

For elliptic problems with P_1-finite elements in X_i,

$$\sum_{i=1}^{N} \|u - u^h\|_{1,\Omega_i}^2 \leq \sum_{i=1}^{N} h_i^2 |\log(h_i)| \, \|u\|_{2,\Omega_i}^2.$$

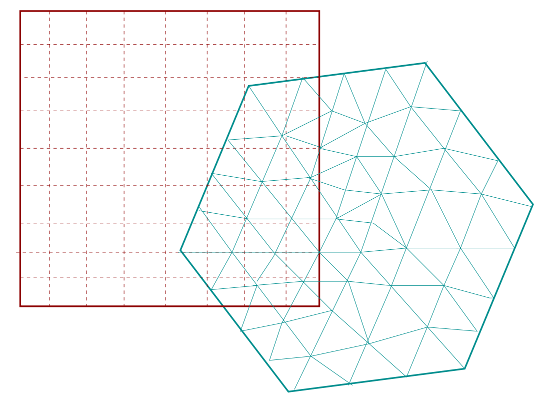
$|\log(h_i)|$: from geometrical nonconformity

Previous DD algorithms for mortar discretization

✔ **Substructuring methods**
(by Achdou, Maday, and Widlund)
Geometrically nonconforming partitions
Condition number bound $(1+\log\frac{H}{h})^2$

✔ **Overlapping Schwarz**
(by Achdou and Maday)

✘ Convergence analysis
✘ Additional coarse space
✘ Condition number bound $(1+(\frac{H}{\delta}))$



New Results

- ✔ **Extension to $3D$ geometrically non-conforming partitions**
- ✔ **Smaller coarse problems**
- ✔ **Simpler analysis**

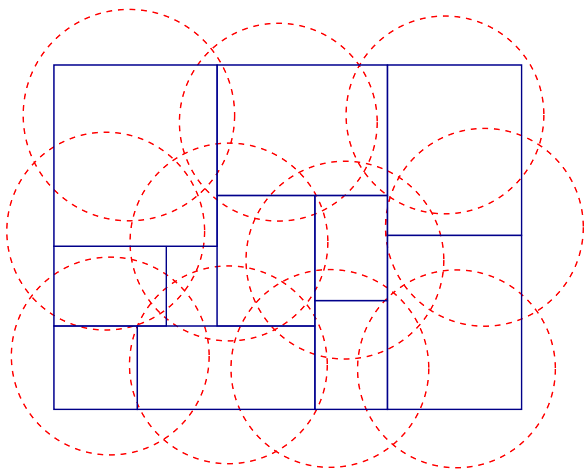
for

- ✔ **Overlapping Schwarz methods**
- ✔ **Dual–Primal FETI methods** (by Farhat et al)
- ✔ **BDDC methods** (by Dohrmann)

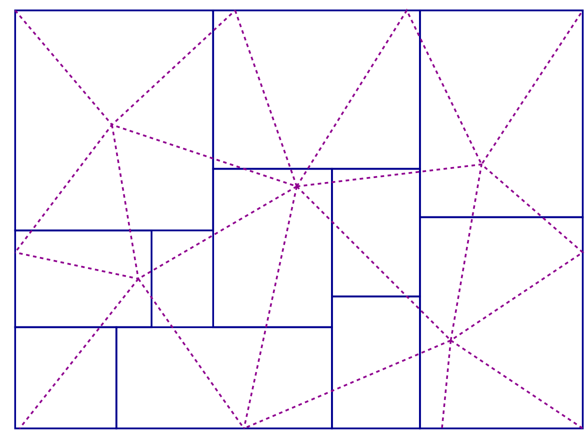
Overlapping Schwarz algorithm for mortar discretization

(Joint work with Olof B. Widlund)

- ✔ **Nonoverlapping subdomain partition** $\{\Omega_i\}_i$ equipped with mortar discretization
- ✔ **Overlapping subregion partition** $\{\widetilde{\Omega}_j\}_j$
- ✔ **Coarse triangulation** $\{T_k\}_k$



overlapping subregions
(local problems)



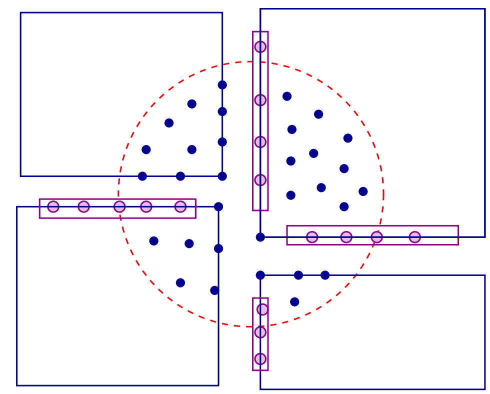
coarse triangulation
(coarse problem)

Subregion (local) problems

✔ Subregion finite element spaces

$$v \in \widetilde{X}_j \subset \widehat{X}$$

v has d.o.fs at the blue nodes.
v at the purple nodes determined by the mortar matching.

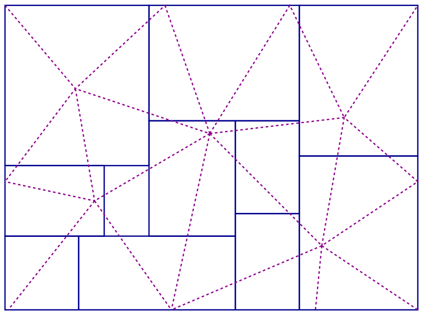


Subregion $\widetilde{\Omega}_j$ (circle)

✔ **Local problems** Find $T_i u \in \widetilde{X}_j$,

$$a(T_i u, v_i) = a(u, v_i), \quad \forall v_i \in \widetilde{X}_j.$$

Preconditioner (a coarse space contained in the mortar finite element space)



Coarse finite element space X^H

- ✔ $I^h(v) : X^H \to X$ defined by

$$I^h(v) = (I_1^h(v), \cdots, I_N^h(v)),$$

  $I_i^h(v)$: nodal interpolant to X_i.

- ✔ **Interpolant** $I^m : X^H \to \widehat{X}$ defined by modifying $I^h(v)$ on the nonmortar side to satisfy the mortar matching.

The coarse problem

- ✔ **Coarse function space**

$$V^H = I^m(X^H) \subset \widehat{X}.$$

- ✔ **Coarse problem** Find $T_0 u \in V^H$,

$$a(T_0 u, v^H) = a(u, v^H), \quad \forall v^H \in V^H.$$

Condition number bound

✔ Condition number estimate

$$\kappa(\sum_{j=0}^{N} T_j) \leq C \max_{j,k} \left\{ (1 + \frac{\widetilde{H}_j}{\delta_j})(1 + \log \frac{H_k}{h_k}) \right\}$$

Note: Additional log-factor from geometrically non-conforming partitions.

$\widetilde{H}_j$: subregion diameter
δ_j: overlapping width
H_k/h_k: the num. of elements across a subdomain Ω_k

BDDC and FETI–DP for the mortar discretization

- ✔ **Form two equivalent linear systems of mortar discretization**

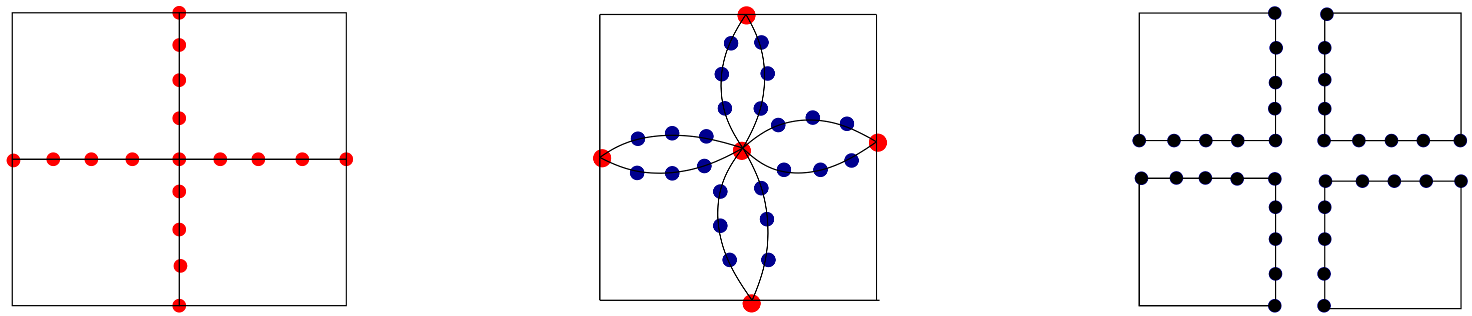
  1. **primal form**
  2. **dual form**

- ✔ **Develop BDDC and FETI–DP**
  BDDC (primal formulation)
  FETI–DP (dual formulation)

- ✔ **Providing preconditioners as efficient as the ones in the conforming case**

$$\kappa(B_{DDC}), \kappa(F_{DP}) \leq C(1 + log(H/h))^2$$

Finite Element Spaces

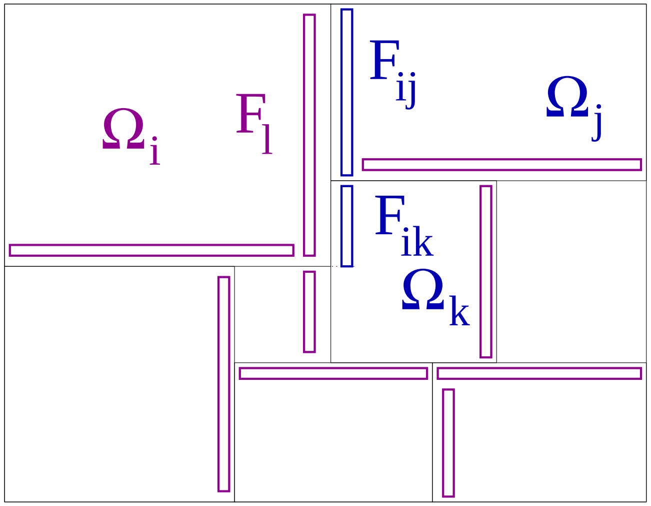


Space $\widehat{W}$ | Space $\widetilde{W}$ | Space $W = \prod_{i=1}^{N} W_i$

$\widehat{W} \subseteq W$ elements satisfying the mortar matching condition.

$\widetilde{W} \subseteq W$ elements satisfying **some of the mortar matching condition,** called **primal constraints**.

Mortar Finite Element Spaces for the geometrically nonconforming case

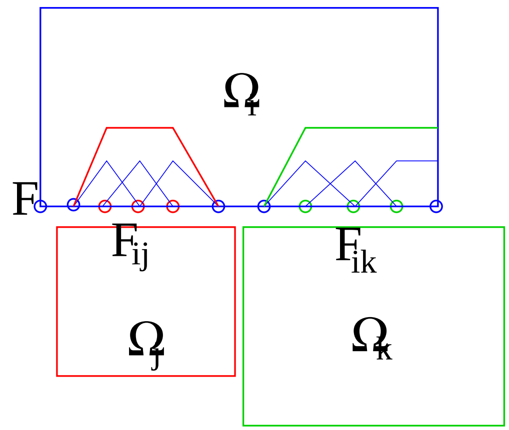


- ✔ Lagrange Multiplier space $M(F_l)$
- ✔ Mortar Matching condition

$$\int_{F_l} (w_i - \phi)\psi \, ds = 0, \quad \forall \psi \in M(F_l)$$

where $\phi = \begin{cases} w_j \; on \; F_{ij}, \\ w_k \; on \; F_{ik}. \end{cases}$

Primal Constraints across $F_{ij} \subset F_l$



$M(F_{ij})$ and $I_{M(F_{ij})}(1)$

- **Primal Constraints**

$$\int_{F_{ij}} (w_i - w_j) I_{M(F_{ij})}(1)\, ds = 0 \qquad (2)$$

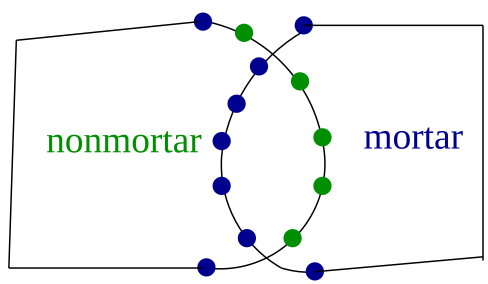
- $M(F_{ij}) \subset M(F)$ (F : nonmortar face)
  span of basis elements supported in $\overline{F}_{ij}$
- $I_{M(F_{ij})}(v)$:the nodal interpolant to $M(F_{ij})$

Change of unknowns (by Klawonn and Widlund)

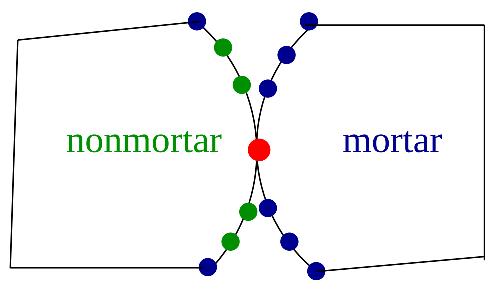
- ✔ To make the primal constraints explicit
- ✔ Much simpler presentation
- ✔ Computationally more stable

On each interface F_{ij}, T_{ij} is defined as

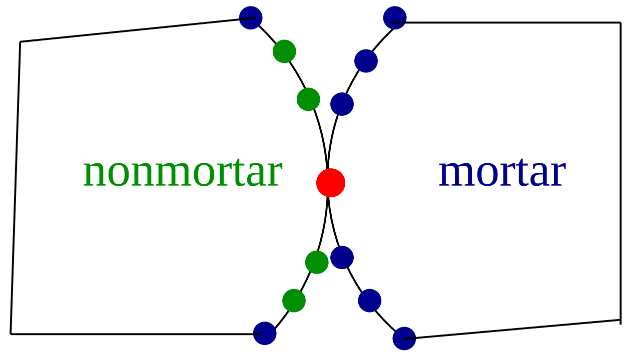
$$w = T_{ij} \begin{pmatrix} w_\Pi \\ w_\Delta \end{pmatrix}, \quad w_\Pi = \int_{F_{ij}} w I_{M(F_{ij})}(1)\, ds.$$



Before transform



After transform



After transform

Primal w_Π

Dual $w_\Delta \to (w_{\Delta,n}, w_{\Delta,m})$

n: nonmortar (green)

m: the others (blue)

Genuine unknowns : w_Π, $w_{\Delta,m}$

Representation of $\widehat{W}$

✔ **Mortar matching condition for $w \in \widetilde{W}$**

$$B_{\Delta,n} w_{\Delta,n} + B_{\Delta,m} w_{\Delta,m} + B_{\Pi} w_{\Pi} = 0$$

$$w_{\Delta,n} = -B_{\Delta,n}^{-1}(B_{\Delta,m} w_{\Delta,m} + B_{\Pi} w_{\Pi}) \tag{3}$$

$w_{\Delta,m}$ and w_{Π} $\qquad$ $w_{\Delta,n}$ (green nodes)

✔ **Mortar map $R_G^t : W_G \to \widehat{W} \subset \widetilde{W}$**

W_G: space of genuine unknowns $(w_{\Delta,m}, w_{\Pi})$.

Mortar Discretization

- ✔ K_i : local stiffness matrices
- ✔ S_i : Schur complement (eliminating interior unknowns)
- ✔ Subassembly at primal unknowns

$$S_i = \begin{pmatrix} S^{(i)}_{\Delta\Delta} & S^{(i)}_{\Delta\Pi} \\ S^{(i)}_{\Pi\Delta} & S^{(i)}_{\Pi\Pi} \end{pmatrix} \Longrightarrow \widetilde{S} = \begin{pmatrix} S_{\Delta\Delta} & S_{\Delta\Pi} \\ S_{\Pi\Delta} & S_{\Pi\Pi} \end{pmatrix}$$

- ✔ **Mortar discretization**

Note that $R_G^t : W_G \rightarrow \widehat{W} (\subset \widetilde{W})$

$$R_G \widetilde{S} R_G^t w_G = R_G \widetilde{g}.$$

$R_G^t w_G \in \widehat{W}$ is the desired solution.

Equivalent dual problem

- ✔ **Constraint minimization problem**

  Primal problem : $R_G\widetilde{S}R_G^t w_G = R_G\widetilde{g}$

  $\min_{w\in\widetilde{W}}\left\{\frac{1}{2}w^t\widetilde{S}w + w^t\widetilde{g}\right\}$ **with** $Bw = 0$**,**

  $B = \begin{pmatrix} B_{n,\Delta} & B_{m,\Delta} & B_{\Pi} \end{pmatrix}.$

- ✔ **Mixed form**

$$\begin{pmatrix} \widetilde{S} & B^t \\ B & 0 \end{pmatrix}\begin{pmatrix} w \\ \lambda \end{pmatrix} = \begin{pmatrix} \widetilde{g} \\ 0 \end{pmatrix}$$

- ✔ **Dual problem**

$$B\widetilde{S}^{-1}B^t\lambda = B\widetilde{S}^{-1}\widetilde{g}$$

BDDC and FETI-DP for Mortar discretization

- ✔ **BDDC algorithm solves**

$$R_G \widetilde{S} R_G^t w_G = R_G \widetilde{g}$$

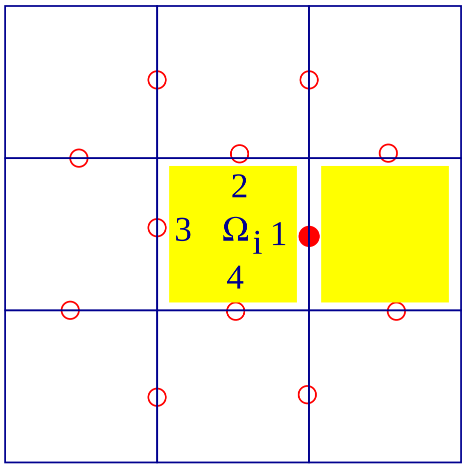
with a preconditioner (Coarse + Local problems)

- ✔ **FETI-DP algorithm solves**

$$B^t \widetilde{S}^{-1} B \lambda = B^t \widetilde{S}^{-1} \widetilde{g}$$

with a preconditioner (Local problems)

Coarse basis elements for the BDDC preconditioner (by Dohrmann)



For each primal unknown (red nodes), we find

1. $\phi_k(x_{\Pi,l}) = \delta_{kl}$
   average one on the face and zero on the other faces
2. minimizing energy $E(\phi_k) = \phi_k^t S_i \phi_k$

$$(\phi_1 \ \phi_2 \ \phi_3 \ \phi_4) = \begin{pmatrix} -S_{\Delta\Delta}^{(i)} S_{\Delta\Pi}^{(i)} \\ I_{\Pi}^{(i)} \end{pmatrix}, \quad I_{\Pi}^{(i)} = I_{4\times 4}.$$

Coarse problem

- ✔ **Coarse finite element space**
  Subassembly of local coarse basis at the primal unknowns

$$\Psi = \begin{pmatrix} -S_{\Delta\Delta}^{-1} S_{\Delta\Pi} \\ I_{\Pi\Pi} \end{pmatrix}$$

- ✔ **Coarse problem**

$$F_{\Pi\Pi} = \Psi^t \widetilde{S} \Psi = S_{\Pi\Pi} - S_{\Pi\Delta} S_{\Delta\Delta}^{-1} S_{\Delta\Pi}$$

Local problems

- ✔ **Local finite element space**

  $W_{\Delta}^{(i)}$: zero at the primal unknowns (zero averages on faces)

- ✔ **Local problem matrix** $S_{\Delta\Delta}^{(i)}$

$$S_i = \begin{pmatrix} S_{\Delta\Delta}^{(i)} & S_{\Delta\Pi}^{(i)} \\ S_{\Pi\Delta}^{(i)} & S_{\Pi\Pi}^{(i)} \end{pmatrix}$$

BDDC preconditioner

✔ **Weighted sum of local and coarse problems**

$$\widehat{M}^{-1} = D\left(\begin{pmatrix} S_{\Delta\Delta}^{-1} & 0 \\ 0 & 0 \end{pmatrix} + \Psi F_{\Pi\Pi}^{-1}\Psi^T\right) D,$$

$$\Psi = \begin{pmatrix} S_{\Delta\Delta}^{-1} S_{\Delta\Pi} \\ I_{\Pi\Pi} \end{pmatrix}$$: space of coarse basis

$$B_{DDC} = \left(R_G^t \widehat{M}^{-1} R_G\right) R_G^t \widetilde{S} R_G$$

We look for D such that

$$\kappa(B_{DDC}) \leq C(1 + log(H/h))^2.$$

FETI-DP preconditioner with Neumann-Dirichlet weight

(Joint work with Chang-Ock Lee)

Our goal is to provide the BDDC algorithm with weight D that performs as good as the FETI-DP algorithm.

✔ **FETI-DP preconditioner**

$$B_\Delta \Sigma_\Delta S_{\Delta\Delta} \Sigma_\Delta B_\Delta^t$$

$$B_\Delta = \begin{pmatrix} B_{\Delta,n} & B_{\Delta,m} \end{pmatrix} \qquad n: \text{nonmortar}$$

Note: $B_{\Delta,n}$ is invertible.

✔ **Neumann-Dirichlet weight**

$$\Sigma_\Delta = \begin{pmatrix} \Sigma_{\Delta,n} & 0 \\ 0 & \Sigma_{\Delta,m} \end{pmatrix} \qquad \begin{array}{l} \Sigma_{\Delta,n} = (B_{\Delta,n}^t B_{\Delta,n})^{-1} \\ \Sigma_{\Delta,m} = 0 \end{array}$$

FETI-DP preconditioner with Neumann-Dirichlet weight

✔ Resulting local problems

$$B_\Delta \Sigma_\Delta S_{\Delta\Delta} \Sigma_\Delta B_\Delta^t \lambda, \quad S_{\Delta\Delta}^{(i)} \Sigma_\Delta^{(i)} (B_\Delta^{(i)})^t \lambda$$

$$S_{\Delta\Delta}^{(i)} \begin{pmatrix} {B_{\Delta,n}^{(i)}}^{-1} \lambda \\ 0 \end{pmatrix},$$

$$S_{\Delta\Delta}^{(i)} = K_{\Delta\Delta}^{(i)} - K_{\Delta I}^{(i)} (K_{II}^{(i)})^{-1} K_{I\Delta}^{(i)}$$

FETI-DP preconditioner with Neumann-Dirichlet weight

- ✔ Condition number bound

$$\kappa(F_{DP}) \leq C(1 + log(H/h))^2$$

- ✔ The most efficient one for problems with jump coefficients

$$-\nabla \cdot (\rho(x)\nabla u) = f$$

$\rho(x) = \rho_i(> 0)$ for $x \in \Omega_i$

The convergence rate is independent of jumps though the preconditioner does not reflect any information of jump.

Connection between FETI-DP and BDDC

- ✔ **New insight into the BDDC preconditioner** (Li and Widlund)

Block Cholesky factorization of $\widetilde{S}$

$$\begin{pmatrix} I & 0 \\ S_{\Pi\Delta} S_{\Delta\Delta}^{-1} & I \end{pmatrix} \begin{pmatrix} S_{\Delta\Delta} & 0 \\ 0 & F_{\Pi\Pi} \end{pmatrix} \begin{pmatrix} I & S_{\Delta\Delta}^{-1} S_{\Delta\Pi} \\ 0 & I \end{pmatrix}$$

$$\widehat{M}^{-1} = D \widetilde{S}^{-1} D,$$

since

$$\widetilde{S}^{-1} = \begin{pmatrix} S_{\Delta\Delta}^{-1} & 0 \\ 0 & 0 \end{pmatrix} + \Psi F_{\Pi\Pi}^{-1} \Psi^T, \quad \Psi = \begin{pmatrix} S_{\Delta\Delta}^{-1} S_{\Delta\Pi} \\ I_{\Pi\Pi} \end{pmatrix}$$

Connection between FETI-DP and BDDC

✔ **F_{DP} and B_{DDC} operators**

$$F_{DP} = (B\Sigma\widetilde{S}\Sigma B^t)B\widetilde{S}^{-1}B^t,$$

$$B_{DDC} = (R_G D\widetilde{S}^{-1}DR_G^t)R_G\widetilde{S}R_G^t.$$

✔ **Jump and Average operators**

$$P_\Sigma = \Sigma B^t B$$

$$E_D = R_G^t R_G D$$

Theorem

If P_Σ and E_D satisfy

$$P_\Sigma + E_D = I$$
$$E_D^2 = E_D, \quad P_\Sigma^2 = P_\Sigma,$$
$$E_D P_\Sigma = P_\Sigma E_D = 0,$$

then the operators B_{DDC} and F_{DP} have the same spectra except the eigenvalue 1.

✔ By Li and Widlund for conforming discretization. The same result first proved by Mandel, Dohrmann, Tezaur in a different context.

✔ **We are able to extend the result to mortar discretization.** (jointly with Max Dryja and Olof Widlund)

Weight D for the BDDC algorithm

- ✔ **The weight D satisfies the assumptions in the Theorem.**

$$D = \begin{pmatrix} D_n & 0 & 0 \\ 0 & D_m & 0 \\ 0 & 0 & D_\Pi \end{pmatrix}, \quad \begin{matrix} D_n = 0 \\ D_m = I \\ D_\Pi = I \end{matrix}$$

- ✔ **The BDDC algorithm with the weight D has the same spectra as the FETI-DP algorithm.**

$$\kappa(B_{DDC}) \leq C(1 + log(H/h))^2$$

Analysis for geometrically non–conforming partitions

The following estimate is used for proving the condition number bound:

$$\|\pi_l(w_i - \phi)\|^2_{H^{1/2}_{00}(F_l)} \leq C\left(1 + \log\frac{H_i}{h_i}\right)^2 \left(|w_i|^2_{S_i} + \sum_j |w_j|^2_{S_j}\right),$$

▷ $\phi = w_j$ on $F_{ij} \subset F_l$, $\int_{F_{ij}}(w_i - w_j)I_{F_{ij}}(1) = 0$
▷ π_l is the mortar projection.
▷ $\phi \in H^{1/2-\epsilon}(F_l)$
▷ F_{ij} is not aligned with triangles in the nonmortar F_l.

In the analysis, we use

- **additional finite element space** $W(F_{ij})$
- **the** L^2**–projection onto** $W(F_{ij})$

Applications to more general PDEs

- ✔ Choice of primal constraints is important to scalability

$$\kappa(B_{DDC}), \kappa(F_{DP}) \leq C(1 + log(H/h))^2$$

1. $2D$ Stokes problem (edge average)

$$\int_{F_{ij}} (v_i - v_j)\psi \, ds = 0, \quad \psi = 1$$

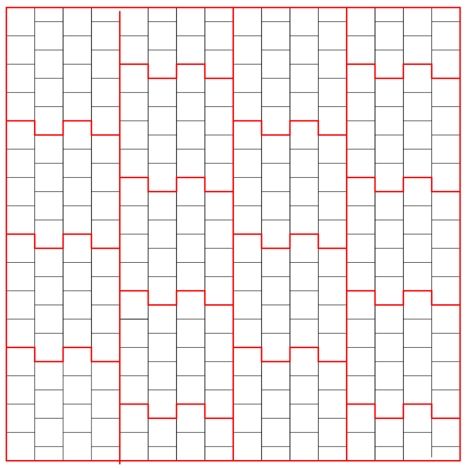
2. $3D$ elasticity problems
   Six primal constraints on each face F_{ij}
   $\{\mathbf{r}_m\}_{m=1}^6$: rigid body motions

$$\int_{F_{ij}} (\mathbf{v}_i - \mathbf{v}_j) \cdot \psi \, ds = 0,$$

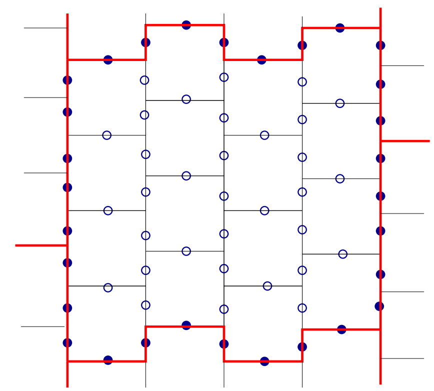
$\psi = I_{M(F_{ij})}(\mathbf{r}_m)$: nodal interpolant

Inexact Coarse problem

(Joint work with Xuemin Tu)
We are able to replace the coarse problem $F_{\Pi\Pi}^{-1}$ by $\widehat{M}_{\Pi\Pi}^{-1}$, a BDDC preconditioner for $F_{\Pi\Pi}$.



Subdomains and subregions



Unknowns at a subregion, $(F_{\Pi\Pi}^{(i)})$

Condition number analysis $(1 + \log(\widehat{H}/H))^2 (1 + \log(H/h))^2$

$\widehat{H}/H$, H/h: subregion, subdomain problem size

Numerical Results : comparison of the BDDC and FETI-DP

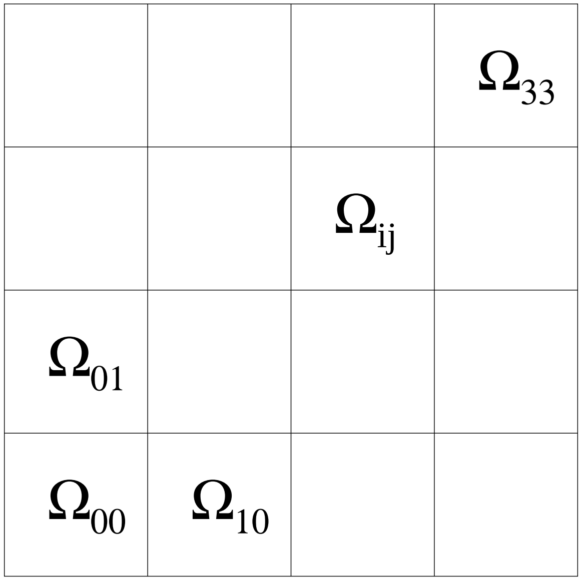
- ✔ Model problem

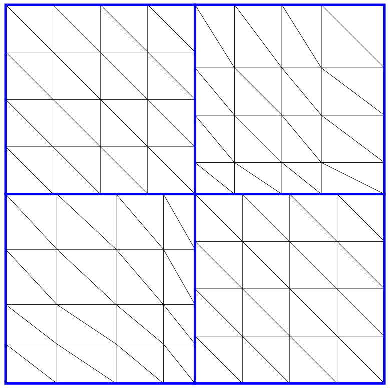
$$-\Delta u(x,y) = f(x,y) \quad (x,y) \in \Omega = [0,1]^2,$$
$$u(x,y) = 0 \quad (x,y) \in \partial\Omega.$$

  Exact solution: $u(x,y) = y(1-y)\sin \pi x$
- ✔ CGM: relative residual norm $\leq$ 1.0e-6
- ✔ N: the number of subdomains
- ✔ H/h: the number of elements on a subdomain edge

Comparison of the BDDC and FETI-DP

✔ Subdomain partition and triangulation





Comparison of the BDDC and FETI-DP

✔ Local problem size (when $N = 4 \times 4$)

H/h	F_{DP}		B_{DDC}	
	λ_{min}	λ_{max}	λ_{min}	λ_{max}
4	1.40	4.09	1.00	4.09
8	1.01	5.72	1.00	5.72
16	1.00	7.72	1.00	7.72
32	1.01	1.00e+1	1.00	1.00e+1
64	1.01	1.28e+1	1.00	1.28e+1

✔ The number of subdomains (when $H/h = 4$)

N	F_{DP}		B_{DDC}	
	λ_{min}	λ_{max}	λ_{min}	λ_{max}
4×4	1.40	4.09	1.00	4.09
8×8	1.37	4.41	1.00	4.41
16×16	1.32	4.49	1.00	4.49
32×32	1.30	4.57	1.00	4.62

Numerical Results : performance of the Neumann-Dirichlet preconditioner

✔ Discontinuous Coefficients

$$-\nabla \cdot (\rho(x) \nabla u(x)) = f(x)$$

where $\rho(x) = \rho_i (> 0)$ for $x \in \Omega_i$.

Preconditioners for F_{DP}

1.Neumann-Dirichlet

$$\widehat{M}_{ND}^{-1} = \begin{pmatrix} {B_{\Delta,n}}^{-1} \\ 0 \end{pmatrix}^t S_{\Delta\Delta} \begin{pmatrix} {B_{\Delta,n}}^{-1} \\ 0 \end{pmatrix}$$

2.Neumann-Neumann

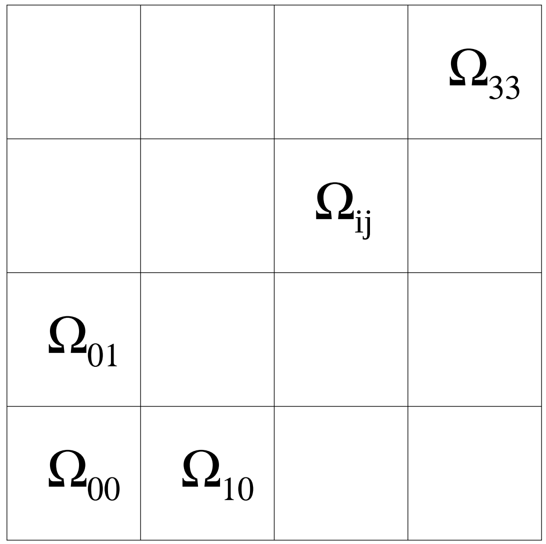
$$\widehat{M}_{NN}^{-1} = (B_\Delta B_\Delta^t)^{-1} B_\Delta S_{\Delta\Delta} B_\Delta^t (B_\Delta B_\Delta^t)^{-1}$$

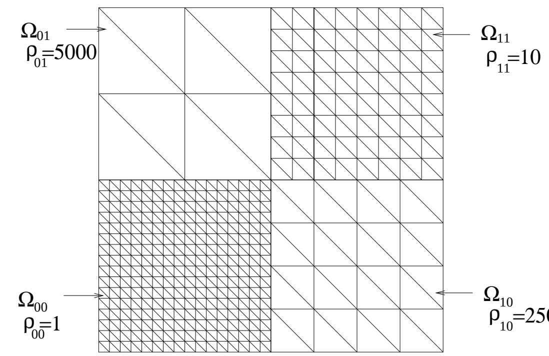
3.Neumann-Neumann with weight

$$\widehat{M}_{NNW}^{-1} = (B_\Delta D_\Delta^{-1} B_\Delta^t)^{-1} B_\Delta D_\Delta^{-1} S_{\Delta\Delta} D_\Delta^{-1} B_\Delta^t (B_\Delta D_\Delta^{-1} B_\Delta^t)^{-1}$$

Note D_Δ depends on ρ_i

Performance of the Neumann-Dirichlet preconditioner





$$\alpha(x,y) = \begin{cases} 1 & (i,j) = (even, even) \\ 250 & (i,j) = (odd, even) \\ 5000 & (i,j) = (even, odd) \\ 10 & (i,j) = (odd, odd) \end{cases}$$

Ratio of meshes: $\frac{h_{ij}}{h_{kl}} \simeq \left(\frac{\rho_{ij}}{\rho_{kl}}\right)^{1/4}$

Performance of the Neumann-Dirichlet preconditioner

N	$\max(H_{ij}/h_{ij})$	$\widehat{M}_{NN}^{-1}$	$\widehat{M}_{ND}^{-1}$	$\widehat{M}_{NNW}^{-1}$
2×2	16	17	3	3
	32	26	3	3
	64	39	4	3
	128	50	4	4
	256	60	4	4
4×4	16	75	4	3
	32	81	4	4
	64	111	4	4
	128	130	4	4
8×8	16	113	3	3
	32	136	4	4
	64	168	4	4

Performance of the BDDC preconditioner for $2D$ geometrically non-conforming case

Scalability w.r.t. the number of subdomains
$H/h = 6, 8, 10$

N	Cond	Iter
16×16	12.36	23
32×32	12.37	24
48×48	12.40	24
64×64	12.41	24
80×80	12.41	25

Scalability w.r.t. the local problem size
$C = \kappa/(1 + \log(H/h))^2$

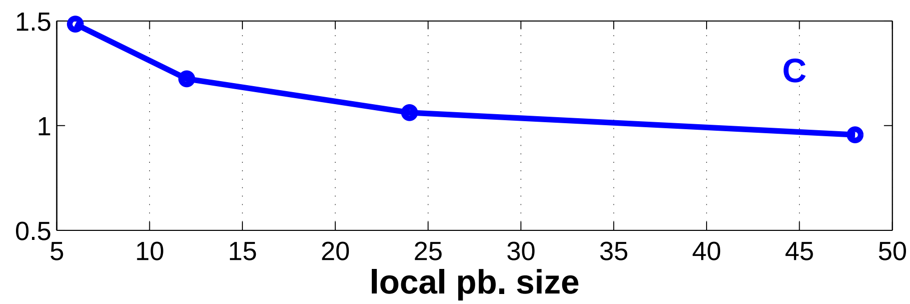


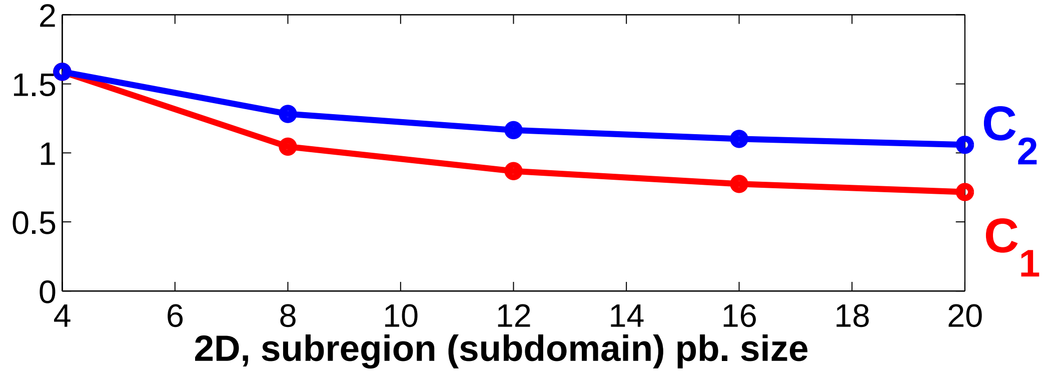
Table 1: $2D$ subregion ($\widehat{H}/H = 4,\ H/h = 4, 5$), $3D$ subregion ($\widehat{H}/H = 3,\ H/h = 3$)

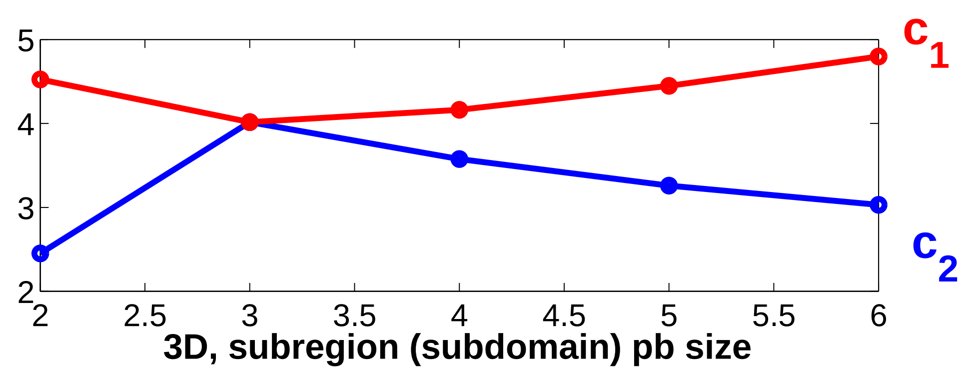
$2D$			$3D$		
Subregion	Cond	Iter	Subregion	Cond	Iter
4^2	9.04	18	2^3	10.65	18
8^2	9.44	20	3^3	17.69	25
12^2	9.45	20	4^3	18.78	28
16^2	9.46	20	5^3	20.07	32
20^2	9.43	19	6^3	21.22	33

Inexact coarse problem (geometrically conforming case, scalability w.r.t. subregion (subdomain) problem size)

$C_1 = \kappa/(1+\log(\frac{\widehat{H}}{H}))^2$, $\frac{\widehat{H}}{H}$: subregion pb. size

$C_2 = \kappa/(1+\log(\frac{H}{h}))^2$, $\frac{H}{h}$: subdomain pb. size





Scalability w.r.t the number of subregions

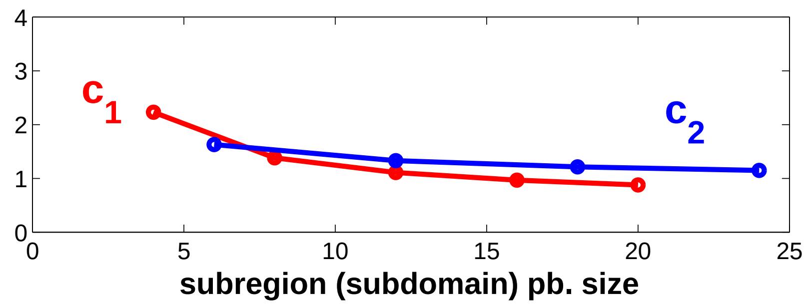
$\widehat{H}/H = 4$

$\widehat{H}/h = 6, 8, 10$

$\widehat{N}$	Cond	Iter
4^2	12.70	26
8^2	12.79	27
12^2	12.81	28
16^2	12.81	29
20^2	12.82	29

Scalability w.r.t the subregion (subdomain) pb. size

$C_1 = \kappa/(1 + \log \frac{\widehat{H}}{H})^2$, $C_2 = \kappa/(1 + \log \frac{H}{h})^2$



Conclusions

We extend the DD algorithms to mortar discretizations on $3D$-geometrically non-conforming partitions.

1. **Overlapping Schwarz algorithm**
2. **FETI-DP with the Neumann-Dirichlet preconditioner**
   - ▷ **Elliptic problems in $2D$, $3D$**
   - ▷ **Stokes problem in $2D$**
   - ▷ **$3D$ compressible elasticity**
   - ▷ **The most efficient for the problems with coefficient jumps**
3. **BDDC algorithm well connected to the FETI-DP**
4. **BDDC with an inexact coarse problem**

The end

Thank you!